1.(a) The budget constraint is: $px + qy \le I$. It does not matter if you use equality.

(b) The consumer's problem:

$$\max_{\{x,y\}} \quad u(x,y) = x^{425}y^{425}$$
$$s.t. \quad px + qy \le I$$

To solve it, let

$$L(x,y,\lambda) = x^{425}y^{425} + \lambda(I - px - qy)$$

From which, derive the FOCs:

$$\frac{\partial L}{\partial x} = 0 \quad \Rightarrow \quad 425x^{424}y^{425} = \lambda p$$
$$\frac{\partial L}{\partial y} = 0 \quad \Rightarrow \quad 425x^{425}y^{424} = \lambda q$$
$$\frac{\partial L}{\partial \lambda} = 0 \quad \Rightarrow \quad px + qy = I$$

Simplify the FOCs to get two equations:

$$\frac{y}{x} = \frac{p}{q}$$
$$px + qy = I$$

From which we can solve:

$$x(p,q,I) = \frac{I}{2p}$$
$$y(p,q,I) = \frac{I}{2q}$$

(c) In the competitive equilibrium, the price must be the same as the marginal cost c, so $p = c$; plug into the demand of *x*, we have the output for *x*: $x(p,q,I) = \frac{I}{2c}$.

(d) The monopolist's revenue is $R = px = p \cdot \frac{I}{2p} = \frac{I}{2}$, which is independent of *p* or *x*, implies that whatever the price is, the revenue is constant; on the other hand, the monopolist's production cost is *cx*, a strictly increasing function of output *x*. So, to maximize the profit, the monopolist should produce *x* as less as possible, given *x* should be positive.

2.(a)

	L	C	R
U	**4.0**, **4.0**	5.4, 3.6	**1.2**, 0.0
M	3.6, 5.4	5.0, 5.0	-4.0, **10.0**
D	0.0, **1.2**	**10.0**, -4.0	1.0, 1.0

In this game, no player has strategy that dominates the other two strategies, so there is no dominant strategy equilibrium.

M is strictly dominated by U; after eliminating M, both C and R are dominated by L; after eliminating C and R, D is dominated by U. So the game after eliminating of strictly dominated strategies is

	L
U	4.0,4.0

The reaction functions are shown in the original game by lower bar.

There is only one pure strategy Nash equilibrium (U,L). It is not Pareto efficient, because the outcome of (M,C) is better than it for each player.

(b)

	L	C	R
U	4.00, 4.00	4.8, **4.20**	0.80, 0.40
M	**4.20**, 4.80	5.00, 5.00	0.67, **5.33**
D	0.40, 0.80	**5.33**, 0.67	**1.00**, **1.00**

There is no strategy strictly dominated by other strategies, so iterated eliminating of strictly dominated strategy does not work in this game; also, there is no dominant strategy equilibrium.

The reaction functions are shown in the game by lower bar.

(D,R) is the only pure strategy Nash equilibrium. It is not Pareto efficient, because the outcomes of (U,L), (U,C), (M,L), (M,C) are better than the outcome of (D,R) for each player.

3 The firm 1's profit function has the form:

$$\pi_1 = px_1 - c_1x_1$$

Plug the demand function $p = 17 - x_1 - x_2$ and marginal cost $c_1 = 3$ into that, we have

$$\pi_1 = (17 - x_1 - x_2)x_1 - 3x_1$$

Derive the FOC, we have

$\frac{\partial \pi_1}{\partial x_1} = 0 \quad \Rightarrow \quad x_1 = 7 - \frac{x_2}{2}$. This is the reaction function of firm 1.

Similarly, the firm 2's profit function is:

$$\pi_2 = px_2 - c_2x_2$$

Given $p = 17 - x_1 - x_2$ and $c_2 = 1$, we have

$$\pi_2 = (17 - x_1 - x_2)x_2 - x_2$$

Derive the FOC, we get the reaction function:

$$\frac{\partial \pi_2}{\partial x_2} = 0 \quad \Rightarrow \quad x_2 = 8 - \frac{x_1}{2}$$

Thus we can solve

$$\begin{cases} x_1 = 4 \\ x_2 = 6 \end{cases}$$