Nash Equilibrium I

- Nash Equilibrium is the heart of economic theory
- reasoning versus learning
- at a Nash equilibrium, there is nothing further to learn
- Each player plays optimally and correctly guesses what the other player will do
- example of the traffic game

Prisoner's Dilemma

prisoners dilemma: two prisoners in separate cells accused of jointly committing a crime

	Player 2	
Player 1	don't confess	confess
don't confess	32,32	28,35
confess	35,28	30,30

tragedy of the commons: two fishermen and a single lake

	Player 2	
Player 1	light fishing	intense fishing
light fishing	32,32	28,35
intense fishing	35,28	30,30

public goods: contribute towards building a bridge

	Player 2	
Player 1	contribute	do not
contribute	32,32	28,35
do not	35,28	30,30

Nash Equilibrium in the Prisoner's Dilemma

	Player 2	
Player 1	don't confess	confess
don't confess	32,32	28,35*
confess	35*,28	30*,30*

- Best responses
- Dominant strategies

Pedro Dal Bo: "Cooperation under the Shadow of the Future: experimental evidence from infinitely repeated games"

http://www.econ.brown.edu/fac/Pedro_Dal_Bo/theshadow.pdf

Table 2: Stage game payoffs in points

PD1					PD2	
		Blue Player			Blue Player	
		C	D		C	D
Red Player	C	65 , 65	10 , 100*		75 , 75	10 , 100*
	D	100* , 10	35* , 35*		100* , 10	45* , 45*

All payoffs in the game were in points. At the end of each session, the points earned by each subject were converted into dollars at the exchange rate 200 points=$1 and paid privately in cash. In addition, subjects were paid a 5 dollar show up fee

Rotating matching

Table 5: Percentage of cooperation by match and treatment*

		Match								
	1	2	3	4	5	6	7	8	9	10
$\delta = 0$	26.26	18.18	10.61	11.62	12.63	12.63	5.56	5.26	5.26	5
Dice										
$H = 1$	26.56	18.23	16.67	17.19	11.98	8.02	6.79	10.49	6.14	6.67
Finite										

*All rounds.

Battle of the Sexes Game

	Opera	Ball Game
Opera	1*,2*	0,0
Ball Game	0,0	2*,1*

No strategies are dominated

this is an example of a coordination game

a pure coordination game: driving on which side of the street?

	Left	Right
Left	1,1	0,0
Right	0,0	1,1

Pigouvian Taxes

We can't compute the Nash equilibrium of the traffic game, so what good does it do us?

Traffic game is like prisoner's dilemma

When you drive you impose a congestion cost on other people

Called a *negative externality*

Just as everyone confesses in the Prisoner's Dilemma, too many people for the social good drive

Pigou 1920: set a tax and charge each commuter for the cost they impose on others

The resulting Nash equilibrium will be Pareto efficient

Prisoner's Dilemma Example

	Player 2	
Player 1	don't confess	confess
don't confess	32,32	28,35*
confess	35*,28	30*,30*

If the other player isn't confessing you impose a cost of 4 on him by confessing; if we charge this as a "confession tax"

	Player 2	
Player 1	don't confess	confess
don't confess	32*,32*	28*,31
confess	31,28*	26,26

Now unique Nash equilibrium (with dominant strategies): don't confess

Computation of a Pigouvian Tax

- Sometimes hard to figure out the costs
- Not so much in traffic game: pretty easy for traffic engineers to compute additional commuting time and for economists to compute the value of the lost time
- Can charge by time and traffic as done using cameras and transponders in London.

Why Don't We Do It?

- Cost of excess congestion greatly exceeds the cost of the war in Afganistan, the cost of September 11, etc.
- Why aren't congestion taxes used to mitigate traffic?
- Problem lies in the political game
- Benefit of voting is small
- Not rational to invest a lot of effort in assessing and/or monitoring political candidates
- Benefits of reduced congestion spread widely

- Because politicians have latitude they are distrusted by voters
- So: a politician offers to raise your taxes to “help” you, what do you think?
- On the left: commuter tax will favor the rich – who can afford the toll – at the expense of the poor – who would be forced into public transportation
- On the right: afraid the government will squander the proceeds of the tax
- Both parties collaborate to prevent an efficient solution to the problem of congestion
- A solution: charge a commuting fee and use the revenue to reduce the local sales tax – which also disproportionately falls on the poor.

