Enforcement in Groups

Groups and the Provision of Public Goods

- over two million farms in the United States
- “farm lobby:” many people
- huge public good problem in getting people to contribute to the public good of lobbying
- everyone wants their group to win the contest - but of course would much prefer that everyone else contribute to the effort while they do not.

Costs and Social Norms

group members independently draw types y_k uniformly distributed on $[0, 1]$

contribute zero effort at zero cost (not participate) or contribute a single unit of effort (participate)

cost of participation is $c(y_i)$, types ordered so non-decreasing

linear cost: $c(y_i) = c_0 + y_i$.

effort for group k is determined by threshold φ_k for participation: the *social norm*

ftypes with $y_i < \varphi_k$ are expected to participate

those with $y_i > \varphi_k$ are not

social norm is followed fraction participating is φ_k

Individual Incentives

chance of influencing outcome negligible so particpate only if cost negative

define $\underline{\varphi} = -c_0$ with max of 1 and min of 0

fraction of committed members (voters)

public good problem

Why do voters vote?

civic duty: sense of obligation

expressive voting: like rooting for a sports team

here: committed voters

also: peer pressure

Coercion

contribution to public goods due to coercion

mandatory voting laws

military draft

penalties for tax evasion

farm lobbies cannot punish non-contributors

even if coercion is relevant rarely the entire reason for public good contributions

huge increase in military enlistments to go fight in Afghanistan after September 11

another form of coercion: peer pressure.

Peer Pressure

keep the good opinion of friends and neighbors

well documented in sociology literature: Coleman

some evidence:

Della Vigna: an important incentive for voters to vote is to show others that they have voted

Gerber: social pressure significantly increases voter turnout;

Palfrey-Pogorelskiy: experimental evidence that communication among voters and in particular communication within parties increases turnout

typically social norms maintained by various forms of social disapproval and ostracism: Elinor Ostrom

The Social Network

group members are organized into a simple social network on the circle

action of a member, whether she has participated or not, is observable by neighbors only

only a noisy signal of the type also observed by neighbors only

those who did not participate signal is $z^i \in \{0,1\}$ where 0 means "good, followed the social norm" and 1 means "bad, did not follow the social norm"

social norm was violated, that is $y_i < \varphi_k$ but member i did not participate, the bad signal is generated for sure

i did not participate but did follow the social norm so that $y_i > \varphi_k$, there is nevertheless a chance θ of the bad signal

neighbors report honestly, else more rounds needed

bad signal: punishment is P_k

Incentive Compatibility

social norm φ_k is *incentive compatible* if and only if $P_k = c(\varphi_k)$

any member with $y \leq \varphi_k$ would be willing to pay the participation cost $c(y)$ rather than face the certain punishment P_k

any member with $y > \varphi_k$ prefers to pay the expected cost of punishment θP_k over the participation cost of voting $c(y) > \theta P_k$

punishment itself a cost to the party

Group Cost

how costly it is for the group to induce additional members other than the committed ones to participate

$\underline{\varphi} = 1$ no problem, so assume otherwise

expected cost of $\varphi_k \geq \underline{\varphi}$ denoted $C(\varphi_k)$ with the convention that $C(\varphi_k) = 0$ for $\varphi_k \leq \underline{\varphi}$.

decompose expected cost $C(\varphi_k)$ into two additive components

turnout cost $T(\varphi_k) = \int_{\underline{\varphi}}^{\varphi_k} c(y)dy$,

monitoring cost $M(\varphi_k) = \int_{\varphi_k}^{1} \theta P_k dy$,

substituting the incentive compatibility condition $P_k = c(\varphi_k)$

$M(\varphi_k) = \theta c(\varphi_k)(1 - \varphi_k)$.

Expected Cost, Duties and Chores

$\varphi_k \geq \underline{\varphi}$ the total cost is $C(\varphi_k) = T(\varphi_k) + M(\varphi_k)$.

define $F = \max\{0, \theta c_0\}$

$\gamma = [(1/\theta) - 1]F + \theta(1 - \underline{\varphi})$

compute expected cost

$C(\varphi_k) = F + \gamma(\varphi_k - \underline{\varphi}) + (1/2)(1 - 2\theta)(\varphi_k - \underline{\varphi})^2$

- numeraire
- when concave/convex?

