Approximate Equilibrium

You Can Fool Some of the People…

You may fool all the people some of the time, you can even fool some of the people all of the time, but you cannot fool all of the people all the time. *Abraham Lincoln.*

- No economist believes “everyone is completely rational all the time”
- We think “most people are pretty rational most of the time”
- But we often ignore the caveat
- What happens when everyone is a little “irrational” and a few people are very “irrational?”

Approximate Equilibrium

1980 Roy Radner introduced notion of approximate Nash equilibrium

σ^i strategy of player i

σ^{-i} strategies of all players except for player i

$\sigma = (\sigma^i, \sigma^{-i})$

$u^i(\sigma)$ utility from strategies

Nash equilibrium $\hat{\sigma}$ such that $u^i(\hat{\sigma}) \geq u^i(\sigma^i, \hat{\sigma}^{-i})$

ε-Nash equilibrium $\hat{\sigma}$ such that $u^i(\hat{\sigma}) + \varepsilon \geq u^i(\sigma^i, \hat{\sigma}^{-i})$

note that this is in expected utility, so allows small probability of large loss or large probability of small loss

Satisficing

Critic of modern theory, Herbert Simon 1956

- Developed notion of *satisficing* behavior (for which he won the Nobel prize)
- People become satisfied and stop attempting to learn if they achieve a desirable goal that falls short of the very best possible
- Behavioral economists talk of him with favor: but ignore his ideas
- Ordinary economists have never heard of him: but use his ideas

Incorporated into modern economics in three ways:

- Habit formation model
- Learning theory
- Approximate optimization

Competitive Equilibrium

- traditional theory of competitive behavior is a model of approximate optimization
- trader always has a little bit of market power – even the smallest wheat trader can change prices a tiny bit in her favor by withholding a some wheat from the market
- practice nobody is going to take the time and effort to figure out how to manipulate a market in order to garner a few cents
- theory of competitive behavior supposes that traders ignore the possibility of such small gains.

Theory of Learning

approximate optimization is also widespread in the modern economic theory of learning

- Foster and Young's 2003 paradigm of the hats it is assumed that a player only try new things if there is evidence of a strategy that works at least a bit better than the status quo
- Fudenberg and Levine [1995] players are assumed to randomize between nearly indifferent alternatives even though this results in slightly less than the optimum payoff. This randomization provides strong protection against an opponent who is cleverer than you are.

Measurement

- given the objective play of other players, and what a player actually did, we can ask "how much more money could that player have earned?"
- measure size of ε
- Nash equilibrium the answer is zero
- measure of "success" for Nash equilibrium should not be whether play "looks like an equilibrium" but whether ε is small