Failures and Failures

- Lots of things are wrong
- This doesn't mean people are irrational
- Or that economists don't know what they are doing
- Many solutions to economic problems are obvious
- Another example: raise the gas tax – taxes pollution, global warming, reduces dependence on foreign oil, etc. etc.
- mandating fuel efficiency standards for cars is dumb – one of unintended consequences lots of SUVs because they aren't covered
- but what to do about the problem of voters not monitoring politicians?

Winston Churchill

No one pretends that democracy is perfect or all-wise. Indeed, it has been said that democracy is the worst form of government except all those other forms that have been tried from time to time.

The Repeated Prisoner's Dilemma

	Player 2	
Player 1	don't confess	confess
don't confess	32,32	28,35
confess	35,28	30,30

- This is a simultaneous move game with a unique Nash equilibrium, and a unique strictly dominant strategy solution at 30, 30.
- The unique non-cooperative solution is Pareto dominated by 32, 32
- with repeated play, incentive are changed by the possibility of punishments and rewards in the future.

Intertemporal Preference

$u_i(t)$ is the utility or payoff received by player *i* in period *t*

the game is repeated indefinitely and that intertemporal preferences are given by average present value

$$U_i = (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}u_i(t)$$

where the common discount factor δ is between 0 and 1.

- a basic feature of repeated games: regardless of the discount factors, the repeated static equilibrium is a subgame perfect equilibrium of the repeated game

Grim Strategies

the *grim strategy* in the repeated game is

- cooperate in the first period
- cooperate in subsequent periods as long as all players have cooperated in every previous period
- cheat in any period in which some player has cheated in any previous period

suppose the other player plays the grim strategy

- payoff to cheating

  $$(1-\delta)\left(35+30\delta+30\delta^2\ldots\right)$$

  $$=(1-\delta)35+30\delta=35-5\delta$$

- payoff to cooperating

  32

- optimal to cooperate if

  $32 \geq 35-5\delta$ or

  $\delta \geq 3/5$

if $\delta \geq 3/5$ both players playing the grim strategy is a Nash equilibrium

Dal Bo Again

Repetition

Infinite horizon

$\delta = 0{,}1/2{,}3/4$ expected length $1{,}2{,}4$

Finite horizon

$H = 1{,}2{,}4$

subjects played all infinite or all finite

done in both orders – increasing length and decreasing length

Theory

δ	PD1	PD2
0	DD	DD
½	DD, DC, CD	DD, CC
¾	All	All

Table 5: Percentage of cooperation by match and treatment*

		Match									
		1	2	3	4	5	6	7	8	9	10
Dice	$\delta = 0$	26.26	18.18	10.61	11.62	12.63	12.63	5.56	5.26	5.26	5
	$\delta = \frac{1}{2}$	28.36	27.12	34.58	35.53	21.60	19.08	29.84	35.96	28.16	50
	$\delta = \frac{3}{4}$	40.44	28.57	27.78	32.92	46.51	33.09	44.05	53.51	42.26	45.83
Finite	$H = 1$	26.56	18.23	16.67	17.19	11.98	8.02	6.79	10.49	6.14	6.67
	$H = 2$	19.79	15.89	14.84	9.64	11.46	10.80	12.04	10.19	6.58	6.67
	$H = 4$	31.64	30.34	30.47	25.52	25.13	23.77	16.36	19.75	14.91	20.83

*All rounds.

Focus on matches 4-10

Table 6: Percentage of cooperation by round and treatment*

		Round											
		1	2	3	4	5	6	7	8	9	10	11	12
Dice	$\delta = 0$	9.17											
	$\delta = \frac{1}{2}$	30.93	26.10	19.87	12.50	12.96							
	$\delta = \frac{3}{4}$	46.20	40.76	38.76	34.58	33.04	27.27	24.75	26.28	29.17	26.04	32.29	31.25
Finite	$H = 1$	10.34											
	$H = 2$	13.31	6.90										
	$H = 4$	34.58	21.55	18.97	10.63								

*Matches four through ten.