Why the Fixed Cost?

if $c_0 > 0$ even the lowest draws of y find it costly to participate

nobody participates cost 0

to get anyone to participate must provide incentives: a punishment of at least c_0

bad signals will arise from those who are legitimately excused – which is to say pretty much everyone, and they all need to be punished

The Key Role of Monitoring Costs

turnout costs costs are convex

$T(\varphi_k) = (F/\theta)\varphi_k + (1/2)(\varphi_k - \underline{\varphi})^2$

turnout costs are convex so with $\theta = 0$ large group always advantaged

monitoring costs are concave

$M(\varphi_k) = \theta(1 - \varphi_k)[(F/\theta) + (\varphi_k - \underline{\varphi})]$

large party advantage arises when $\underline{\varphi} > 0$ and $\theta < 1/2$

small party advantage arises for a medium prize when $F > 0$ and $\theta > 1/2$

Models of Group Behavior

- rule consequentialism

widely used in the voting literature often known as the ethical voter model

implicitly used in the great many political economy models that treat a group as a single individual.

- partial altruism

also been used in the voting literature as well as on work on conflict.

Group Utility

a single group still denoted by k of size $\eta_k = 1$ in which a single member of the group is of size Δ

group members i choose effort levels $\phi^i \in [0, 1]$

group has a social norm or target effort level φ_k

all except i follow the social norm group effort level

$(1 - \Delta)\varphi_k + \Delta\phi^i$

utility of an group member i

$u(\phi^i, \varphi_k) = v((1 - \Delta)\varphi_k + \Delta\phi^i) - \phi^i$

ϕ^i is the cost of effort provision, u is concave.

utility of entire group

$U(\phi^i, \varphi_k) = v((1 - \Delta)\varphi_k + \Delta\phi^i) - ((1 - \Delta)\varphi_k + \Delta\phi^i)$

Rule Consequentialism

eeach group member asks what would be in the best interest

of the group what social norm is most advantageous for the

group?

then each “does their bit” choosing $\phi^i = \varphi_k$

Harsanyi, Coate-Conlin, Romer, Hooker, Riker-Ordeshook, Feddersen-Sandroni, Li-Majumdar

supposed to capture the idea that it is unethical to free ride

in principle rule consequentialism can be decentralized so that each group member independently calculates what they are supposed to contribute

First Order Condition for Rule Consequentialism

maximize

$U(\varphi_k, \varphi_k) = v(\varphi_k) - \varphi_k$

first order condition $v'(\varphi_k) = 1$

Altruism

cares only about the total utility of the group

maximizes $U(\phi^i, \varphi_k)$ with respect to ϕ^i

first order condition

$\Delta v'((1-\Delta)\varphi_k + \Delta\phi^i) - \Delta = 0$

at symmetric equilibrium $v'(\varphi_k) = 1$

same as rule consequentialism

Coordination Game

3,3	0,0
0,0	1,1

difference between rule consequentialism: 3,3 and

pure altruism: either 3,3 or 1,1

rule consequentialists like peer enforcers can solve coordination problems

Partial Altruism

weight λ to own utility

maximize

$\lambda u(\phi^i, \varphi_k) + (1-\lambda)U(\phi^i, \varphi_k)$.

This objective function is also concave and the first order condition is

$\Delta\lambda v'((1-\Delta)\varphi_k + \Delta\phi^i) - \lambda)$

$+\Delta(1-\lambda)v'((1-\Delta)\varphi_k + \Delta\phi^i) - \Delta(1-\lambda) = 0$

which at a symmetric equilibrium is

$v'(\varphi_k) = 1 + \lambda\frac{1-\Delta}{\Delta}.$

Cases of Partial Altruism

$$v'(\varphi_k) = 1 + \lambda \tfrac{1-\Delta}{\Delta}.$$

$\lambda = 0$ same as pure altruism or rule consequentialism

$\lambda = 1$ is $v'(\varphi_k) = 1/\Delta$

goes to zero as the group gets large

less altruism (larger λ) and larger groups (smaller Δ) decrease φ_k below the purely altruistic level

altruism model examples: Schram-Sonnemans, Fowler, Fowler-Kam,

Edlin-et-al, Faravelli-Walsh}, Evren, Jankowski

Peer Enforcement

violators $\phi^i \neq \varphi^k$ punished for sure

non-violators $\phi^i = \varphi_k$ are punished with probability θ.

