Self Control, Risk Aversion, and the Allais Paradox

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Behavioral Economics

- The paradox of the inner child in all of us
- More behavioral models than you can shake a stick at
- Models should do more than simply organize the data from a given experiment.
- It is much better to have a small number of models that explain a large number of facts than the reverse.
- Ideally, a model should not only predict the data to which it was fit, but also make correct predictions about outcomes in other settings, including experiments that have not yet been run

The Dual Self Model

- A model designed to explain hyperbolic discounting
- The self-control framework of Gul and Pesendorfer [2001], Fudenberg and Levine [2006]
- Introduces a tradeoff between commitment and self-control
- Earlier work: implication for risk aversion over laboratory small stakes
- Earlier work: some evidence self-control costs are convex not linear
- This leads to violation of the independence axiom
- Here: examines quantitatively if it explains the Allais paradox

Shiv and Fedorikhin [1999]

memorize either two- or a seven-digit number

walk to table with choice of two desserts: chocolate cake or fruit salad

pick a ticket for one dessert

report number and dessert choice in a different room

seven-digit number: cake 63% of time

two-digit number: cake 41% of time

(statistically as well as economically significant)

our interpretation: cognitive resources used for self-control are substitutes for cognitive resources used for memorizing numbers plus increasing marginal cost of cognitive resource usage

An Implication

replace desserts with lotteries giving a probability of a dessert reduces temptation, so with convex costs fewer subjects should give in to temptation of chocolate cake

as far as we know this hasn't yet been done…

This behavior would violate independence axiom

We will see that Allais paradox rests on a similar violation of the independence axiom.

Cost of Self-Control

The long-run self maximizes the expected discounted present value of the utility of the short-run selves subject to a cost of self-control g

$$U_{RF} = \sum_{t=1}^{\infty} \delta^{t-1} [u_t - g(d_t + \overline{u}_t - u_t)]$$

This cost depends on the *temptation utility* $\overline{u}_t$ for the short-run self.

The actual realized utility that the long-run self allows the short-run self is u_t, and there may be cognitive load due to other activities, d_t.

we argue *g* is typically convex

In our calibrations of the model, we will take the cost function to be quadratic: $g(v_t) = \gamma v_t + (1/2)\Gamma {v_t}^2$.

Self-Control with a Cash Constraint

periods $t = 1, 2, \ldots$divided into two sub-periods

bank subperiod and *nightclub* subperiod

state $w \in \Re_+$ wealth at beginning of bank sub-period

“bank” subperiod, no consumption, wealth w_t divided between savings s_t (remains in bank) and cash x_t carried to nightclub

(*the model we calibrate allows for spending on durables…)*

consumption not possible in bank, so short-run self indifferent between all possible choices, and long-run self incurs no cost of self control

in nightclub consumption $0 \le c_t \le x_t$ determined, with $x_t - c_t$ returned to bank at end of period

$w_{t+1} = R(s_t + x_t - c_t)$ no borrowing possible, and no source of income other than return on investment.

Mental Accounting

"Pocket cash" rations consumption and so reduce the temptation to the sort-run self.

In Fudenberg and Levine [2006] the notion of a bank and pocket cash were taken literally.

In practice there are many strategies that individuals use to reduce the temptation for impulsive expenditures.

The view we take here is that "pocket cash" is determined by mental accounting of the type discussed by Thaler [1980], and not necessarily by physically isolating money in a bank- it is the amount agent feel "entitled" to spend.

This means "pocket cash" is not directly observable.

In the calibrations we will calculate it from consumption and savings data.

Choice of Venue

Basic model can explain small-stakes risk aversion but to explain its extent need implausible parameters.

Extend the model to create an additional wedge between SR and LR marginal utility of consumption.