Ultimatum Bargaining

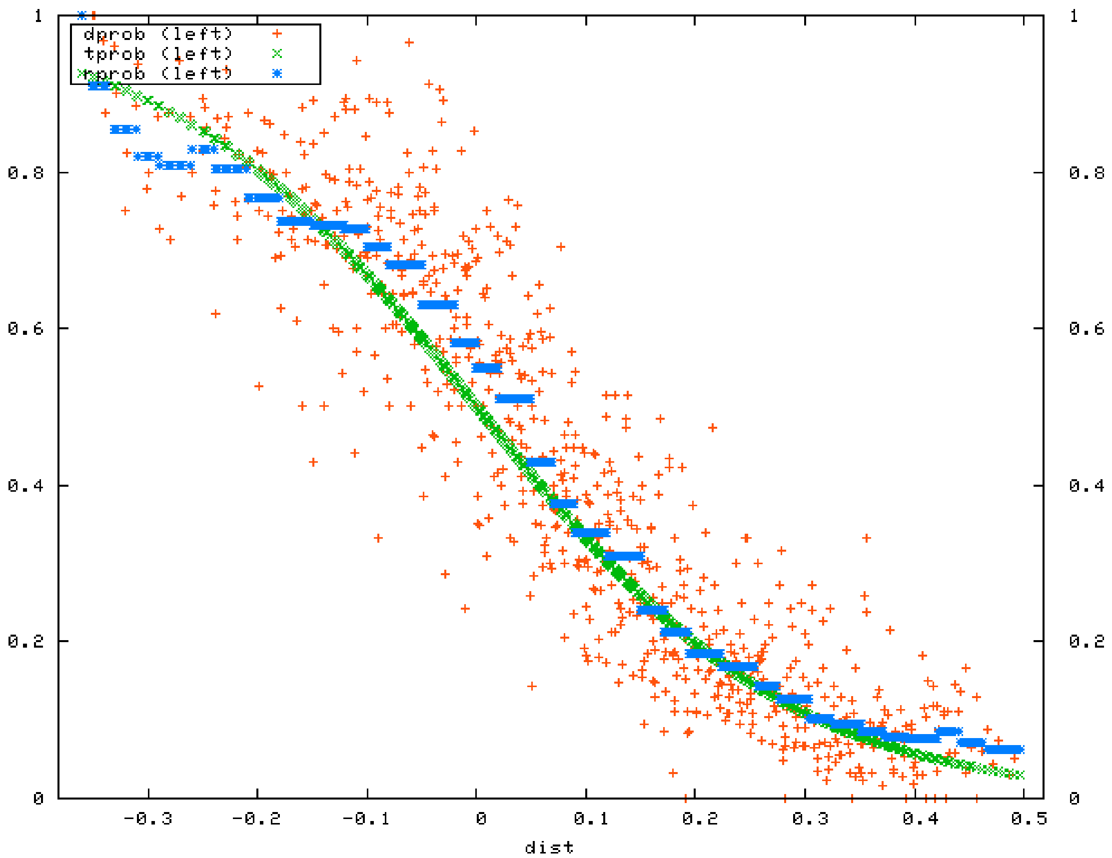
- Fudenberg and Levine [1997]
- losses to players playing less than a best-response as averaging $0.99 per player per game out of the $10.00 at stake
- most of the money is not lost by second players to whom we have falsely imputed selfish preferences, but rather by first movers who incorrectly calculate the chances of having their offers rejected
- however, a first player who offers a 50-50 split may not realize that she could ask for and get a little bit more without being rejected, nor if she continues to offer a 50-50 split, will she learn of her mistake.
- comparing the prediction of subgame perfection to the data indicates that players offered $5.00 when they should have offered $0.05. Yet a more reasonable measure of the success of the theory is that players lose only $0.99 out of the possible $10.00 that they can earn.

Equilibrium: The Weak versus the Strong

- problem with ε-equilibrium is not that it makes inaccurate predictions
- makes too many predictions
- ultimatum bargaining game is a perfect example
- with $\varepsilon = \$0.99$ half of the offers at $5.00 is an approximate equilibrium – and so is all the offers at $0.05.

- a theory that is sometimes weak and sometimes strong can be useful if it lets us know when it is weak and when it is strong
- with narrow range of predictions – as in the voting game, or in games such as best shot or competitive bidding – the theory is useful and correct.w
- with a broad range of predictions – as in ultimatum bargaining - the theory is correct, but not as useful.

Voting Redux



Quantal Response Equilibrium

- ignore psychological forces entirely and just assume that costly deviations from equilibrium are less likely than inexpensive ones
- when incentive are weak play is less predictable.
- *quantal response equilibrium* (or QRE) introduced by McKelvey and Palfrey in 1995
- built on the standard logistic choice model introduced to economics by McFadden [1980].

Formal Definition of QRE

$\sigma_i(s_i)$ is the probability with which player i plays the strategy s_i

$\lambda > 0$ be a parameter of the choice function

first define propensities with which strategies are layed

$p_i(s_i) = \exp(\lambda u_i(s_i, \sigma_{-i}))$

strategies that yield higher utilities have higher propensities of being played

QRE equilibrium probabilities are given by normalizing the propensities to add up to one.

$$\sigma_i(s_i) = p_i(s_i) / \sum_{s_i'} p_i(s_i').$$

Observations

- contains an unknown preference parameter λ
- $\lambda = 0$ play is completely random
- as λ becomes large, the probability of playing the “best” response approaches one
- λ kind of index of rationality.
- in the voting experiment we can estimate a common value of λ for all players.
- corresponding equilibrium probabilities of play are given by the green curve
- does an excellent job of describing individual play
- it makes roughly the same predictions for aggregate play as Nash equilibrium.

Limitations of QRE

- captures only the cost side of preferences
- recognizes – correctly – departures from standard “fully rational” selfish play are more likely if less costly in objective terms
- does not attempt to capture benefits of playing non-selfishly
- does not well capture, for example, the fact that under some circumstances players are altruistic, and in others spiteful.

