Political Contests

Trade versus Contests

- economics is largely about mutual gains to trade
- politics is largely about transfer payments – about conflict

conflict is complicated

- many groups of different types
- effort provision that determines the outcome of conflict can have many forms: money, votes, boycott, strikes, peaceful demonstration, violence
- many political prizes – offices at different levels, referenda

we do not have a “general equilibrium theory” of conflict

Turnout, Polls, and Elections

they key to elections: voter turnout; effort provision

- polls do a good job predicting how people are going to vote
- unexpected outcome like Brexit or Trump is generally because polls do a poor job of predicting whether people are going to vote or not

example: Spanish national March 14, 2004 in the aftermath of the Atocha bombings

- incumbent People's Party was favored to win by around 6 percent but was voted out of office
- it was not that fed up People's Party supporters voted for other parties
- furious opposition voters turned out in much greater than expected numbers

Simple Contest

think of country like Greece where the political party that wins the election gets a lot of government jobs to reward its followers

- two parties: large L and small S
- government jobs worth V
- parties have a fixed set of members
- relative size of two parties $\eta_L > \eta_S > 0$
- turnout (effort) by party $k \in \{L, S\}$ is fraction of members sent to polls $0 \leq \varphi_k < 1$ (note trick to get rid of endogenous tie-breaking rule)
- sending a voter to the polls costly: normalize the cost to 1 per unit of voter sent to the polls
- party that sends the most voters to the polls wins the prize; tie, prize is split

The Game

election a game between two players - the parties

payoff to party k

win: is $V - \eta_k \varphi_k$

lose: $\varphi_k \eta_k > \varphi_{-k} \eta_{-k}, \text{ it is } -\varphi_k \eta_k$

tie: $V/2 - \varphi_k \eta_k$

this is called an an all-pay auction

number of voters $b_k = \varphi_k \eta_k$ sent to the polls is the bid and the highest bidder wins

both the winner and loser have to pay their bid, hence “all-pay”

Why Game Theory?

- the same game could be a lobbying game where the bids are money
- general framework where groups compete by submitting bids to win a prize is the workhorse model of political economy
- in lobbying/bribery it may be that only the winner pays
- we know small groups are effective in lobbying and large groups in voting
- perhaps small groups are more effective in winner-pays auctions and large groups more effective in all-pay auctions?

The All-Pay Auction

the all-pay auction with complete information has a unique Nash equilibrium

three key characteristics

- equilibrium is not in pure strategies so outcome of the election necessarily unpredictable.
- large party never does worse than small party in expected utility, may do better
- higher stakes favor large party

Mixed Equilibrium

- no pure strategy equilibrium means the outcome of the election cannot be predicted in advance, it must be uncertain
- upsets such as Brexit or Trump are to be expected
- no pollster or political scientist can make it otherwise
- uncertainty principle for elections
- why pollsters are often wrong
- theory also works for wars, strikes,public demonstrations and other conflicts in which both sides pay regardless of whether they win or lose

How the All-Pay Auction Works

Theorem: *In equilibrium there cannot be a positive probability of a tie and there is no equilibrium in pure strategies.*

Proof of no tie:

tie is at $b < V$ each wants to bid a bit higher

tie at V each would rather bid 0

Proof of no pure strategy equilibrium:

pure strategies and no tie one party loses and should bid 0

but if one party bids 0 the other should bid the smallest number bigger than zero and there is no such number

Willingness to Bid

highest bid willing and able to pay

$W_k = \min\{V, \eta_k\}$ urns out all of its voters).

medium stakes election: $V \leq \eta_S$ so both parties have the same willingness to bid

high stakes election: $V > \eta_S$ so the large party has higher willingness to bid

Theorem: *The large party has expected payoff of $V - W_S$ and small party get 0. In a high stakes election the small party bids 0 with probability $1 - \eta_S/V$ and the large party bids W_S with probability $1 - \eta_S/V$. All remaining probability of either party is a uniform density on $(0, W_S)$ of height $1/V$.*

