Behavior Anomalies in Game Theory

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Linchpins of Economic Theory

- Utility theory
- Equilibrium theory

Utility Theory

- Not a great deal of content
- Motion of planets can be described by utility theory
- Incorporates things you might not think it can: self-control for example
- Transitivity and the axioms of revealed preference – works exceptionally well
- Expected utility theory – works pretty well
- Selfish preferences – works better than you might think

Equilibrium Theory

- Controversial
- Yet it works well

Most documented anomalies are with the decision theory side, not the equilibrium side, as we shall see

The Basics – Normal Form Games

The normal form of a game describes the game by listing the strategies available to each player and the payoffs each player gets for any strategy profile by all players.

The normal form in the case of two-players is usually described by a matrix.

Example: the classical Prisoner's Dilemma Game

	cooperate	defect
cooperate	2,2	0,3
defect	3,0	1,1

a unique dominant strategy equilibrium (defect,defect)

more generally we consider Nash equilibrium in which each player is choosing the best strategy given correct anticipation of the strategies of other players

Mixed Strategy Equilibria

in some games uncertainty is intrinsic to Nash equilibrium

for example the Holmes-Moriarity game (matching pennies)

	canterbury	dover
canterbury	-1,1	1,-1
dover	1,-1	-1,1

The only Nash equilibrium is in mixed strategies: both players randomize 50-50

Note the use of expected utility theory in evaluating mixed strategies

Coordination Games

A game with both mixed and pure Nash equilibria: Battle of the Sexes

	opera	baseball (p)
opera (p)	x,1	0,0
baseball	0.0	1,x

Where x>1. Note that the mixed strategy equilibrium is

$$px = 1 - p$$

$$p = 1/(1 + x)$$

the more you like your alternative the less frequently you play it

Question: Do people mix correctly?

Answer: It depends.

- In simple problems, yes.
- In complicated problems, it is an acquired skill (soccer players, tennis players, Japanese baseball pitchers)
- Inexperience players tend to have too much negative correlation – don't understand length of runs in random data

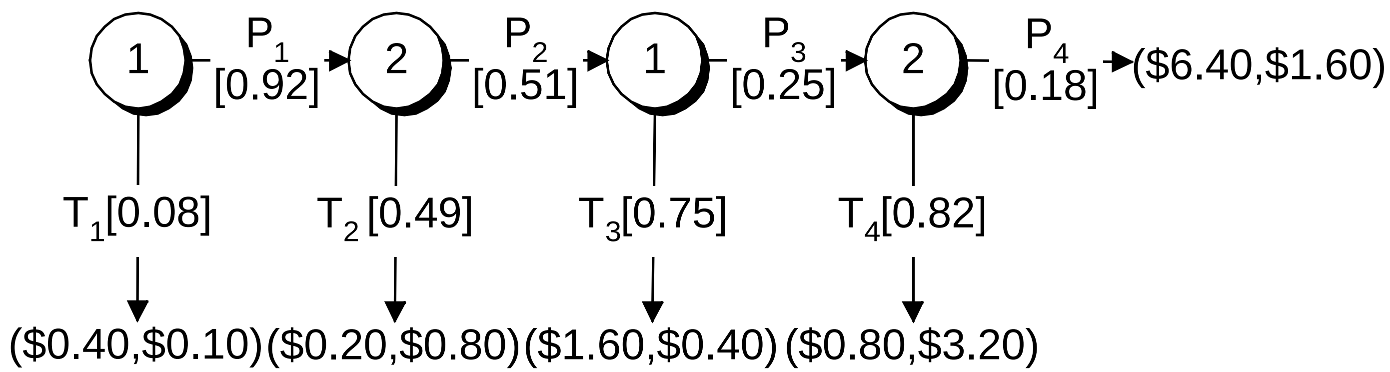
The Basics – Extensive Form Games

The extensive form describes the game by means of a game tree with explicit random moves by a player called Nature. Payoffs take place at the end of the tree. Simultaneous moves and other lack of information is represented by aggregating decision nodes into information sets. A strategy for a player consists of a contingent plan for playing at each information set. By listing the strategies and corresponding expected utility for all players we can construct a normal from from an extensive form.

Note the use of expected utility theory in computing the normal form.

Subgame Perfection

Conceptually if our concept of equilibrium is Nash equilibrium we may wish to consider subgame perfect equilibrium in which we insist that there be a Nash equilibrium in every subgame.