Auctioning a Jar of Pennies

- surefire way to make some money
- put a bunch of pennies in a jar
- get together a group of friends
- auction off the jar of pennies
- with about thirty friends that you can sell a $3.00 jar of pennies for about $10.00

Winner's Curse

- friends all stare at the jar and try to guess how many pennies there are.
- Some under guess – they may guess that there are only 100 or 200 pennies. They bid low.
- Others over guess – they may guess that there are 1,000 pennies or more. They bid high.
- Of course those who overestimate the number of pennies by the most bid the highest – so you make out like a bandit.

Nash Equilibrium?

- According to Nash equilibrium this shouldn't happen
- Everyone should rationally realize that they will only win if they guess high
- they should bid less than their estimate of how many pennies there are in the jar
- they should bid a lot less – every player can guarantee they lose nothing by bidding nothing.
- in equilibrium, they can't on average lose anything, let alone $7.00.

QRE

- Recognize that there is small probability people aren't so rational
- Very different prediction
- some most possible profit anyone can make by getting the most number of pennies at zero cost: call this amount of utility U
- some least possible profit by getting a jar with no pennies at the highest possible bid: call this amount of utility u
- QRE says ratio of probability between two bids that give utility U, u is $\exp[\lambda(U - u)]$
- whatever is the difference in utility between two strategies it cannot be greater than that between U and u
- probability of highest possible bid is at least $p > 0$
- depends on how many bids are possible, not on how many bidders or their strategies

QRE with Many Bidders

- each bidder has at least a p probability of making the highest possible bid
- becomes a virtual certainty that one of the bidders will (unluckily for them) make this high bid
- with enough bidders, QRE assures the seller a nice profit.

Break Left? Or Right?

Matching Pennies

- each player has penny, and secretly places it heads up or heads down
- two pennies match – both heads or both tails – one player, the matching player, wins both
- two pennies do not match her opponent wins both pennies
- example of a zero sum game: one player’s gain is the other’s loss

The Final Problem

- Conan Doyle's "The Final Problem" written in 1893
- Holmes on a train bound for Dover and Moriarity pursuing Holmes on another train
- only stop is at Canterbury
- both get off at the same stop Moriarity catches Holmes (the "pennies" match) and Moriarity wins
- get off at different stops Holmes wins

How Smart Was Moriarity?

Conan Doyle not a good game theorist

in the story:

> Holmes reasons that Moriarity thinks he is going to Dover, so he gets off at Canterbury while Moriarity continues to Dover and loses the game

- why does not the supposedly brilliant mathematician Moriarity understand Holmes reasoning so get off at Canterbury himself?
- why does not Holmes anticipating this get off at Dover?

Mixed Strategies

- game has a unique Nash equilibrium
- requires that players choose randomly
- each has a 50% chance of getting off at Canterbury or Dover
- so each has a 50% chance of winning the game no matter what the other player does
- each can do no better than a 50% chance of winning

Do People Choose Randomly?

problem of evading capture does not occur only in novels.

- best selling book released by RAND Corporation: 1955 table of random numbers.
- Folklore has it that at least one captain of a nuclear submarine kept it by his bedside to use in plotting evasive maneuvers.

More familiar are sporting events

- soccer player kicking penalty must keep goal keeper in the dark about whether he will kick to the right or the left of the goal
- tennis player must be unpredictable as to which side of the court she will serve to,
- football quarterback must not allow defense to anticipate run or pass, or whether the play will move to the right or the left
- baseball catcher must keep batter uncertain as to how his pitcher will deliver the ball

How Do Athletes Do It?

- once in Japan catchers equipped with mechanical randomization devices to call the pitch
- later ruled unsporting and banned from play
- good tennis players in important matches do it right
- professional soccer players do it right

Laboratory Study: Holt and Goeree [2001]

variations of Matching Pennies

	50% (48%)	50% (52%)
50% (48%)	80,40	40,80
50% (52%)	40,80	80,40

Asymmetric Matching Pennies

increased(from 80 to 320) or decreased (from 80 to 44) tpayoff to Player 1 in the upper left corner

theory: changes Player 2’s equilibrium play, but Player 1 should continue to randomize 50-50.