Mixed Strategies

a cdf over bids G_k

non-decreasing function on $(-\infty, \infty)$ with $G_k(b) = 0$ for $b < 0$ and $G_k(1) = 1$

right continuous

if it fails to be left continuous the height of the jump at b is the probability of the bid – the *atom*

at points of continuity of G_k the probability of the bid is zero

High and Low Bids

one party must get 0 and both parties must bid arbitrarily close to W_S

- since the small party gets near zero by bidding near W_S it is the small party that gets zero
- since the large party gets at most $V - W_S$ by bidding near W_S it cannot get more than this
- the large party can get near $V - W_S$ by bidding a bit more than W_S so it cannot get less than this

Low Bids

$\underline{b}$ the lowest bid by either party

$b < \underline{b}$ we have $G_k(b) = 0$ for both parties while for $b > \underline{b}$ we have $G_k(b) > 0$ for at least one of the parties

cannot lead to tie with positive probability

k faces opponent with zero probability of bidding $\underline{b}$

Lemma: k *must get 0 in equilibrium*

Proof of the Lemma

$-k$ has a continuous G_{-k} at $\underline{b}$

suppose k is not bidding near $\underline{b}$

for some $b > \underline{b}$ we have $G_k(b) = 0$ hence $G_{-k}(b) > 0$

since bids by $-k$ in $(0, b]$ lose for certain they are not made so have $G_{-k}(0) = G_{-k}(b) > 0$

implies $\underline{b} = 0$ and that G_{-k} is discontinuous there, a contradiction.

so for $b > \underline{b}$ we have $G_k(b) > 0$

what do these bids earn?

as $b \to \underline{b}$ we have $G_{-k}(b) \to 0$

bids by k in $(\underline{b}, b]$ lose with probability at least $1 - G_{-k}(b) \to 1$ and earn at most $G_{-k}(b)V \to 0$

High Bids

highest bid is less than W_S the party getting an expected payoff of zero should bid a shade higher

one party cannot have a higher highest bid than the other, since the party with the higher highest bid could lower its bids, saving cost and still winning with probability 1

so both parties must bid near W_S.

Conceding Elections

large party gets $V - \eta_S$ in a high stakes election and bids close to zero
those bids get the same

probability wins must be close to $1 - \eta_S / V$

so small party must bid zero with that probability.

No Atoms and No Gaps

Lemma: $G_k(b)$ *is continuous and strictly increasing on* $(0, W_S)$*.*

No Gaps

not strictly increasing this means there is a gap where k does not bid
gap for one party implies same gap for the other: absolutely no point bidding in a range where the other party does not bid

top b of a hypothetical gap one party $-k$ does not have an atom

so k should not bid above but close to \$b\$: better to bid at the bottom of the gap

so gap goes all the way to W_S which we know is not the case

No Atoms

k has atom at b then $-k$ should not bid just below, implying a gap

Equilibrium Strategies

since G^k continuous the utility for party k from the bid b is $G_{-k}(b)V - b$

since G_k is continuous and strictly increasing, utility must be constant for any b

directly compute that $G_L(b) = b/V$ and $G_S(b) = 1 - W_S/V + b/V$.

The Tripartite Auction Theorem

- truthful equilibrium in a menu auction-pay
- weak dominance equilibrium in a second price sealed bid auction-pay

high value wins and pays the cost of matching the bid of the low value

exactly the same as the all-pay auction-pay

- success or failure of small groups not to be found in the type of auction
- small groups are not successful in lobbying because it is a second price auction but unsuccessful in voting because it is an all-pay auction
- we do see why lobbying leads to more certain results than elections

Revenue Equivalence

this tripartite auction theorem has nothing to do with the well-known revenue equivalence theorem

- about bidders utility in an auction with a commonly known value
- revenue equivalence is about auctioneers sellers utility with privately known values

(Relatively) Small Groups

country	% agriculture	farm subsidy hours
Switzerland	0.8	23
Japan	1.2	19
U.S.	1.3	11
Norway	1.6	17
EU	1.7	14
Canada	1.7	8
Australia	2.4	2

% agriculture: percent of value added in the agricultural sector

farm subsidy hours: hours worked per capita to pay farm subsidies

note the large absolute size: more than 200,000 farms in Canada

Small Groups Voting

California special elections for US House due to death or another job – in every case the party with the largest number of registered voters won