Unique subgame perfect equilibrium take-take-take-take

Contrast with actual play

Ultimatum Bargaining

player 1 proposes how to divide $10 in pennies

player 2 may accept or reject

Nash: any proposal by player 1 with all poorer proposals rejected and equal or better proposals accepted

Subgame Perfect: first player gets everything

Raw US Data for Ultimatum

x	Offers	Rejection Probability
$2.00	1	100%
$3.25	2	50%
$4.00	7	14%
$4.25	1	0%
$4.50	2	100%
$4.75	1	0%
$5.00	13	0%
	27	

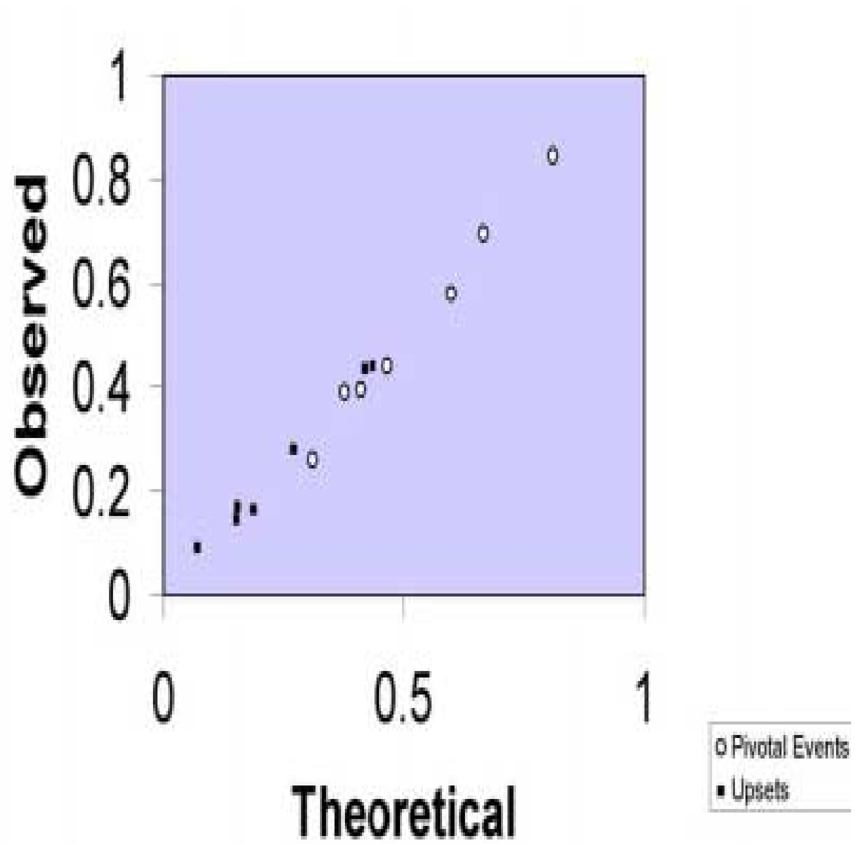
US $10.00 stake games, round 10

Observations:

1. anomalies involve decision theory not game theory: in centipede results are driven by altruism, in ultimatum results are driven by spite
2. losses are small: in centipede expected loss per player per game is $0.22, but taking account of lack of information, only $0.02 per player per game
3. losses are small: in ultimatum expected loss per player per game is $0.99, but taking account of lack of infomration, only $0.34 per game

Game where the theory performs well

“cost” of participation in a vote uniformly distributed on 0-55. Benefit of your party winning the election is 100.



Competitive Environments

The theory generally does well in competitive environments

Theory also predicts that preferences toward others should not make much difference here

Preference Anomalies

- interpersonal preferences
- attitudes towards risk
- non-linearity puzzles in interpersonal preferences

Risk Aversion in the Laboratory

In laboratory experiments we often observe what appears to be risk averse behavior over small amount of money (typical payment rates are less than $50/hour, and play rarely lasts two hours)

How can people be risk averse over gambles involving such an insignificant fraction of wealth?

Rabin [2000]: Risk aversion in the small leads to impossible results in the large

“Suppose we knew a risk-averse person turns down 50-50 lose $100/gain $105 bets for any lifetime wealth level less than $350,000, but knew nothing about the degree of her risk aversion for wealth levels above $350,000. Then we know that from an initial wealth level of $340,000 the person will turn down a 50-50 bet of losing $4,000 and gaining $635,670.”

Allais Paradox

Decision problem 1:

1 x 1Q for sure **[most common choice]**

(or)

.1 x 5Q, .89 x 1Q, .01 x 0Q

Decision problem 2:

.1 x 5Q, .9 x 0Q **[most common choice]**

(or)

.11 x 1Q, .89 x 0Q

So $u(1) > .1u(5) + .89u(1) + .01u(0)$ or $u(5) < 1.1u(1) - .1u(0)$

And $.1u(5) + .9u(0) > .11u(1) + .89u(0)$ or $u(5) > 1.1u(1) - .1u(0)$

Token Donation Paradox

Number of tokens donated to the “common” in a public good contribution game

γ	n	$m_i > 0$	$m_i > 1/3$	$\overline{m}$
0.3	4	0.00	0.00	0.00
0.3	10	0.23	0.10	0.07
0.75	4	0.58	0.33	0.29
0.75	10	0.55	0.30	0.24