	12.5% (16%)	87.5% (84%)
50% (96%)	320,40	40,80
50% (4%)	40,80	80,40

	87.5% (80%)	12.5% (20%)
50% (8%)	44,40	40,80
50% (92%)	40,80	80,40

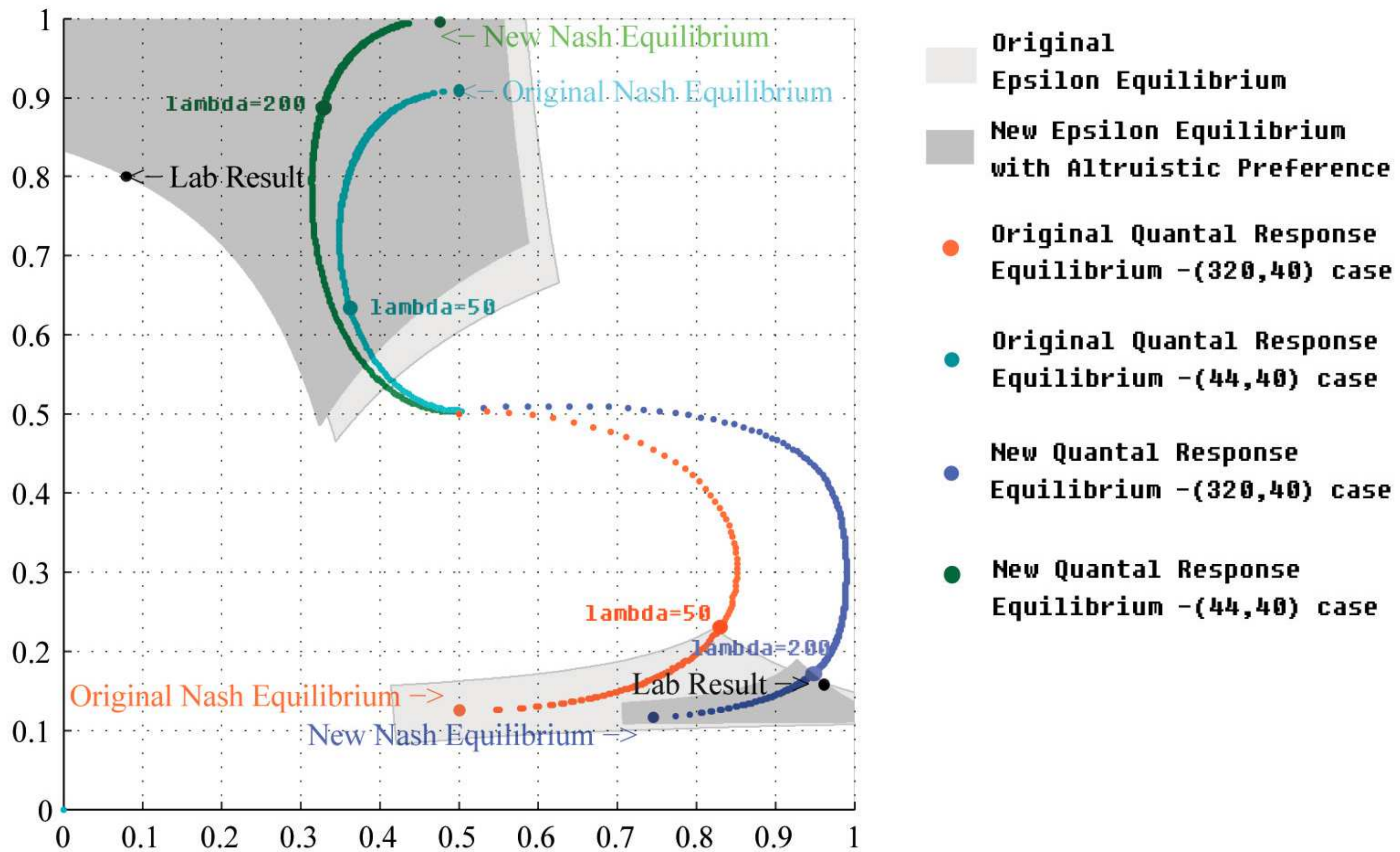
How Does the Theory Do?

- theory does about as badly as it can
- theory predicts equal probability between the two rows
- actuality is that one row is played pretty much all the time.
- unlike the other experiments this one involves players who are inexperienced in the sense that they only got to play the game once
- but can our tools help us understand what happened?