in a large group the benefit of deviating is the cost savings ϕ^i

group utility accounting for monitoring costs

$U(\varphi_k, \varphi_k) - \theta\varphi_k = v(\varphi_k) - \varphi_k - \theta\varphi_k$

first order condition $v'(\varphi_k) = 1 + \theta$

rather similar to partial altruism $\theta = \lambda(1 - \Delta)/\Delta$

Internalization of Social Norms

$\theta = 0$ same outcome as rule consequentialism or pure altruism

"internalization of social norms" meaning people punish themselves for violating the social norm so the monitoring cost is zero

What are Altruism Models About?

nobody believes that farm lobbies are based on the altruism of farmers for far away famers

Esteban and Ray

> An equivalent (but somewhat looser) view is that α [λ in our notation] is some reduced-form measure of the extent to which within-group monitoring, along with promises and threats, manages overcome the free-rider problem of individual contribution.

Now see a formal reason this is true

Indivisibility and Monitoring

- examine the case where the effort is indivisible
- in voting a natural assumption: either a member votes or does not vote but does not cast half a vote
- lobbying often the group asks for a fixed levy of time, effort, or money, and treating the level of contribution of exogenous the issue for members is then whether or not to participate
- allow for *ex post* differences at the time the participation decision is made
- on election day a group member is in the hospital, a member of a lobbying group is suffering financial distress
- look at extensive margin (how many participate) rather than intensive margin (how much each contributes)

Why not Split a Large Group?

with a positive fixed cost why doesn't the larger group “act like a smaller group” by appointing a smaller subgroup to act on its behalf?

a subgroup of size $M_k < N_k$ will only receive a share of the prize: $(M_k/N_k)V$

so raw willingness of the subgroup to pay is

$$M_k\underline{\varphi} + \frac{(M_k/N_k)V - M_kF}{c} = \frac{M_k}{N_k}\left(N_k\underline{\varphi} + \frac{V - N_kF}{c}\right) = \frac{M_k}{N_k}r_k$$

a fraction M_k/N_k of the raw willingness of the entire group to pay.

problem involves “renegotiation” subgroup will collude not to do it

Repeated Monitoring in a Group

$N > 2$ identical members $i = 1, \ldots, N$ of a collusive group

group plays one time primitive game in period 0

which members choose actions $a^i \in A$ a finite set

expected payoff of a member $u(a^i, a^{-i})$.

let a^R be common action of members

shorthand: as $u(a^i, a^R) = u(a^i, a^R, \ldots, a^R)$

assume that there is at least one symmetric static Nash equilibrium:
$a^R \in A$ for all $a^i \in A$ we have $u(a^i, a^R) \le u(a^R, a^R)$

The Question and Peer Monitoring

sustainability of actions a^R which are possibly not Nash equilibria through incentive compatible peer monitoring

based on Kandori's information systems approach

members audit each others behavior

accounts for the self-referential nature of punishment equilibria by supposing a potentially unlimited number of audit rounds $t = 1, 2, \ldots$

Signals

signals of behavior in the primitive games and in the subsequent auditing rounds

actions primitive game generate a signal of individual play $z^i \in \{0, 1\}$

0 is bad and 1 is good

probability of the bad signal 0 about member i is $\pi_0(a^i, a^R)$

Audit Rounds

sequence of audit rounds $t = 1, 2, \ldots$

players matched in pairs as *auditor* i and *auditee* j

matching: members located on circle – identify member 0 with member N and member $N + 1$ with member 1

assume that $j = i - 1$: each member audits the member to his left

in round $t \geq 1$ auditor i assigned to audit member j chooses whether or not to conduct the audit

Audit Signals and Punishments

depending on whether audit is conducted or not bad-good signal $z_t^i \in \{0, 1\}$ generated

audit: bad signal probability π

no audit: bad signal probability $\pi^p \geq \pi$

audit conducted: privately observe signal $z_{t-1}^j \in \{0, 1\}$ of auditee in previous round

signal is 0 (bad) auditee is punished

punishment has cost to the auditee of $P > 0$

cost to the auditor of audit is $\theta_t P \geq 0$

stationarity: $\theta_t = \theta$ for $t > 1$

initial audit can have different cost

The Super Game

first: meeting in which members agree on a scheme to maximize the utility of group members

agree on a common action a^R and for each round $t = 0, 1, \ldots$ beginning with the primitive round 0 a probability δ_t that the next audit round will take place

