Midterm Exam I
Answer Key

1) a) Budget constraint: $px + qy = I$

b) • Recognizing $u = x^{1/2}y^{1/2}$ is Cobb-Douglas utility;

$$x = \left(\frac{\alpha}{\alpha+\beta}\right)\frac{I}{p} \quad \text{and} \quad y = \left(\frac{\beta}{\alpha+\beta}\right)\frac{I}{q} \quad \text{where}$$

$$\alpha = \beta = 1/2 \Rightarrow x = \frac{I}{2p}, \; y = \frac{I}{2q}.$$

• Alternatively:

$$MRS = \frac{\partial u/\partial x}{\partial u/\partial y} = \frac{\left(\frac{y^{1/2}}{2x^{1/2}}\right)}{\left(\frac{x^{1/2}}{2y^{1/2}}\right)} = \frac{2y}{2x} = \frac{y}{x} = \underset{\substack{\uparrow \\ \text{slope of the budget line}}}{\frac{p}{q}}$$

$\Rightarrow qy = px$

substituting into the budget constraint

$\Rightarrow 2px = I \Rightarrow x = \frac{I}{2p}$ and similarly

$2qy = I \Rightarrow y = \frac{I}{2q}$

c) At the competitive equilibrium, price = marginal cost.

Using prices given by inverse demand, we have;

$p^* = \frac{I}{2x^*} = c \Rightarrow x^* = \frac{I}{2c}$ and

$q^* = \frac{I}{2y^*} = c \Rightarrow y^* = \frac{I}{2c}$

$p^* = q^* = c$

$x^* = y^* = \frac{I}{2c}$

d) Revenue = Price × Quantity

$$R = p \cdot x$$

We use inverse demand for p to get,

$$R = \frac{I}{2x} \cdot x = \frac{I}{2}$$

⇒ Revenue does not vary with output.

⇒ A profit maximizing monopolist with fixed revenue should strive to lower costs $(c \cdot x)$.
Therefore, the monopolist will decrease x as much as he can, without reaching $x=0$ consequently increasing prices to infinity.

2) a)

	L	R
U	6,6	-10,7*
D	7*,-10	-1*,-1*

- (D,R) is a dominant strategy equilibrium and a Nash equilibrium
- (D,R) is <u>not</u> Pareto efficient.

· Reduced forms

-

	R
U	-10, 7
D	-1, -1

⇒

	U
D	-1,-1

-

	L	R
D	7,-10	-1,-1

⇒

	U
D	-1,-1

b)

	L	R
U	5*,4*	-1,2
D	2,-5	2*,3*

- There are no dominant strategy equilibria.
- (U,L) and (D,R) are Nash equilibria.
- (U,L) is also Pareto efficient.
- The game can not be reduced.

c)

	L	C	R
U	2,2	3,1	4,5*
M	3,2	4,6*	5,3
D	4*,2	5*,-10	7*,3*

- There are no dominant strategy equilibria
- (D,R) is the only Nash Equilibrium.
- (D,R) is [redacted] Pareto efficient

- Reduced forms:

	C	R
U	3,1	4,5
M	4,6	5,3
D	5,-10	7,3

⇒

	C	R
D	5,-10	7,3

⇒

	R
D	7,3

or

	L	C	R
D	4,2	5,-10	7,3

⇒

	R
D	7,3

3) Let π_1 = profit of firm 1, π_2 = profit of firm 2

$x = x_1 + x_2$ where x_1 and x_2 are produced by firm 1 and 2 respectively.

$$\pi_1 = (p \cdot x_1) - (MC \cdot x_1) = [11 - 2(x_1 + x_2)]x_1 - x_1$$

$$= 10x_1 - 2x_1^2 + 2x_1x_2$$

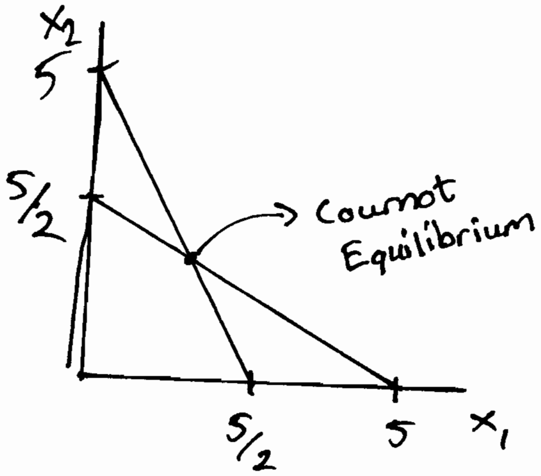
$$\pi_2 = 10x_2 - 2x_2^2 - 2x_1x_2$$

$$\partial\pi_1/\partial x_1 = 10 - 4x_1 - 2x_2 = 0$$

$$\Rightarrow x_1 = \frac{5 - x_2}{2}$$

$$\partial\pi_2/\partial x_2 = 10 - 4x_2 - 2x_1 = 0$$

$$\Rightarrow x_2 = \frac{5 - x_1}{2}$$



rearranging,

$$x_1 = 5 - 2x_2 \overset{\text{at equilibrium}}{=} \frac{5 - x_2}{2}$$

$$\Rightarrow x_2^* = 5/3$$

$$x_1^* = 5/3$$

$$\Rightarrow p^* = 11 - 2\left(\frac{5}{3} + \frac{5}{3}\right) = \frac{13}{3}$$