Table 7: Distribution of outcomes by stage game and treatment*

	$\delta = 0$		$\delta = \frac{1}{2}$		$\delta = \frac{3}{4}$	
	PD1	PD2	PD1	PD2	PD1	PD2
CC	2.98	0.27	**3.17**	**18.83**	20.68	25.64
CD & DC	20.83	13.98	**28.57**	**25.50**	30.34	26.03
DD	76.19	85.75	68.25	55.67	48.98	48.33

*Matches four through ten, and all rounds.

Reciprocal Altruism

- final period of the two period games with a definite ending
- against an experienced player: who has already engaged in six or more matches
- in one shot game chance of cooperation 6.4%
- cheat in first period probability of cooperation in the final period 3.2%
- cooperate probability of cooperation in final round 21%

Crashes

- Doesn’t the crisis prove people are irrational?

Running for the Exits

- Nash equilibrium and the Prisoner's Dilemma are about the difference between individual rationality – the assumption of economists and
- Group rationality (Pareto efficiency) which only occurs under specific circumstances

	Everyone else	
You	orderly	rush
orderly	9	0
rush	10*	5*

Market Panics

- If the market is going to crash it is perfectly rational to try to get your money out first
- Another way to say it: sell to the people who are still uniformed
- If you think your bank is going to fail rational to get your money out as quickly as possible
- if everyone does that the bank will fail
- standard model of bank runs: Diamond and Dybvig 1983
- no facts or details about the crisis inconsistent with models of rational behavior.

Rational Expectations and Crashes

"Economics is useless because it can't predict crises"

- Suppose that we could predict crises
- Some model or oracle or something is proven right again and again.
- It tells us "Next week the stock market will fall 20%"
- What should you do?
- Don't wait a week before selling!
- So the crash won't occur next week – it will occur right now
- And the model/oracle or whatever is suddenly wrong
- It's not that economists can't predict crises
- Nobody can

Uncertainty Principle for Economics

an intrinsic interaction between the forecaster and the forecast – if forecaster is believed

not like predicting the weather

chances of a hurricane don't depend on whether or not we think there is going to be a hurricane.

Only the rational expectations model (Nash equilibrium) accounts for the relationship between forecaster and forecast

The rational expectations model asks what happens if the forecast is believed

anyone else in the economy – which if people believe his forecasts will have to be the case

Failures of Economics?

Krugman 2009:

> the profession's blindness to the very possibility of catastrophic failures in a market economy. During the golden years, financial economists came to believe that markets were inherently stable — indeed, that stocks and other assets were always priced just right. There was nothing in the prevailing models suggesting the possibility of the kind of collapse that happened last year.

Some earlier literature

- 1979 "A Model of Balance-of-Payments" shows how under perfect foresight crises are ubiquitous when speculators swoop in and sell short
- deficiency in supposing crises are perfectly foreseen
- fixed in 1983 by Steve Salant: rational expectations leads to speculation and unexpected yet catastrophic price drops

non-economists are always shocked, shocked

1992, nine years after Salant

> Sweden's abandonment Thursday of its battle to defend the krona, in a grudging capitulation to currency speculators, bodes ill for Europe's other weak currencies and threatens to send new waves of turbulence through the European Monetary System.
>
> The central bank, which jacked interest rates to an astronomical 500 percent to stave off devaluation during the European currency crisis in September, raised rates to 20 percent Thursday morning, from 11.5 percent, in a last attempt to bolster the krona, only to concede defeat hours later.
>
> “The speculative forces just proved too strong,” Prime Minister Carl Bildt said in announcing that Sweden would let the krona float.

Krugman?

Odd that a Nobel prize winning economist wouldn't know a famous much cited paper from 1979 about the ubuiquity of crises – especially considering he wrote it