$1 - \delta_t$ probability that the game ends after round t determined endogenously by the group.

auditing rounds take place quickly so no discounting beyond that induced by δ_t

Incentive Compatibility

group is bound by incentive constraints – only incentive compatible plans can be chosen

a plan $a^R, \delta_t|_{t=0}^{\infty}$ is *peer feasible* if the individual strategies of playing a^R in the primitive round and always conducting an audit in the audit rounds is a Nash equilibrium of the super-game induced by the continuation probabilities δ_t

at the initial meeting group may either choose a peer feasible plan, or it may choose a static Nash equilibrium of the primitive game together with $\delta_0 = 0$. Among these plans the group chooses the plan that maximizes the *ex ante* expected utility of the members

Enforceability

a^R is *enforceable* if there is some punishment scheme based on the signal such that a^R is incentive compatible

there must be some punishment P_1 such that for all a^i we have
$u(a^R, a^R) - \pi_0(a^R, a^R)P_1 \geq u(a^i, a^R) - \pi_0(a^i, a^R)P_1$

$\sigma_0(a^i, a^R) \equiv \pi_0(a^i, a^R) - \pi_0(a^R, a^R)$ called *signal increase*

The Gain Function

gain function $\tilde{G}(a^i, a^R)$

for $\sigma_0(a^i, a^R) = 0$

and $u(a^i, a^R) = u(a^R, a^R)$ gain function is $\tilde{G}(a^i, a^R) = 0$

and $u(a^i, a^R) \neq u(a^R, a^R)$ gain function is
$\tilde{G}(a^i, a^R) = [u(a^i, a^R) - u(a^R, a^R)] \cdot \infty$

for $\sigma_0(a^i, a^R) \neq 0$

$$\tilde{G}(a^i, a^R) = \frac{u(a^i, a^R) - u(a^R, a^R)}{\sigma_0(a^i, a^R)}.$$

Characterization of Enforceability

Lemma: *The group action a^R is enforceable with the punishment $P_1 \geq 0$ if and only if*

$$\max_{\sigma_0(a^i,a^R)\geq 0} \tilde{G}(a^i,a^R) \leq P_1 \leq \min_{\sigma_0(a^i,a^R)<0} \tilde{G}(a^i,a^R)$$

If $\max\{0, \max_{\sigma_0(a^i,a^R)\geq 0} \tilde{G}(a^i,a^R)\} \leq \min_{\sigma_0(a^i,a^R)<0} \tilde{G}(a^i,a^R)$ define

$G(a^R) \equiv \max\{0, \max_{\sigma_0(a^i,a^R)\geq 0} \tilde{G}(a^i,a^R)\}$ otherwise $G(a^R) = \infty$

a^R is enforceable for some P_1 if and only if $G(a^R) < \infty$

Peer Feasibility

audit signal increase $\sigma = \pi^P - \pi$

Theorem: *If the action* a^R *is not static Nash it is peer feasible for some* $\delta_t|_{t=0}^{\infty}$ *if and only if* $P \geq G(a^R)$, $\theta_1/\sigma \leq 1$ *and* $\theta/\sigma < 1$, *in which case the group optimally chooses the termination probabilities*

$\delta_0 = G(a^R)/P, \delta_{t>0} = \theta/\sigma.$

The corresponding utility attained by each member is

$U = u(a^R, a^R) - \left(\pi_0(a^R, a^R) + \theta_1 + \frac{\theta_1(\theta+\pi)}{\sigma-\theta}\right) G(a^R).$

Summary of Optimal Auditing

utility net of minimum punishment cost

$v(a^R) = u(a^R, a^R) - \pi_0(a^R, a^R)G(a^R)$ t

unit cost of auditing

$C = \theta_1 + \frac{\theta_1(\theta+\pi)}{\sigma-\theta}$

optimum peer feasible utility from action a^R is $U = v(a^R) - CG(a^R)$

Optimal Plan

Either don't audit or optimally audit

If C is large choose static Nash

Theorem: *The optimal* a^R *has* $v(a^R)$ *and* $G(a^R)$ *weakly decreasing in* C

as the unit cost of auditing declines, it becomes optimal to accept larger gains to deviation in exchange for higher group net utility in the primitive game