Analysis of Levine and Zheng [2010]

- vertical axis is the frequency with which Player 1 chooses the *Top* row;
- horizontal axis the frequency with which Player 2 chooses the *Left* column
- laboratory results shown by the black dots labeled *Lab Result*
- upper left dot corresponds to second matrix (44 game)
- lower right dot corresponds to first matrix (320 game)
- theoretical prediction of Nash equilibrium – that Player 1 (and only Player 1) randomizes 50-50 – are labeled as *Original Nash Equilibrium*

The Picture



<- New Nash Equilibrium
<- Original Nash Equilibrium
lambda=200
<- Lab Result
lambda=50
lambda=50
lambda=200
Lab Result ->
Original Nash Equilibrium ->
New Nash Equilibrium ->
Original Epsilon Equilibrium
New Epsilon Equilibrium with Altruistic Preference
Original Quantal Response Equilibrium -(320,40) case
Original Quantal Response Equilibrium -(44,40) case
New Quantal Response Equilibrium -(320,40) case
New Quantal Response Equilibrium -(44,40) case

What Happened?

- all approximate equilibrium with losses are no greater than those actually suffered: light gray shaded region
- QRE corresponding to differing levels of λ: light blue and red curves
- start at that begin at the respective Nash equilibria when $\lambda = \infty$
- move towards complete random 50-50 as $\lambda \to 0$
- dark gray region and green and dark blue curves are similar but when there is altruism

Analysis: 320 Game

- prediction of quantal response: tendency toward the middle
- in the 320 game Player 2 plays *Left* in Nash equilibrium 12.5% of the time
- we see that in actuality 16% rather than 12.5% of Player 2's play *Left*.
- substantial impact on the incentives of Player 1
- "too many" player 2's playing *Left* best thing for Player 1 is play *Top* and try to get 320
- this is what we see
- as λ decreases QRE play shifts towards to the right

Analysis: 44 Game

- “too many” player 2’s play *Right* – 20% rather than 12.5% –
- tilts the Player 1’s towards *Down*
- initial effect of decreasing λ is to move QRE towards the lab result
- when λ too small QRE approaches a pure 50-50 randomization.
- happens “too soon”: play in the QRE “starts back” towards 50-50 before it gets to the laboratory result
- much more pronounced in the 44 game than the 320 game

Altruism

- potentially important in the 320 game: Player 2 by giving up 40 can increase the payoff of Player 1 by 280
- don’t have to be that generous to take such an opportunity
- also can explain why “too many” Player 2’s play *Left*
- assume a combination of errors due to quantal response and some altruistic players
- can explain the 320 game quite well: curve combining the two effects passes more or less directly through the laboratory result

QRE and Altruism

- combining altruistic players with quantal response errors can explain only about half the laboratory result in the 44 game
- approximate equilibrium regions help us understand what is going on
- in 320 game the approximate equilibrium region wide but not tall
- many possible strategies by player 1 that consistent with a relatively small loss
- few strategies by player 2: Player 2 must play *Right* with between about 0% and 20% probability

QRE and Altruism in the 44 Game

- approximate equilibrium has little to say
- only that Player 1 should play *Top* more frequently than *Bottom* and Player 2 should play *Right* more frequently than *Left*
- in 320 game incentives are relatively strong: making a wrong choice can lose between 40 and 280
- 44 game making a wrong choices can lose between 4 and 40
- when incentives less strong the set of approximate equilibrium larger
- less able to make accurate predictions of how players will play

Finance Theory and Noise Traders

- central to theory of financial markets: extent to which “informationally efficient,”
- how well do they incorporate information available to investors about economic circumstances?
- If you cannot fool anybody ever the tiniest bit of information would typically be revealed nearly instantaneously
- Conundrum: nobody could profit from inside information so nobody would bother to acquire any in the first place

Abraham Lincoln's Theory

- you surely can fool some of the people some of the time
- in modern form in dissertation of Anat Admati in *Econometrica* in 1985
- picked up by Fischer Black: he avoided joining his co-author Myron Scholes on the stand to receive the Nobel prize in Economics by dying too soon

Black's description of noise traders: small but important irrational component of the market

- published in 1986 there have been 1328 follow-on papers
- ridiculous to assert as many commentators do that the central finding in modern finance theory is that markets are informationally efficient