Choice of nightclubs indexed by quality of nightclub $c^* \in (0, \infty)$

"target" level of consumption expenditure

low value of $c\,*$ cheap beer bar

high value of $c\,*$ expensive wine bar

base preference of short-run self $u(c, c^*)$

$u(c, c) = \log c$, ($\log(0) = -\infty$)

$u(c, c^*) \leq u(c, c)$: best to choose nightclub of same index as intended consumption.

convenient functional form

$$u(c,c^*) = \log c^* - \frac{(c/c^*)^{1-\rho} - 1}{\rho - 1}. \; (\rho > 1)$$

reduced form preferences for long-run self are (w/o durable)

$$U_{RF} = \sum_{t=1}^{\infty} \delta^{t-1} \left[u(c_t, c_t^*) - g(u(x_t, c_t^*) - u(c_t, c_t^*)) \right]. \qquad (2.2)$$

no cost of self-control in bank so choose $c_t^* = c_t = x_t = (1-\delta)w_t$

same as solution without self-control

utility as function of wealth: $U_1(w_1) = \dfrac{\log(w_1)}{1-\delta} + K$

Uncertainty and Unforeseen Choices

unexpectedly the short-run self at the nightclub is offered a choice between an amount z_1 today and an amount θz_1 tomorrow, where $\boldsymbol{\theta} > R$

high cost of self-control: SR self insists on z_1 today

low cost of self-control: LR self forces θz_1 tomorrow

commitment for a future date: LR chooses θz_1 at the later date

replace certain rewards with probability p of rewards: reduces temptation and so cost of self-control

linear cost of self-control, irrelevant

convex cost of self-control, can have reversal, take the z_1 for certain reward, θz_1 for the risky reward

Data from Keren and Roelsofsma [1995] shows that this is exactly what happens

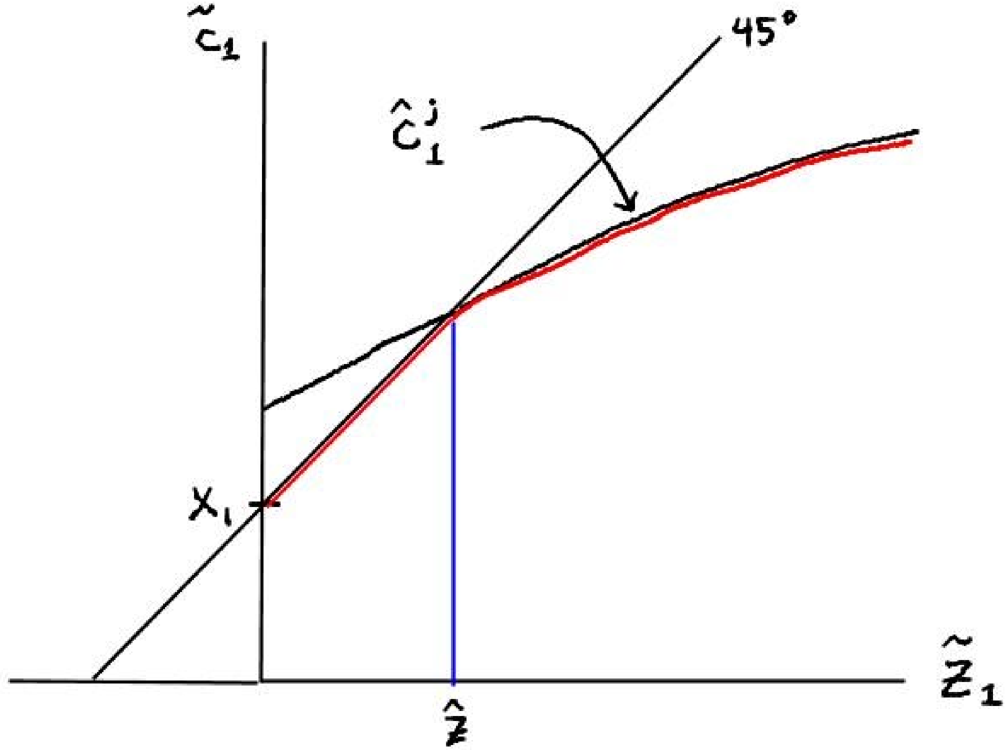
		Probability of reward	
		1.0	0.5
A	$175 now	0.82	0.39
	$192 4 weeks	0.18	0.61
B	$172 26 weeks	0.37	0.33
	$192 30 weeks	0.63	0.67

This dependence of the choices on the probability of reward is not consistent with quasi-hyperbolic preferences.