Theorem: C *is increasing in* $\theta_1, \theta, \pi, 1/\sigma$*.*

A Public Good Contribution Game

might be attempting to corrupt a politician or it could be a consortium bidding on a contract.

each group member chooses between two actions $a^i \in A = \{0, 1\}$ is utility cost of contributing to the public good

contribution $a^i = 1$ this results in benefit to the group of $s > 1$ divided equally among all N members

$u(a^i, a^R)$

	$a^R = 0$	$a^R = 1$
$a^i = 0$	0	$s - (s/N)$
$a^i = 1$	$(s/N) - 1$	$s - 1$

assume that $\pi_0(a^i, a^R)$, π and π^p do not depend on group size

Public Good Theorem

Theorem: *Abbreviate* $\sigma_0 = \sigma_0(0,1) = \pi_0(0,1) - \pi_0(1,1)$. *Define*

$$\overline{N}(s,P) =$$

$$\begin{cases} s/\left(1 - \frac{\sigma_0(s-1)}{\pi_0(1,1)+C}\right) & \text{for} \quad s \le 1 + [\pi_0(1,1)+C]\cdot \min\{P, 1/\sigma_0\} \\ s/\left(1-\sigma_0 P\right) & \text{for} \quad s \ge 1 + [\pi_0(1,1)+C]P,\ P < 1/\sigma_0 \\ \infty & \text{for} \quad s \ge 1 + [\pi_0(1,1)+C]/\sigma_0,\ P \ge 1/\sigma_0 \end{cases}.$$

For $N \le s$ *the group contributes full effort, requires no costly auditing, and achieves utility* $U = u(1,1) = s-1$. *For* $s < N \le \overline{N}(s,P)$ *and* $\theta_1/\sigma \le 1, \theta/\sigma < 1$ *the group employs costly auditing, contributes full effort and achieves utility*

$$U = s - 1 - [\pi_0(1,1)+C][1-(s/N)]/\sigma_0.$$

For $N > \overline{N}(s,P)$ *or* $\theta_1/\sigma > 1$ *or* $\theta/\sigma \ge 1$ *the group contributes no effort and achieves utility* $U = 0$.

Interpretation of the Theorem

peer discipline not available if $\theta_1/\sigma > 1$ or $\theta/\sigma \geq 1$

- standard public good problem: group contributes full effort as long as individuals have adequate incentive to provide effort: $N \leq s$.
- once group becomes larger it ceases to provide effort

peer discipline is available when $\theta_1/\sigma \leq 1, \theta/\sigma < 1$

- full effort in the range $s < N \leq \overline{N}(s, P)$
- once group becomes larger it ceases to provide effort

if $\overline{N}(s, P)$ is finite qualitatively this similar to the pure public goods case

comparative statics of $\overline{N}(s, P)$ have expected monotonicity properties: lower cost of peer discipline as measured by smaller $\pi_0(1, 1) + C$ and larger σ_0 increase the size of group that can sustain effort

The Infinite Case

$\overline{N}(s, P) = \infty.$

requires:

- punishment be adequately large for the given initial signal quality - $P \geq 1/\sigma_0$
- s be sufficiently large: $s \geq 1 + [\pi_0(1,1) + C]/\sigma_0$

very different than public good case: contributions no matter how large the group is

Empirics of Very Large Groups

about two million farms in the United States

- similar to the paradox of voting: not very plausible that the individual lobbying efforts of a single farmer increase the chances of farm subsidies enough to be individually worthwhile
- we observe farm subsidies of similar per-farm value across countries with very different sizes: Japan and the United States, for example

Theory in Very Large Groups

suppose peer discipline technology and the benefit per farmer of farm subsidies s are roughly the same in the different countries

if $\overline{N}(s,P)$ is finite, then in countries with few farmers $N \leq \overline{N}(s,P)$ we should find lobbying effort and farm subsidies, while in countries with many farmers $N > \overline{N}(s,P)$ we should find no lobbying and no farm subsidies.

$\overline{N}(s,P) = \infty$ covers this fact: full effort is provided independent of group size, so no matter the number of farmers or size of country, the amount of per capita public good achieved should be roughly similar - as it is.