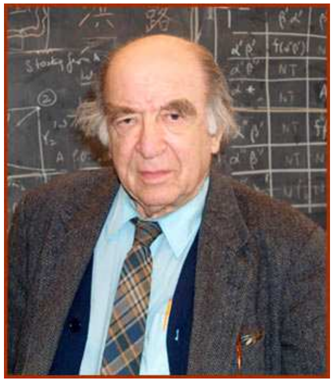
# Introductory Lecture

## *What this class is about*

- economic science as it exists today
- intrinsically a mathematical subject
- the heart of economics is mechanism design theory
- the goal of the class is to give you a limited working knowledge of mechanism design theory

## *The Monopoly Pricing Problem*

- The catering company Big Eats has the exclusive right to sell pizza on the campus of Big U.
- How much should it charge for each pizza?
- Each pizza will cost *c$* to produce and distribute.
- Market research indicates that the number of units that will be sold $x$ depends upon the price $p$ according to the relation $x = d(p)$, where a higher price results in fewer sales.
- This is the simplest example of a mechanism design problem: here the choice is between different prices that can be charged. Deeper analysis would consider more elaborate pricing schemes: auction the pizzas to the highest bidder, allocate the pizzas by means of a contest and so forth.
- Illustrates the interplay between an economic problem (what should we do with the pizzas?) and mathematical methods.

## *Solution to the Problem of Monopoly*

$p$ is price, $x$ is output, $c$ is unit cost

profit $\pi = px - cx$; this is what Big Eats cares about

demand $x = d(p)$ or inverse demand $p = f(x)$

profit again $\pi = f(x)x - cx$

for a maximum: marginal profit equals zero

$$\frac{d\pi}{dx} = f'(x)x + f(x) - c = 0,\ f(x)\left[\frac{f'(x)x}{f(x)} + 1\right] = c$$

$\eta \equiv \dfrac{d\log x}{d\log p} = \dfrac{d\log x}{d\log f(x)} = \dfrac{1/x}{f'(x)/f(x)} = \dfrac{f(x)}{f'(x)x}$ the price elasticity of demand

$p\left[\dfrac{1}{\eta} + 1\right] = c$ or $p - c = -p/\eta$

## *Discussion of the Solution*

$p - c = -p/\eta$

$\eta$ is negative so the markup $p - c$ is positive

- monopoly vs. "competition": the more "elastic" is output [large absolute $\eta$] with respect to price the smaller the markup
- competition: raise price a tiny amount lose entire market: infinite elasticity
- the more "inelastic" is output [small absolute $\eta$] with respect to price, the bigger the markup: monopolists like inelasticity, you can increase your price a lot without having much effect on your sales
- game theoretic perspective: we are taking into account how "other players" respond to our "strategy": the more we charge, the less the "other players" are willing to pay