- farmers obviously not winning elections
- must be successful at lobbying
- why does not the 90% plus of the people in the economy who are not farmers form an anti-farm lobby and prevent the farmers from picking their pocket?
- note the difference with under the table bribes

Chores

- is it worth it to take the time and effort to find, learn about, join and support an anti-farm lobby in hopes of getting an extra 11 hours a year worth of wages?
- hardly worth it to the lobby to vet me, process my application and so forth if I am only going to contribute the equivalent of a few hours a year
- substantial fixed cost in joining an organization:
- cannot simply write a check for 32 cents to the “anti-farm lobby” as an effective way to lobby against them
- considerable cost incurred even as I contributed absolutely nothing to the lobbying effort

fixed cost of effort provision: a *chore*

Duties

- a ballot referendum against farm subsidies
- costly to go to the polling place: but it is my civic duty
- satisfaction of having discharged my duty might more than offset the direct cost of participating
- lobbying is a chore
- voting is a duty
- broad meaning of duty: a political demonstration or protest might be an enjoyable event

Duties Versus Chores

effort $\varphi_k \in [0,1]$ and $q \in \{0,1\}$ with $q = 1$ if $\varphi_k > 1$

the tie-breaking rule is now endogenous

per capita fixed cost of $F \geq 0$ of organizing the group

individual level of duty $1 > \underline{\varphi} \geq 0$ with f cost of not performing duty

per capita cost of effort provision

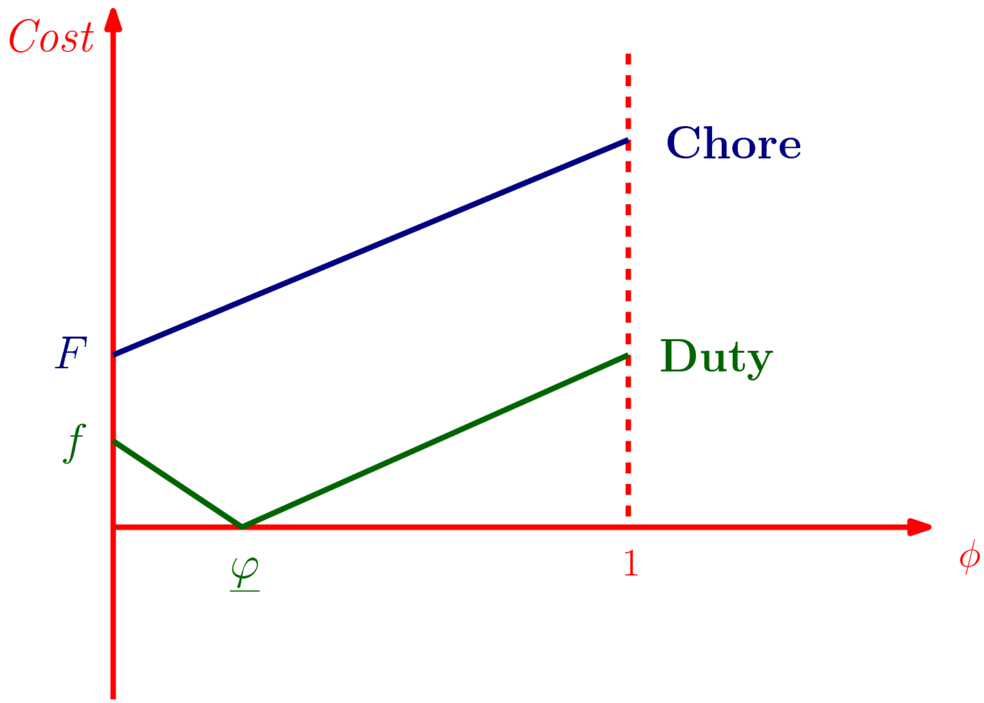
$C(\varphi_k) = qF + f \cdot \max\{0, \underline{\varphi} - \varphi_k\} + \max\{0, \varphi_k - \underline{\varphi}\}.$

two polar cases

effort a *duty*: $\underline{\varphi} > 0$ and $F = 0$

effort a *chore*: $\underline{\varphi} = 0$ and $F > 0$

Effort Cost



Willingness to Bid

always willing to provide $\eta_k \underline{\varphi}$ units of effort

additional cost of additional effort is $\eta_k F + \eta_k(\varphi_k - \underline{\varphi})$

equate to V and solve for effort $B_k = \varphi_k \eta_k$

desire to bid

$B_k = \eta_k \underline{\varphi} + V - \eta_k F.$

willingness to bid

$B_k < \eta_k \underline{\varphi}$ then $W_k = \eta_k \underline{\varphi}$

$B_k > \eta_k$ then $W_k = \eta_k$

$\eta_k \underline{\varphi} \leq B_k \leq \eta_k$ then $W_k = B_k$

get the benefit of duty f regardless of whether you win or lose, so does not enter