Risky Drinking: Nightclubs and Lotteries

Suppose at door to nightclub you are unexpectedly offered a choice between two lotteries, A and B with returns $\tilde{z}_1^A, \tilde{z}_1^B$ (losses not to exceed pocket cash)

Assume that no further lotteries at nightclubs are expected in the future



$\tilde{c}_1$
45°
$\hat{c}_1^j$
X_1
$\tilde{Z}_1$
$\hat{z}$

Calibration

Department of Commerce Bureau of Economic Analysis, real per capital disposable personal income in December 2005 was $27,640. will use three levels of income $14,000, $28,000, and $56,000.

do not use currently exceptionally low savings rates, but higher historical rate of 8% (see FSRB [2002])

gives us consumption from income; then wealth is consumption divided by subjective interest rate.

Some expenditures not subject to temptation: housing, durables, medical.

adjust basic model of utility by assuming it is separable (and logarithmic) between "durable" consumption c^D that not subject to temptation, with weight on "tempting" or "nightclub" consumption equal to "temptation factor" τ

National Income and Product Accounts Q4 2005

personal consumption expenditure $8,927.80.

$1,019.60 durables, $1,326.60 housing, and $1,534.00 medical care

gives temptation factor $\tau = 0.57$.

subjective interest rate real market rate, less growth rate of per capita consumption

Shiller [1989]

average growth rate of per capita consumption has been 1.8%

average real rate of returns on bonds 1.9%

real rate of return on equity 7.5%

use three values: 1%, 3%, and 5%

prefer 1% as that is what Gabaix and Laibson use in a compatible model of lock-in that is consistent with the equity premium puzzle time horizon of short-run self

most plausible period based on evidence from the psychology literature seems to be about a day

similar results with horizons up to a week.

Percent interest *r*		y_1 =14K		y_1 =28K		y_1 =56K	
annual	daily	w_1	x_1	w_1	x_1	w_1	x_1
1	.003	1.3M	20	2.6M	40	5.2M	80
3	.008	.43M		.86M		1.7M	
5	.014	.30M		.61M		1.2M	

So use 3 values of pocket cash: \$20,\$40, \$80.

Measuring Self-control Costs

in our model consumption cutoff between high MPC of 1.0 and low MPC of order $\tau(1-\delta)$ given by

$$\hat{c} = (x_1)^{\frac{\rho-1}{\rho}} \left[\frac{\tau(1-\delta)}{\delta}(1+\gamma)[w_2] \right]^{1/\rho}$$
$$\approx x_1 (1+\gamma)^{1/\rho}$$

Define $\mu_1 = (1+\gamma_1)^{1/\rho}$

This is the cutoff relative to income, will report this rather than marginal cost of self-control

Theory of the Consumption Function

how does marginal propensity to consume "tempting" goods change with unanticipated income?

Older literature on permanent income hypothesis

study using 1972-3 CES data Abdel-Ghany et al [1983]

examine marginal propensity to consume semi- and non-durables out of windfalls

windfalls = "inheritances and occasional large gifts of money from persons outside family...and net receipts from settlement of fire and accident policies"

windfalls less than 10% of total income MPC is 0.94

windfalls more than 10% of total income MPC of 0.02

reason for 10% unclear so take it as a general indication

Cutoff at 10% of annual income corresponds to $\mu_1 \approx 70$.

Rabin Paradox

A $(.5 : -100, .5 : 105)$

B to get nothing for sure

Many people choose B

With standard preferences this implies that agents will reject an even gamble (lose \$4,000, win \$635,670).

Our model predicts this large gamble is accepted. (logarithmic preferences over lifetime wealth.)

Our model predicts unexpected small winnings will be spent, so in this range the agent looks like someone with wealth equal to pocket cash and risk aversion coefficient ρ

Rabin gamble $(.5 : -100, .5 : 105)$ chosen to make a point

Estimating Risk Aversion

Actual laboratory risk aversion much greater.

From Holt and Laury [2002] and pocket cash = $20, $40, $80, we estimated ρ for two different percentiles

		$20	$40	$80
ρ 50^{th}		1.06	1.3	1.8
ρ 85^{th}		2.1	2.8	4.3

So all but poorest ($14K income) agents have more SR risk aversion than consistent with sr preferences being log.

