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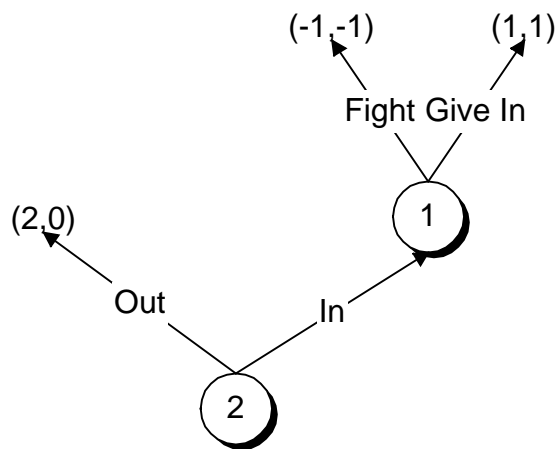
# Stackelberg Equilibrium

- precommitment

to be effective a precommitment must be

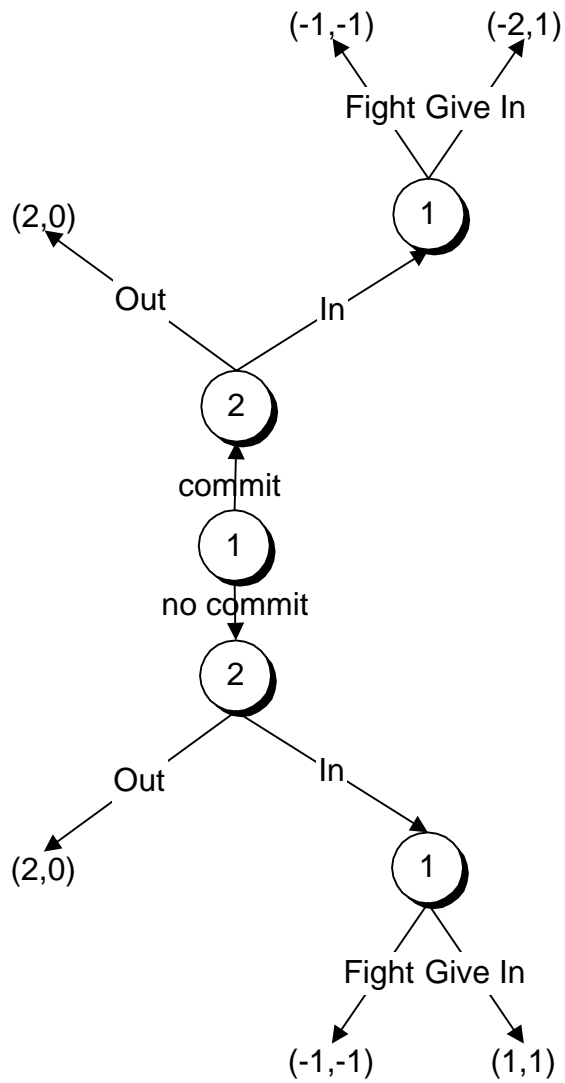
- public
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## The Chain Store Game



player 1 is the *Stackelberg leader*

gives rise to a new extensive form



## ***Stackelberg Leadership in Duopoly***

$$p = a - bx$$

$$a = 17, c = 1, b = 1$$

so that the competitive solution is 16 units of output, the monopoly solution is 8 units of output, the Cournot solution  $10 \frac{2}{3}$