Advantage and Disadvantage

$B_k < \eta_k \underline{\varphi} = 0$ for both k: neither group submits a bid

assume hereafter $V > \eta_S F$

advantage

- group with the least willingness to bid: disadvantaged d
- group with the highest willingness to bid: advantaged $-d$

size of prize (is the large group willing to outbid the entire small group?)

- medium: $V < F\eta_L + \eta_S$
- large: $V > F\eta_L + \eta_S$

Key Results

- The level of utility of the two groups is the same regardless of whether the prize is allocated by an all-pay, first-price or second-price auction.
- Only an advantaged group can receive a positive level of utility and always does so.
- The small group is advantaged for a chore with a low to medium prize, the large group is advantaged for a duty, and for a chore with a large prize.

Size of the Prize

- for the small group to be advantaged the prize must not be too large
- for the farm subsidies the prize is relatively modest: hours per non-farmer not months
- defeat in the U.S. Congress of the “Stop Online Piracy Act” appears to be because they asked so much it was worth it to the larger group of everyone else to make a bid
- (a broad grass-roots lobbying effort against the bill killed it)

Auctions with Duties and Chores

Theorem: *The small group is advantaged in a chore with a low to medium prize. Otherwise, the large group is advantaged.*

large: $V > F\eta_L + \eta_S$

large party definitely advantaged

Medium Prize and Average Cost

cost to k of a bid b

$$\eta_k C(b/\eta_k) = b\,(C(b/\eta_k)/(b/\eta_k))$$

party with the lower cost for b is with lower average cost is lower

small party must exert more effort for a given bid

$C(\varphi_k)$ convex (duty) average cost is increasing and the large party is advantaged

$C(\varphi_k)$ concave (chore) average cost is decreasing and the smal party is advantaged

Tripartite Auction Theorem

cost to advantaged group of matching disadvantaged, if

$W_d \geq \eta_{-d}\underline{\varphi}$ then cost $W_d - \eta_{-d}\underline{\varphi} + \eta_{-d}F$

$W_d < \eta_{-d}\underline{\varphi}$ then cost zero

surplus: difference between the value of the prize and the cost of matching the bid of the disadvantaged group if positive, zero otherwise

Tripartite Auction Theorem: *In the second-price, first-price and all-pay auction, the disadvantaged group gets 0 and the advantaged group gets the surplus. It follows that the expected effort provided is the same for all three mechanisms.*

Intermediate Prize Structure of Equilibrium

Theorem: *For a chore with $0 < W_L < \eta_S q = 0$ the small group is advantaged. For a duty with $\eta_L \underline{\varphi} < W_S < \eta_S$ the large group is advantaged. In both cases surplus is $(\eta_L - \eta_S)(F + \underline{\varphi})$. In the all-pay case each group provides minimal effort $\eta_k \underline{\varphi}$ with equal probability and otherwise play uniformly on $(\eta_L \underline{\varphi}, W_d)$. With a chore the small group provides minimal effort with $q = 1$ and the large group with $q = 0$ and the tie-breaking rule is that if the small group chooses $q = 0$ it loses.*

wins for sure since it has more committed members. Hence the large group has a higher probability of winning. In the case of a chore if both groups opt out the definition of opting out requires that the advantaged group pay the fixed cost and the disadvantaged group not to. Hence also in the case of a chore the advantaged group has a higher chance of winning.