Allais Paradox

Kahneman and Tversky [1979] version of Allais Paradox

A_1 $(.01 : 0, .66 : 2400, .33 : 2500)$

B_1 2400 for certain

$A_2 = (.67 : 0, .33 : 2500)$

$B_2 = (.66 : 0, .34 : 2400)$

paradox: choose B_1 and A_2. This violates the independence axiom

Base Case

annual interest rate $r = 1\%$

annual income is $28,000

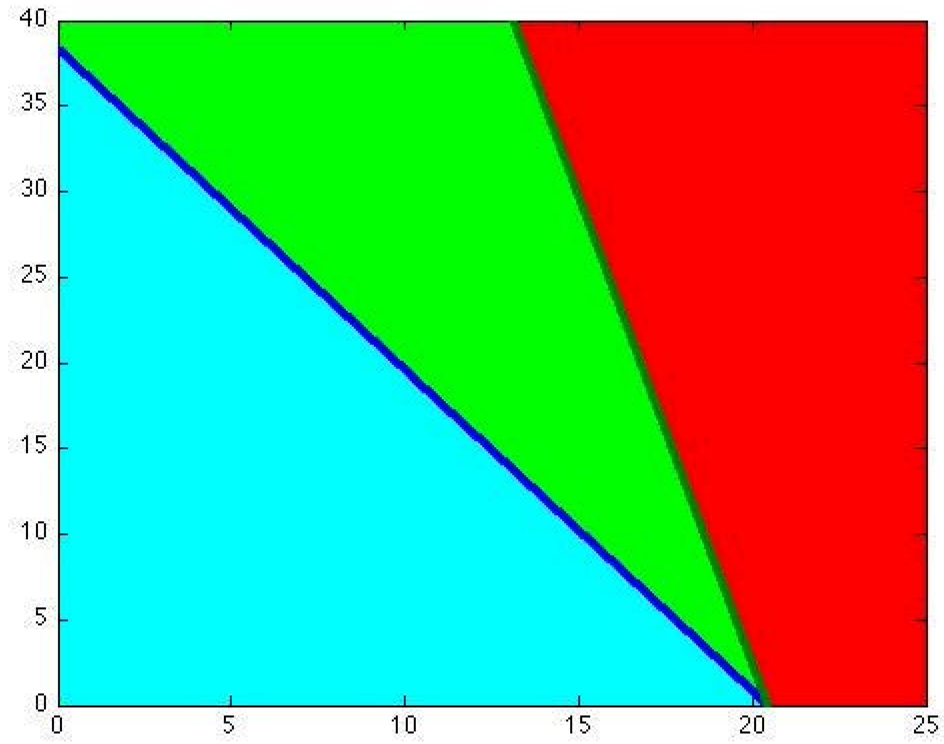
wealth is $860,000

short-run self's horizon a single day

pocket cash and chosen nightclub are $x_1 = c_1^* = 40$.

$\rho = 1.3$

Allais Self-Control Parameters



Paradox occurs in green region.
Blue: low cost of control, pick *A* both times; Red: pick B both times.

Summary of Self-Control Costs

income	$x_1 = c_1*$	ρ	$\mu_1(\gamma_1*)$
14000	20	1.06	16.65
14000	20	2.10	7.56
28000	40	1.30	8.49
28000	40	2.80	4.13
56000	80	1.80	4.57
56000	80	4.20	2.47

The Delayed Allais Paradox

	Now	3 month delay
A. 1.00 chance of 9 euros	0.58	0.43
B. 0.80 chance of 12 euros		
A. 0.10 chance of 9 euros	0.22	0.21
B. 0.08 chance of 12 euros		

Cognitive Load

experiment by Benjamin, Brown and Shapiro [2006] shows the impact of cognitive load on risk preferences

Chilean high school juniors

Chose between lotteries both under normal circumstances and under the cognitive load of having to remember a seven digit number.

key fact: students responded differently to choices involving increased risk when the level of cognitive load was changed

real not hypothetical reward; safe option was 250 pesos

paid in cash at end of session

1 $US= 625 pesos; average weekly allowance including lunch money around 10,000 pesos

Fraction Choosing Risky Option

50-50 gambles

The table summarizes the fraction of the population taking the risky choice.

(The numbers in parentheses are the the number of subjects.)

650/0 versus 250		650/0 versus 300/200	
No load (13)	Load (21)	No Load (15)	Load (22)
70%	24%	73%	68%

Adding load increases marginal cost of self control-> less benefit to winnings that would be saved, so go for sure payoff.

Effect smaller when the “safe” option is also risky.

CONCLUSION

Macro is all good