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## ***Norms and Scale***

- How do social norms such as tit-for-tat or reciprocity function in a large group?
- Juvenal in the first century A.D.: “Sed quis custodiet ipsos custodes?”
- translation: “Who shall guard the guardians?”
- the answer is not complicated; tit-for-tat is a lousy strategy but –
- they shall guard each other

## ***The Meeting Economy***

- Players meet randomly in a large economy, but have limited information about the past play of opposing players
- More relevant to economics
- More sensible setting for learning arguments
- Much easier to analyze evolution because of smaller strategy space

## *An Example*

Overlapping generations with many players in each generation

Young player are randomly matched against old players

Only the young have a move – give a gift to old person or not

Gift worth  $x > 1$  to old person; costs 1 to give the gift

A simplification of the classical prisoner's dilemma:

The dominant strategy is not to give the gift

But everyone better off if everyone gives gifts

Note the large population; absence of any future connection to your opponent – a strong example of the “paradox” of norms and scale

## ***Information About Past Play***

An “information system” (Kandori) assigns each young person a flag based on the action they took and the flag of the old person they met

Note that this could be viewed as an “informational rule of thumb” even when more information is available

## *Tit-for-tat*

Tit-for-tat looks just at the action that was taken: the strategy is to take the same action as your opponent took when they were young

This is an equilibrium: if everyone plays this way it is best to give a gift, because this way you get a gift for sure; not giving a gift means that you will not get a gift for sure

since  $x > 1$  it is better to give and receive than not give and not receive

But tit-for-tat does not give a positive reason for carrying out punishment

if you meet someone who did not give a gift, you should still give them a gift

- Tit-for-tat is not an equilibrium when the “information system” is noisy
- This was already a problem in repeated games, well understood by game-theorists, but perhaps not by the world at large.

## ***Team information system/strategy***

Information system:

- Two flags: red and green
- Gift to green flag or no gift to red flag >> green flag
- Gift to red flag or no gift to green flag >> red flag

Strategy:

- Give gift to green flag

If you meet a green flag:

- gift you get  $x - 1$
- no gift you get 0

If you meet a red flag

- no gift you get  $x$

- gift you get  $-1$

So it is **strictly** optimal to gift against green (your team) and no gift against red (the other team)

Notice that this is a **strict** Nash equilibrium if there is noise (so that there are some red flags)

the punishment chain is never broken

the players who are offended do not need to carry out their own punishments

Judges punish criminals; and they punish each other for failing to punish criminals, including other judges...



## ***Folk Theorem***

The “Folk Theorem:” lots of equilibria

Notice that never gift no matter what the flags is also a strict Nash equilibrium

There is nothing egalitarian in this

Dal Bo shows that if there are several levels of gift, one player can be “King”

Anyone who “follows the law” gets a small gift

The law requires giving the King a large gift

Note the self-referentiality of these social norms

the rule is: you are rewarded for following the rule

## *The Question*

- Can evolutionary arguments pin down a particular equilibrium in a repeated PD type of setting?
- Can evolutionary arguments be given a reasonable economic interpretation, and if so, what does this mean for the repeated PD type of setting?
- How do information systems evolve?

## *The Model*

overlapping generation of three-period lived players

young, old, observer

constant and large population  $n$

three generations are alive at each time

each young player randomly matched with an old player, that old player is the observer in the young player's next match

young player must choose between altruism (A) and selfishness (S)

decision of young player  $i$  of generation  $t$  is denoted by  $a_t^i$

young player selfish, both players receive zero

young player altruistic young gets  $-1$ , old gets  $x > 1$

## ***Sources of Information for Young Players***

- statistical information about past play of other generations
- personal information about particular old person matched with from the observer

## ***Personal Information***

observer can deliver one of two messages, “red flag” or “green flag”

set of messages is  $\{r, g\}$

strategy for the observer depends on the play of the old player when young and message the message sent about his opponent

a map  $\eta: \{A, S\} \times \{r, g\} \rightarrow \Delta(\{r, g\})$

$\omega > 0$ : message delivery is noisy  $\eta \in \{\omega, 1 - \omega\}$

finite set  $N$  of available information systems (observer strategies)

all maps with  $\eta \in \{\omega, 1 - \omega\}$

$b_t^i$  vector of messages sent by the different information systems

## ***Sequence of Play***

young player receives statistical information about past play

chooses a *stage game strategy*  $s^i = (\eta, a(r), a(g))$

observer chooses one information system  $\eta$

map  $a$  from the message provided by  $\eta$  the set of actions  $\{A, S\}$

## ***Statistical Information About Past Play***

common pool of anonymous observations about past matches

each observation of the form  $\phi_t^i = (b_t^i, s_t^i)$

$b_t^i$  vector of messages about old player;  $s_t^i$  strategy of young player

set of possible observations  $\Phi$

**we assume that the strategies of young players are observable, not merely their actions**

pool of observations of fixed size  $k$  written as a fixed length vector

$\theta_t \in \Phi^k$ ; pool updated by randomly replacing  $1 \leq m < n$  observations with  $m$  randomly chosen matches

## ***Beliefs***

young have

- beliefs about message vector of old person matched with
- beliefs about distribution of strategies used by young when old

young assume they are sufficiently small component of the economy that own decision has negligible effect on latter distribution

beliefs formed from assumption that distribution of strategies constant over time and equal to the empirical frequencies  $\sigma(\theta_t)$  in pool

and



distribution of messages about previous period old people given by empirical frequencies in pool  $\beta_0(\theta_t)$

probabilities of messages for current old  $\beta(\theta_t)$  derived from  $\beta_0(\theta_t), \sigma(\theta_t)$  using the Markov message process

in  $\beta$  every message has probability at least  $\omega$ , regardless of other information systems.

