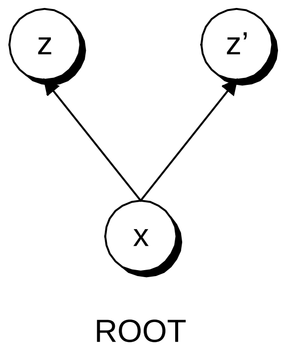
Extensive Form Games I

Definition of Extensive Form Game

a finite game tree X with nodes $x \in X$

nodes are partially ordered and have a single root (minimal element)

terminal nodes are $z \in Z$ (maximal elements)



Players and Information Sets

player 0 is nature

information sets $h \in H$ are a partition of $X \setminus Z$

each node in an information set must have exactly the same number of immediate followers

each information set is associated with a unique player who “has the move” at that information set

$H_i \subset H$ information sets where *i* has the move

More Extensive Form Notation

information sets belonging to nature $h \in H_0$ are singletons

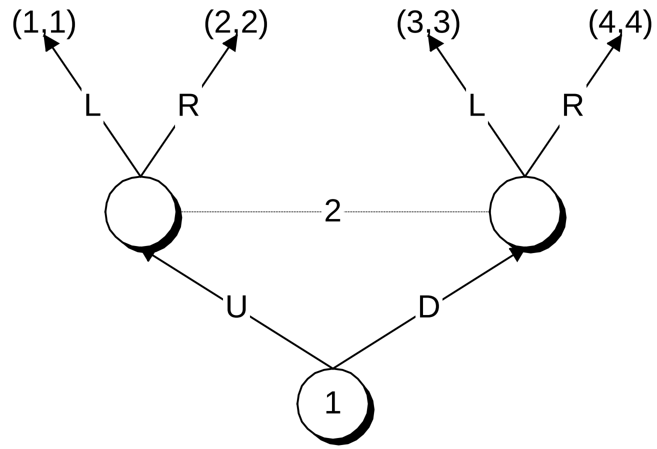
$A(h)$ feasible actions at $h \in H$

each action and node $a \in A(h), x \in h$ is associated with a unique node that immediately follows x on the tree

each terminal node has a payoff $r_i(z)$ for each player

by convention we designate terminal nodes in the diagram by their payoffs

Example: a simple simultaneous move game



Behavior Strategies

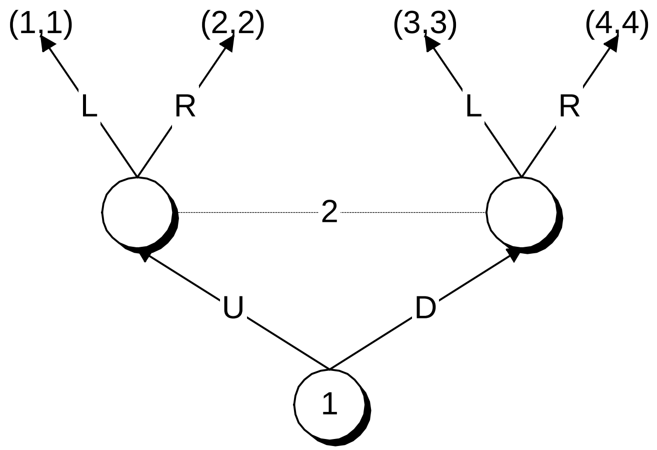
a *pure strategy* is a map from information sets to feasible actions $s_i(h_i) \in A(h_i)$

a *behavior strategy* is a map from information sets to probability distributions over feasible actions $\pi_i(h_i) \in P(A(h_i))$

Nature's move is a behavior strategy for Nature and is a fixed part of the description of the game

We may now define $u_i(\pi)$

normal form are the payoffs $u_i(s)$ derived from the game tree

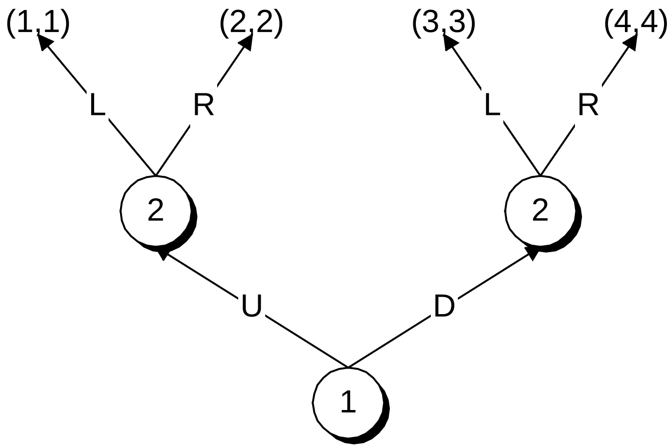


	L	R
U	1,1	2,2
D	3,3	4,4

Kuhn's Theorem:

every mixed strategy gives rise to a unique behavior strategy

The converse is NOT true



1 plays .5 U

behavior: 2 plays .5L at U; .5L at R

mixed: 2 plays .5(LL),.5(RR)

2 plays .25(LL),.25(RL),.25(LR),.25(RR)

however: if two mixed strategies give rise to the same behavior strategy, they are *equivalent*, that is they yield the same payoff vector for each opponents profile $u(\sigma_i, s_{-i}) = u(\sigma'_i, s_{-i})$

