Self-Confirming Equilibrium and the Lucas Critique

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“The fact that nominal price and wages tend to rise more rapidly at the peak of the business cycle than they do in the trough has been recognized from the time when the cycle was first perceived as a distinct phenomenon. The inference that permanent inflation will therefore induce a permanent economic high is no doubt equally ancient, yet it is only recently that this notion has undergone the mysterious transformation from obvious fallacy to the cornerstone of economic policy.”

Lucas [1976].

Off Path Superstitions

- Equilibrium is the result of non-equilibrium learning
- Implies players better informed about the equilibrium path than about off the equilibrium path
- Incorrect beliefs about off-path play may persist
- We call a belief that is objectively false a “superstition”
- “econometric policy evaluation:” behavioral parameter estimated under current government policy may not be invariant to changes in that policy
- “self-confirming:” good policies may fail to be adopted because of superstitions

Peasant Dictator

peasant moves first: plant corn or eat the corn seed

eats the corn: dictator get 0, peasant gets 1

plants the corn: dictator decides between high and low tax

high tax: dictator gets 4, peasant gets 0

low tax: dictator gets 1, peasant gets 3

obvious time consistency problem (Kydland and Prescott [1977]): would like to commit to low tax otherwise peasant eats the corn

"Lucas Critique" Model

variation on peasant-dictator game: dictator can commit before the peasant moves

three policies

always set low tax

always set high tax

set low tax unless there is a "war" in which case set the high tax

probability of a "war" is 50-50

objectively ascertainable whether or not there is a "war" (can determine if dictator follows commitment)

a "war" occurs only after the decision to grow is made

[historically dictators have committed through institutional arrangements such as requiring a vote of parliament to raise taxes]

Analysis of Lucas Critique Game

commit to low tax: peasant grows, dictator gets 1, peasant gets 3

commitment to high tax: peasant eats, dictator gets 0 peasant gets 1

commitment to tax only in case of “war”: peasant grows: dictator gets 2.5, peasant gets 1.5

optimum for dictator: tax only in case of “war”

suppose this is played for many generations and an enterprising econometrician comes along, call him AWP

he regresses government income on the tax rate

because taxes are high and low 50% of the time there is variation in the RHS

OLS: low taxes -> government income of 1

high taxes -> government income of 4

AWP recommends to the dictator that to maximize government revenue the best policy is to set a high tax all the time

Once the policy is announced: the peasant stops growing corn and starts eating it and government revenue falls to zero

the Lucas Critique: a structural relation (between taxes and revenues) estimated under one policy regime (high tax only when “war”) leads to a recommendation of a regime change (high tax always) that in turn results in a change in the structural relation

Lucas Critique Critique

- problem mitigated since policy makers will eventually discover new policy is a mistake
- problem a manifestation of deeper issue

in an equilibrium we see only the equilibrium path

knowledge about off the equilibrium path is either conjecture (such as the econometrician who conjectures the structural relationship will not change) or based on limited evidence from previous deviations from the equilibrium path

no cheap shortcut for genuine understanding of causality – if we knew the model we would understand the peasant would stop growing as soon as taxes were always set high

next: the case opposite the one considered by Lucas

The FDI Games

two countries: East and West

two economic policies: give monopolies to foreign investors, or force foreign competitors to compete

[you may wish to think of the monopolies as patent/copyright protection – which originated in the late Middle Ages as a way to induce skilled artisans to relocate]

single multinational investor must place two units of FDI in each of the two countries, may invest one unit in each, everything in one, or not invest

normalize payoffs: no investment in a country generates zero for country and investor

a unit of investment under monopoly: investor 2, country 1

a unit of investment under competition: investor 1, country 3

to generate variation in policy:

10% probability that one of the governments is “socialist” and refuses to grant monopolies

one sub-game perfect equilibrium: all non-socialist governments grant a monopoly w/ investor splitting investment, or if a country does not grant a monopoly invest entirely in the other country

add to the game an international agency the “WIPO”

WIPO cares only about welfare: sume of country plus investor payoff

Suppose in the equilibrium above

WIPO econometricians estimate relationship between welfare and monopoly using data on each individual country

Not offering monopoly: welfare is 0

Offering a monopoly results in welfare 3 90% of the time (other country not socialist) and welfare 6 10% of the time: welfare is 3.3

So offering monopoly is recommended

Of course the policy of never offering a monopoly would in fact result in a welfare of 4 in each country

Note: assumed that countries not simultaneously socialist, or else we’d get the relevant data

Self-Confirming Equilibrium

Strategies and mixed strategies s_i, σ_i, beliefs μ_i over other players strategies, utility $u_i(s_i, \mu_i)$

$\bar{X}(\hat{\sigma})$ information sets of the game tree hit with positive probability when $\hat{\sigma}$ is played

Definition 3.2: $\hat{\sigma}$ is a *unitary self-confirming equilibrium* if for each player i there are beliefs μ_i and for each s_i with $\hat{\sigma}_i(s_i) > 0$ such that

(a) s_i is a best response to μ_i and

(b) μ_i is correct at every $x \in \bar{X}(\hat{\sigma})$

By way of contrast, *Nash equilibrium* strengthens (b) to hold for all information sets, not just those on the equilibrium path

Alesina-Angeletos

government moves: high redistribution (H) or low redistribution (L)

representative individual: invest (1) or not invest (0)

nature: good luck (G) or bad luck (B) with equal probability

normalize the base payoff of investor to 0

- premium of 2 for good luck
- benefit of investment 2
- cost of investment 1

under high redistribution if you get 1 minus any cost of investment

government gets same utility as the investor plus “fairness bonus” of ½ for the high redistribution policy.

subgame perfection

low redistribution -> invest, both get 2

high redistribution -> no invest, investor 1, government 1.5

so…low redistribution

self-confirming

high redistribution -> no invest, investor 1, government 1.5

government believes that low redistribution will also result in no investment resulting in investor 1, government 1

Sargent-Williams-Zha Inflation Model

"opposite" of the "Lucas critique"

policy maker chooses high or low inflation

representative consumer choose high or low unemployment

policy maker gets: 2 for low unemployment 0 for high unemployment, 1 for low inflation, 0 for high inflation

representative consumer will choose low unemployment no matter what

subgame-perfect equilibrium: policy maker chooses low inflation and gets 3

self-confirming equilibrium: high inflation gets 1, believes low inflation leads to high unemployment

The Effectiveness of Learning

two armed bandit

certain arm returns 0

μ^* actual return on uncertain arm

μ prior return on uncertain arm; α number of "prior successes" β number of "prior failures"

δ discount factor

λ expected present value of optimal policy relative a known second arm

yellow-shaded rows: Lucas case

uncertain arm has positive prior mean, but true mean negative

blue-shaded rows: self-confirming case

uncertain arm has negative prior mean, but true mean positive

integrated by monte-carlo with 10,000 trials

Actual	Prior				Effectiveness
μ^*	μ	α	β	δ	λ
-0.2	0.2	3	2	0.5	.102
-0.2	0.2	3	2	0.9	.392
-0.5	0.5	3	1	0.5	.061
-0.5	0.5	3	1	0.9	.475
-0.9	0.2	3	2	0.9	.706
0.2	-0.2	2	3	0.5	.000
0.2	-0.2	2	3	0.9	.506
0.5	-0.5	1	3	0.5	.000
0.5	-0.5	1	3	0.9	.000
0.9	-0.2	2	3	0.9	.965