## *An Example with Linear Demand*

$p = a - bx$

monopoly

$\pi = (a - bx)x - cx = (a - c)x - bx^2$

$$\frac{d\pi}{dx} = (a - c) - 2bx = 0$$

$x = \dfrac{a - c}{2b}$ the monopoly output

competitive equilibrium

$p = c$

$a - bx = c$

$x = \dfrac{a - c}{b}$ twice the monopoly output

## *Graphical Analysis*

revenue = $px = f(x)x$

marginal revenue = $MR = \frac{d}{dx}\text{revenue}$

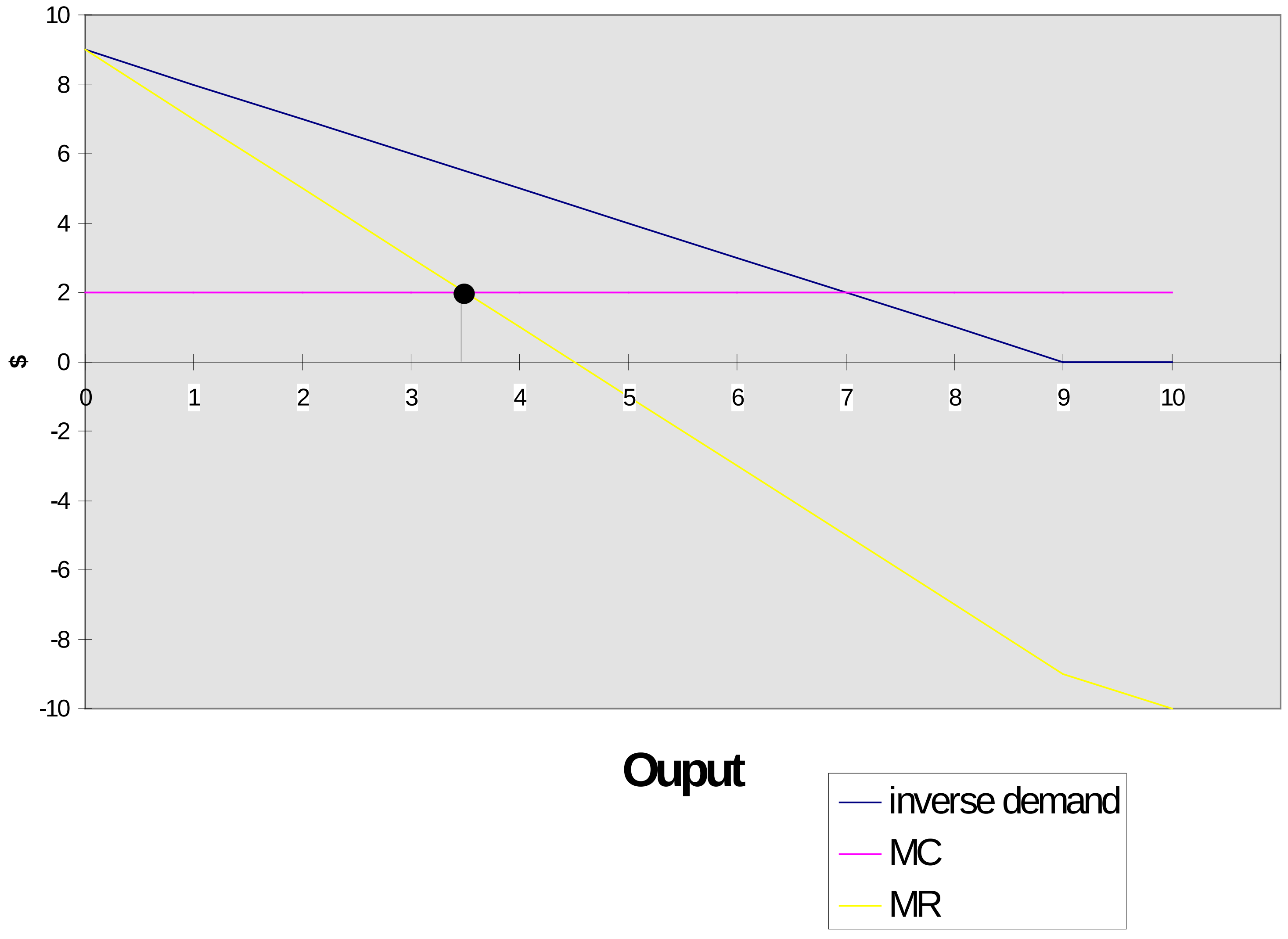
cost = $cx$

marginal cost= $MC = \frac{d}{dx}\text{cost} = c$

$f'(x)x + f(x) = c$ or $MR = MC$

take a=9, b=1, c=2

# Optimum of the Monopolist



## *Returns to Scale*

$\text{total cost} = cx + dx^2 \,/\, 2$

$\text{average} = c + dx \,/\, 2$

$\text{marginal} = c + dx$

- if $d = 0$ constant returns to scale
- if $d > 0$ decreasing returns to scale
- if $d < 0$ increasing returns to scale

## *Example Revisited*

$$p = a - bx$$

monopoly

$$\pi = (a - bx)x - cx - dx^2 / 2$$

$$= (a - c)x - (b + d / 2)x^2$$

$$\frac{d\pi}{dx} = (a - c) - 2(b + d / 2)x = 0$$

$$x = \frac{a - c}{2b + d}$$

competitive equilibrium

$$a - bx = c + dx$$

$$x = \frac{a-c}{b+d}$$

- when $d > 0$ (decreasing returns to scale) monopolist produces more than ½ competition
- when $d < 0$ competitor earns negative profit

$$\text{average} = c + dx/2$$

$$\text{marginal} = c + dx$$

when $d < 0$

average cost > marginal cost

so price = marginal cost < average cost

means you lose money on each unit you sell