$$\text{profits } \pi_i = [17 - (x_i + x_{-i})]x_i - x_i$$

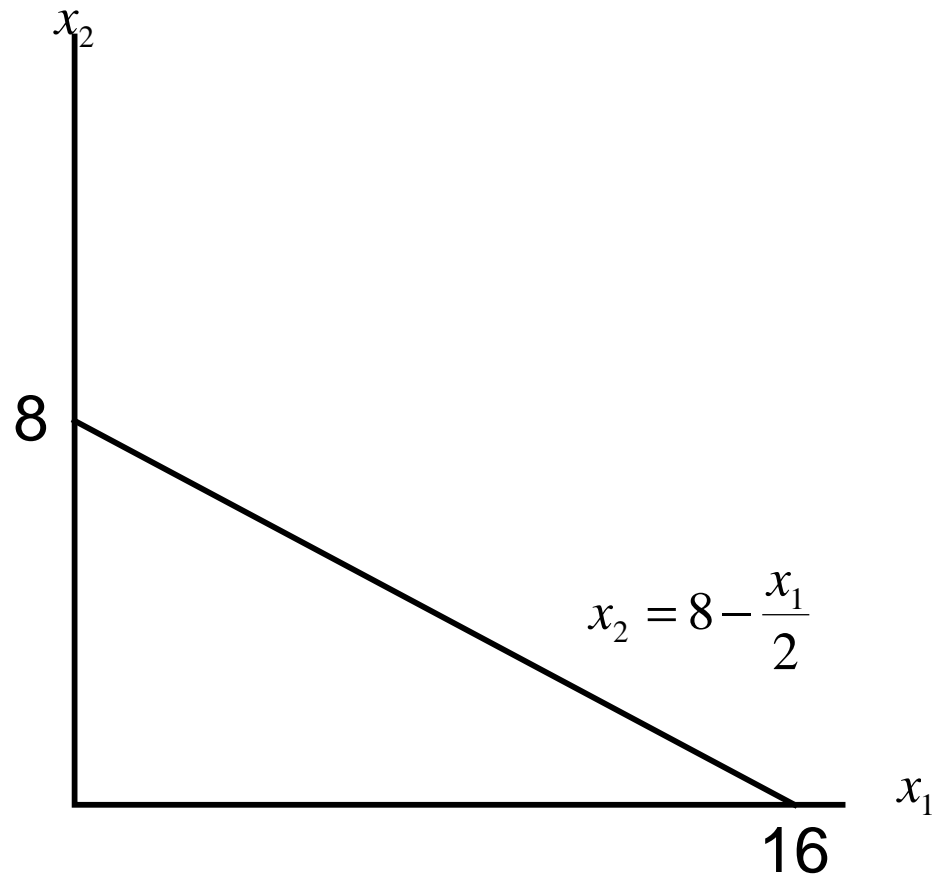
firm 1 is the Stackelberg leader

solve the game by *backwards induction*

- firm 1 precommits to producing  $x_1$  units of output
- what does firm 2 do?
- subgame perfection says that firm 2 must play a best response, or equivalently, must be on its reaction function.

recall that the best-response function for firm 2 is

$$x_2 = 8 - \frac{x_1}{2}$$





## ***Formal Solution of the Stackelberg Problem***

maximize

$$\pi_1 = [16 - (x_1 + x_2)]x_1$$

subject to

$$x_2 = 8 - \frac{x_1}{2}$$

may solve by Lagrange multipliers, or by direct substitution

$$\begin{aligned}\pi_1 &= \left[ 16 - \left( x_1 + 8 - \frac{x_1}{2} \right) \right] x_1 \\ &= \left[ 8 - \frac{x_1}{2} \right] x_1\end{aligned}$$

$$\frac{d\pi_1}{dx_1} = \left[ 8 - \frac{x_1}{2} \right] - \frac{x_1}{2} = 0$$

so that at the optimum  $x_1 = 8$  which is the same as the monopoly solution

## Summary of the Stackelberg Equilibrium

	Stackelberg	Cournot	Monopoly	Competition
$x_1$	8	5 1/3	8	8
$x_2$	4	5 1/3	0	8
$x$	12	10 2/3	8	16
$p$	5	6 1/3	9	1
$\pi_1$	32	28.4	64	0
$\pi_2$	16	28.4	0	0
$\pi$	48	56.9	64	0