

Duopoly

Marginal cost remains c and inverse demand remains

$$a - bQ$$

Instead of a monopoly there are two firms, firm 1 and firm 2, and firm i produces Q_i , that is

$$Q = Q_1 + Q_2$$

for illustrative purposes

$$a = 17, b = 1, c = 1$$

so that the competitive solution is 16 units of output and the monopoly solution is 8 units of output

restrict the firms to produce either 4 units of output, or 5 units of output

Profits

Profits of firm i

$$(a - bQ)Q_i - cQ_i = [16 - (Q_1 + Q_2)]Q_i$$

	Firm 2	
Firm 1	4	5
4	32,32	28,35
5	35,28	30,30

This is a *simultaneous move matrix game*

Prisoner's Dilemma Game

prisoner's dilemma: two prisoners in separate cells accused of jointly committing a crime

		Player 2	
		don't confess	confess
Player 1			
don't confess		32,32	28,35
confess		35,28	30,30

Tragedy of the Commons

two fishermen and a single lake

		Player 2	
		light fishing	intense fishing
Player 1			
light fishing		32,32	28,35
intense fishing		35,28	30,30

Public Goods

contribute towards building a bridge

		Player 2	
		contribute	do not
Player 1	contribute	32,32	28,35
	do not	35,28	30,30



Pareto Dominance

		Player 2	
		contribute	do not
Player 1			
contribute		32,32	28,35
do not		35,28	30,30

- $(32,32)$ *pareto dominates* $(30,30)$
- all players are at least as well off, and one is strictly better off
- question: does $(32,30)$ pareto dominate $(30,30)$
- does $(32,29)$?

Externalities and Free Riding

Compare (32,32) to (28,35). Here we say that player 2 is being a *free rider*; player 2 gets the benefit of the bridge, but does not have to pay for it

We also say that by switching from contributing to not contributing, player 2 generates a *negative externality* for player 1 by reducing player 1's payoff; player 2 does not bear the full cost of his action, part of the loss is suffered by player 1

Conversely we say that by switching from (28,35) to (32,32) player 2 generates a *positive externality* for player 1

Pareto Efficiency (Optimality)

an outcome that is NOT pareto dominated by any other outcome

	Player 2	
Player 1	contribute	do not
contribute	32,32	28,35
do not	35,28	30,30

- (32,32), (35,28), (28,35) are pareto efficient
- (30,30) is not
- why?

Dominant Strategies

		Player 2	
		contribute	do not
Player 1	contribute	32,32	28,35
	do not	35,28	30,30

the strategy “do not contribute” is *dominant* because it gives at least as high a payoff regardless of opponent’s play, and sometimes higher

[warning: do not confuse the dominance of a strategy with pareto dominance]

Weak versus Strict Dominance

weak dominance = dominance

strict dominance a higher payoff no matter what the opponent does
in statistics a strategy that is not weakly dominated is called
admissable

- (32, 28) are payoff to contribute
- (35, 30) strictly dominates
- How about (35,28)? (35,27)?

Tragedy of the Commons

When both players play their dominant strategies the outcome is (30,30) which is Pareto dominated by (32,32)

basic conflict between individual and social objectives

	Player 2	
Player 1	contribute	do not
contribute	32,32	28,35
do not	35,28	30,30

Do People Behave So Selfishly?

Pedro Dal Bo had participants play a Prisoner's Dilemma game ten times against different opponents

The final four times they played less than 5% contributed to the public good

We will see later that the “against different opponents” is pretty crucial here

Next: The Second Price Auction

Auctions

- A central question in economics: how are prices set.
- In monopoly the question is how much money can the monopolist extract from buyers?
- A common method of price setting is to sell items by means of an auction.



Two Types of Auctions

- *English auction* - announced bids, sold to highest bidder at the price bid (oral, first-price)
- *Sealed bid second price* – each buyer submits a single bid at the same time, sold to highest bidder at the second highest bid.
- Sealed bid second price = English auction – why?

What to Bid in a Sealed Bid Second Price Auction

your value of the item v

the highest bid of someone else p (you do not know this)

what do you get if you win? $v - p$

what do you get if you tie with k others? $(v - p)/k$

what do you get if you lose? 0

when would you like to win? $v - p > 0$

when would you like to lose? $v - p < 0$

how can you do this? Bid v - this weakly dominates everything else

the point is: the price you pay does not depend on your bid so there is no point in bidding low to get a “good price”

this is a bit unfamiliar to most people

BDM Mechanism

This ticket is worth \$2.00 to you.

You can sell it.

Name your offer price.

A price will be posted shortly

The posted price was drawn randomly between:

[\$ 0 and \$ 6]

If your offer price is **below** the posted price then you sell your ticket at the posted price.

If your offer price is **above** the posted price then you do not sell your ticket but you do collect the \$2.00 value of the ticket.

You can view the posted price after you have named your price.

Indicate the appropriate amount .

My offer price is **below** the posted price.

Pay me the posted price of \$ _____.

My offer price is **above** the posted price.

Pay me \$ 2.00.

Used to Elicit Values

Willingness to pay versus willingness to accept

- public goods surveys
- how much would you pay to avoid pollution?
- how much would we have to pay you to accept pollution?
- or value of a life, or climate change

Problem: people make mistakes (buy low, sell high – even when done for money)

they learn from their mistakes and permanently change their behavior

but they often must learn

annoying noise experiment

What is a Mechanism?

A *mechanism* is a game we design for players to play

examples:

- which is better for the seller? A first price sealed bid or second price sealed bid auction? (for a broad range of situations doesn't matter)
- the BDM mechanism to elicit value
- a matching mechanism, such as the clearing mechanism used to allocate students to universities in the UK, or to allocate kidney donors to kidney recipients

market design is the science of developing mechanisms to solve pricing and allocation problems

Concepts

- **duopoly**
- **game**, simultaneous move game, matrix game
- **Prisoner's Dilemma game**
- tragedy of the commons, **public good**
- **Pareto dominance, Pareto efficiency**
- **free riding**, negative **externality**, positive externality
- strategic dominance, weak dominance
- auction, English auction, sealed bid second price auction
- BDM mechanism
- **mechanism, market design**

Skill

given information about a game

find the payoff matrix, which strategies are strictly and/or weakly dominated, which outcomes are Pareto efficient

given a comparison of two outcomes in a game

determine whether there is free riding and a positive or negative externality