## ***Strategic Behavior***

*intentional behavior*

maximizes sum of payoffs over two periods of life, given beliefs

in case of tie use fixed tie-breaking rule depending only on aggregate statistic

solution denoted  $\rho(\theta_t)$

young players sometimes make errors

probability of intentional behavior  $\rho(\theta_t)$  is  $1 - \varepsilon$

every other stage-game strategy chosen with probability  $\varepsilon(\#S - 1)$

## *Dynamics*

intentional and random play of the players with the random replacement of observations induces Markov process  $M$  on the state space  $\Phi^k$

$M^k$  strictly positive since each observation has positive probability and there is positive probability of replacing all observation in pool over  $k$  periods

so  $M$  is ergodic with unique stationary distribution  $\mu^\varepsilon$

because of behavioral errors transition probabilities polynomials in  $\varepsilon$   
apply Theorem 4 from Young [1993]

## ***Stochastically Stable States***

$\mu = \lim_{\varepsilon \rightarrow 0} \mu^\varepsilon$  exists: the *stochastically stable distribution* places weight only on states with positive weight in stationary distributions for  $\varepsilon = 0$

characterize stochastically stable distribution using Ellison [1995]

## ***Pure Steady States***

$\varepsilon = 0$ , some example of stationary distribution

*pure steady states*: all players play same pure stage game strategy;  
observation pool records only this pure strategy

consider all feasible initial distributions of flags

a pure steady state is a Nash equilibrium: each player's play is optimal  
given the actual strategies played by opposing players

## ***Selfish Strategy***

all players choose to be selfish regardless of the message

## ***Tit-for-Tat***

information system gives a  $1 - \omega$  chance of a green flag for altruism and a  $1 - \omega$  chance of a red flag for selfishness

altruistic towards green and selfish towards red

interesting case  $1 < (1 - 2\omega)x$

best to be altruistic regardless of the flag of the old player

so tit-for-tat is not pure steady state

Note role of flag noise

## ***Green-team Strategy***

if you have a green flag you are on the green team

altruistic towards team members, selfish toward non-members

conversely anyone who behaves in accordance with the strategy is admitted to the team, anyone who does not is expelled

if the strategy is followed  $1 - \omega$  chance of receiving  $x$  next period

any deviation gains at most  $-1$  in the current period, results in only  $\omega$  chance of  $x$  next period

this is a pure steady state for  $1 < (1 - 2\omega)x$



## *The Main Theorems*

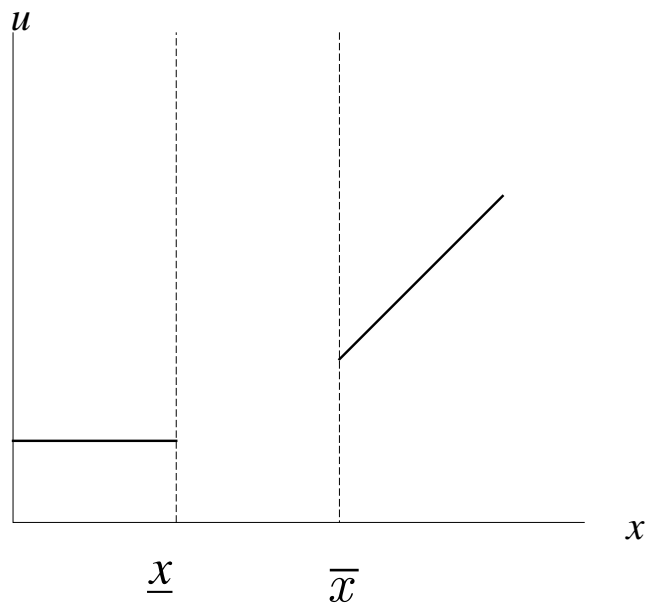
$\Theta^D \subset \theta^k$  states where all observations are of the one of the strategies always selfish

**Proposition 1:** If  $x < \underline{x}$  then  $\mu(\Theta^D) = 1$ .

## Cooperation

$\Theta^R$  states where all observations are red team strategy, and  $\Theta^G$  green team strategy

**Proposition 2:** If  $x > \bar{x}$  then  $\mu(\Theta^R) = \mu(\Theta^G) = 1/2$ .



Utility as a Function of Benefit of Altruism

## *Anti Team*

why  $\frac{1}{2}$ -dominance fails even for large  $x$

consider *green anti team*: give gift to green, but give opposite flags of green team

population 50-50 between green team and green anti-team

flags 50-50 in each team

green team: gives gift 50% of time; gets gift 50% of time

red anti-team: gives gift 50% of time; gets gift 50% of time

but red anti-team doesn't do very well against itself: all red flags, no gifts

selfish does a bit better than anyone against 50-50 between green team and green anti-team

gets gift 50% of the time, and never gives a gift

## ***Why Half Dominance Fails***

Does it make sense that stupid strategies should successfully invade by making some other strategy look good?