# Mixed Strategy Equilibria

## *Matching Pennies*

| | H | T |
|---|---|---|
| H | 1*,-1 | -1,1* |
| T | -1,1* | 1*,-1 |

- This game does not have a Nash equilibrium: each player wants to do the opposite of the other
- Suppose instead of choosing H or T for sure, each player flips a coin to determine what to do

Call H, T *pure* strategies

A *mixed* strategy is a probability distribution over pure strategies

## *Solving the Matching Pennies Game*

$p_1$ probability that 1 chooses H

$p_2$ probability that 2 chooses H

$$u_1(p_1, p_2) =$$
$$p_1 p_2 + (1 - p_1)(1 - p_2) - (1 - p_1)p_2 - p_1(1 - p_2)$$

$$u_2(p_1, p_2) =$$
$$-p_1 p_2 - (1 - p_1)(1 - p_2) + (1 - p_1)p_2 + p_1(1 - p_2)$$

reaction function of 1:

if $p_2 < 1/2$ then $p_1 = 0$

if $p_2 > 1/2$ then $p_1 = 1$

if $p_2 = 1/2$ then indifferent

reaction function of 2:

if $p_1 < 1/2$ then $p_2 = 1$

if $p_1 > 1/2$ then $p_2 = 0$

if $p_1 = 1/2$ then indifferent

if $p_1 = p_2 = 1/2$ then both players are indifferent

this is a *mixed strategy Nash equilibrium*

## *Remarks*

- Not easy to give a recipe for finding mixed Nash equilibria
- To mix a player must be indifferent, this is the usual method of solving: find the strategies for player 2 that makes player 1 indifferent and vice versa
- Every finite game has a  mixed Nash equilibrium

## *Coordination Game*

| | L | R |
|---|---|---|
| U | 1*,1* | 0,0 |
| D | 0,0 | 1*,1* |

Two pure equilibria, but also a mixed equilibrium where both players play 50-50.

- Interpretation of mixed equilibrium in terms of uncertainty

## *Battle of the Sexes*

| | L | R |
|---|---|---|
| U | 2*,1* | 0,0 |
| D | 0,0 | 1*,2* |

Two pure equilibria.   Is there a mixed equilibrium?

Player 1's utility from playing U $2p_2$

Player 1's utility from playing D $1 - p_2$

Player 1's indifference $2p_2 = 1 - p_2$

solve to find $p_2 = 1/3$

Similarly we can solve for player 2's indifference and find $p_1 = 2/3$

So each player puts more weight on the strategy he likes best

Probability of U,L is 2/9, of D,R is 2/9

Probability of U,R is 4/9, of D,L is 1/9

## *Kitty Genovese Problem*

Description of the problem

Model of the problem

*n* people all identical

benefit is someone calls the police is *x*

cost of calling the police is 1

Assumption: $x > 1$

Look for symmetric mixed strategy equilibrium where *p* is probability of each person calling the police *solution*

p is the symmetric equilibrium probability for each player to call the police

each player *i* must be indifferent between calling the police or not

if *i* calls the police, gets *x*-1 for sure.

If *i* doesn't, gets 0 with probability $(1-p)^{n-1}$, gets *x* with probability $1-(1-p)^{n-1}$

so indifference when

$$x-1 = x\left(1-(1-p)^{n-1}\right)$$

solve for *p*

$$p = 1 - (1/x)^{1/(n-1)}$$

probability police is called

$$1 - (1-p)^n = 1 - \left(\frac{1}{x}\right)^{\frac{n}{n-1}}$$

$$1 - (1-p)^n = 1 - (1/x)^{n/(n-1))}$$

x=10