Types of Equilibria in the All-Pay Auction

stakes	condition	advantaged group	
		duty	chore
high	$W_S = \eta_S, W_L > \eta_S$	large	large
medium	$\eta_S > W_S > \eta_L \underline{\varphi}, W_L > \eta_L \underline{\varphi}$	large	small
low	$W_S < \eta_L \underline{\varphi}$ or $\eta_S F < V < \eta_L F$	large	small
very low	$V < \eta_S F$	none	none

Remark: understanding all the equilibria is important if the prize is endogenous

Bidding at the Bottom

duty: bidding only the committed members $b_k = \eta_k \varphi_k$

chore: bidding zero and for the advantaged group paying the fixed cost $q_{-d}(0) = 1$ and for the disadvantaged group not paying the fixed cost $q_d(0) = 0$

High stakes

effort constraint cannot bind on the large group since then it is advantaged and only has to bid η_S

$W_S = \eta_S, W_L > \eta_S$: high stakes

- duty: desire to pay of the small group exceeding ability to pay
- chore: desire to pay of the large group exceeding ability of the small group to pay

large group always advantaged

High Stakes: Description of Equilibrium

same uniform distribution on $[\eta_L \underline{\varphi}, \eta_S]$

both groups have positive probability of bidding at the bottom

large group also has a positive probability of bidding η_S and winning for sure if there is a tie.

probability of the large group bidding at the bottom is determined by the small group earning zero

duty: small group must earn zero for bidding $\eta_L \underline{\varphi}$ so

$$G_L(\eta_L \underline{\varphi})V = (\eta_L - \eta_S)\underline{\varphi}$$

chore: small group must earn zero for bidding a bit more than 0 and paying the fixed cost $G_L(0)V = \eta_S F$

High Stakes: Left Overs

probability p_L "left over" from the uniform

$1 - p_L = \frac{\eta_S - \eta_L \underline{\varphi}}{V}$

small group must bid 0 with probability p_L

large groups bids the top η_S with probability $p_L - G_L(0)$

duty:

$$\frac{V-(\eta_S-\eta_L\underline{\varphi})-(\eta_L-\eta_S)\underline{\varphi}}{V} = \frac{V-\eta_S(1-\underline{\varphi})}{V}$$

chore:

$$\frac{V-(\eta_S-\eta_L\underline{\varphi})-\eta_S F}{V} = \frac{V-\eta_S(1+F)}{V}$$

Medium Stakes

$\eta_S > W_S > \eta_L \underline{\varphi}, W_L > \eta_L \underline{\varphi}$: medium stakes

both groups active in bidding

- duty: desire to pay of the small group being less than its ability to pay but greater than the committed bid of the large group
- chore: desire to pay of the large group less than the ability of the small group to pay but positive
- can also be described as the interior case

advantage depends on cost are duty or chore: large group advantaged for duty, small for chore

Medium Stakes: Description of Equilibrium

same uniform distribution on $[\eta_L \underline{\varphi}, W_d]$

both groups also bid with the same positive probability at the bottom; in the case of a chore the small group bids positive fixed cost the large group does not, and the small group wins ties, but loses them with “high enough” probability if it does not pay the fixed cost

probability bottom bid determined by disadvantaged group earning zero

duty: $G_L(\eta_L \underline{\varphi})V = (\eta_L - \eta_S)\underline{\varphi}$

chore: $G_S(0)V = \eta_L F$

Low stakes: Duty

$W_S < \eta_L \underline{\varphi}$

both groups bid just committed members: pure strategy equilibrium

large group advantaged and wins for sure

Low stakes: Chore

$\eta_S F < V < \eta_L F$

large group bids zero and does not pay fixed cost

small group advantaged

two types of equilibrium:

small group pays fixed cost and wins ties when it does and loses with “high enough” probability when it does not

small group does not pay the fixed cost and wins for sure

Very Low Stakes: Chore

$V < \eta_S F$

neither group willing to pay fixed cost so neither does

tie-breaking rule is arbitrary

Low and Very Low Stakes: Chore

equilibria in which nobody pays the fixed cost

in applications it may be appropriate not to use a tie-breaking rule but to say that the contest does not take place and that both groups get 0

in this case the low stakes equilibrium where the small group does not pay the fixed cost is ruled out

Why Lobbyists Win

- they do not always: pharmaceutical patents, SOPA
- patent stakes are higher than copyright stakes
- small groups (special interests) have an advantage for chores: but only if the stakes are not too high