Subgame Perfection

some games seem to have too many Nash equilibria

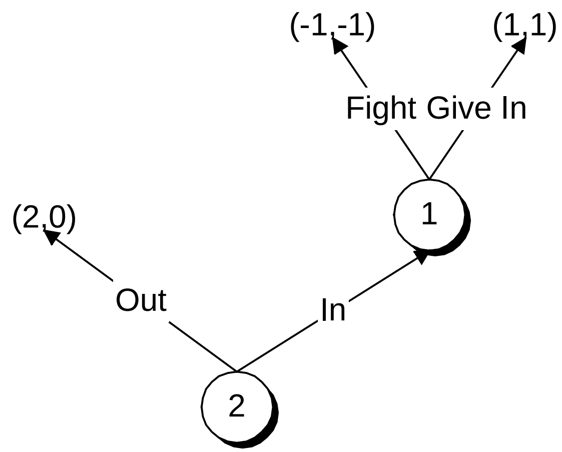
Ultimatum Bargaining

player 1 proposes how to divide $10 in pennies

player 2 may accept or reject

Nash: any proposal by player 1 with all poorer proposals rejected and equal or better proposals accepted

Chain Store



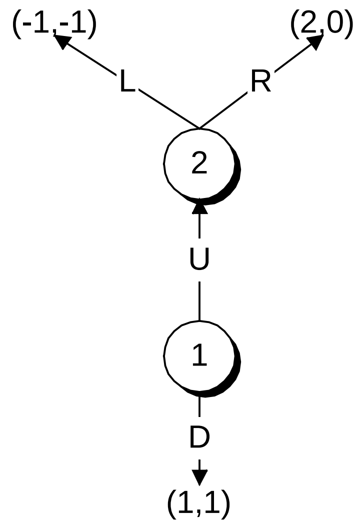
	out	in
fight	2*,0*	-1,-1
give in	2,0	1*,1*

Subgame Perfection

A *subgame perfect Nash Equilibrium is a Nash equilibrium in every subgame*

A subgame starts at a singleton information set

Selten Game



	L	R
U	-1,-1	2*,0*
D	1*,1*	1,1

equilibria:

UR is subgame perfect

D and .5 or more L is Nash but not subgame perfect

Application to Rubinstein Bargaining

the pie division game: there is one unit of pie; player 1 demands p_1

player 2 accepts or rejects

if player 2 rejects one period elapses, then the roles are reversed, with player 2 demanding p_2

common discount factor $0 < \delta < 1$

Nash equilibrium: player 1 gets all pie, rejects all positive demands by player 2; player 2 indifferent and demands nothing

conversely: player 2 gets all the pie

wait 13 periods then split the pie 50-50; if anyone makes a positive offer during this waiting period, reject then revert to the equilibrium where the waiting player gets all the pie

subgame perfection: one player getting all pie is not an equilibrium: if your opponent must wait a period to collect all pie, he will necessarily accept demand of $1-\delta-\varepsilon$ today, since this give him $\delta+\varepsilon$ in present value, rather than δ the present value of waiting a period

Rubinstein's Theorem:

there is a unique subgame perfect equilibrium

players always make the same demands, and if they demand no more than the equilibrium level their demands are accepted

to compute the unique equilibrium observe that a player may reject an offer, wait a period, make the equilibrium demand of *p* and have it accepted, thus getting δp today; this means the opposing player may demand up to $1-\delta p$ and have the demand accepted; the equilibrium condition is

$p = 1-\delta p$ or $p = \dfrac{1}{1+\delta}$

notice that the player moving second gets

$\frac{\delta}{1+\delta}$ and that as $\delta \to 1$ the equilibrium converges to a 50-50 split

a problem: if offers are in pennies, subgame perfect equilibrium is not unique

How to prove the equilibrium is unique:

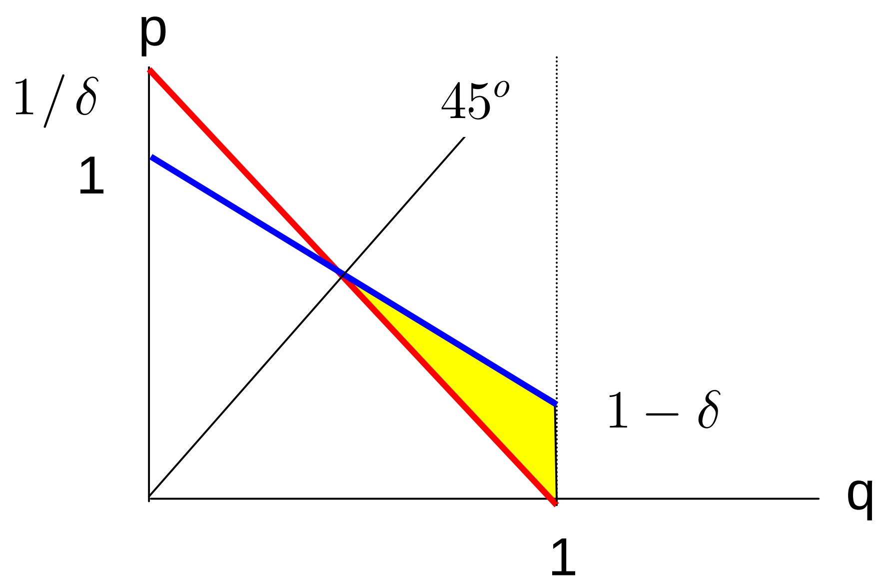
let p be such that any higher demand will be rejected in every equilibrium

let q be such that any lower demand will be accepted in every equilibrium

if you accept p you get $1 - p$ versus at least δq by rejecting, so $1 - \delta q$ or less will be rejected in any equilibrium and $p \leq 1 - \delta q$

if you accept q you get $1 - q$ versus at most δp by rejecting so $(1 - q)/\delta$ will be accepted in any equilibrium and $p \geq (1 - q)/\delta$

moreover $p \geq q$


p
$1/\delta$
1
45^o
$1-\delta$
q
1