# EXCESS DEMAND AND GENERAL EQUILIBRIUM

**July 1, 1996**

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Our model economy has *n* traders and *m* goods. Denote by $x_j^i$ the consumption by trader *i* of good *j*. We also let $x^i$ denote the vector or bundle of goods consumed by trader *j*. Trader *i*'s preferences for consuming different goods are given by her utility function $u^i(x^i)$. Initially trader *i* is endowed with $\bar{x}_j^i$ of good *j*.

There is no production in this economy and it lasts only one period. Traders simply exchange goods with each other. We presume "the law of one price," that is, traders scope out opportunities to the extent that each good is sold (and purchased) at only one price. Denote by $p_j$ the price of good *j*, and let *p* be the list of all prices of all goods, or the **price vector**. We also presume competitive behavior, that is, traders do not perceive that they can have any influence over these market prices. Our theory of the result of trading in this economy is that it will result in a **competitive equilibrium**. Competitive equilibrium prices, which we denote by $\hat{p}$ to distinguish them from arbitrary prices, are (by definition) prices at which every trader can simultaneously satisfy her desire to trade at those prices.

Mathematically this definition of competitive equilibrium prices may be formulated in terms of excess demand. Denote by $x_j^i(p,m)$ the demand by trader *i* for good *j* when prices are *p* and money income is *m*. In the pure exchange economy money income is generated by selling off the endowment. (Remember, it doesn't cost anything extra to sell your endowment then buy it back, since the prices at which you buy and sell are the same.) In addition it is convenient (but not necessary) to deal with the demand to buy rather than *x* which is the demand to consume. This leads us to define excess demand as

$$z_j^i(p,\bar{x}^i) \equiv x_j^i(p,\sum_{j=1}^m p_j \cdot \bar{x}_j^i) - \bar{x}_j^i.$$

For example, in the case of a Cobb-Douglas utility function for two goods we have

$$\begin{aligned} x_1^1(p_1,p_2,m) &= \alpha m / p_1 \\ z_1^1(p_1,p_2) &= \alpha(p_1\bar{x}_1^1 + p_2\bar{x}_2^1)/p_1 - \bar{x}_1^1 \\ &= \alpha\frac{p_1}{p_2}\bar{x}_2^1 + (1-\alpha)\bar{x}_1^1. \end{aligned}$$

This covers individual excess demand, that is, how much each trader wants to buy as a function of prices. However, we are concerned with market excess demand: that is, the total amount that all consumers want to buy. Market (or aggregate) excess demand is simply the sum of all the individual excess demands:

$$z_j(p) \equiv \sum_{i=1}^{n} z_j^i(p).$$

How is competitive equilibrium defined in terms of excess demand? Excess demand represents the result of consumer optimization. At the equilibrium prices it must be possible for all consumers to optimize at the same time. This means that in the market for each good demand to buy cannot exceed zero, since there is no production or outside agent to provide supply to the market. In other words, one traders excess demand must be anothers excess supply. Mathematically, if $\hat{p}$ are competitive equilibrium prices
$z_j(\hat{p}) \leq 0$ for every good $j = 1, 2, \ldots, m$.

## Properties of Excess Demand

To use excess demand to study competitive equilibrium, we must begin by understanding its properties. Individual demand has two key properties: it is **homogeneous of degree zero** and it satisfies the budget constraint (**Walras's law**). These properties are inherited by individual excess demand functions, so that for each individual *i*

$z_j^i(\lambda \cdot p) = z_j^i(p)$ (homogeneity of degree zero )

$\sum_{j=1}^{m} p_j \cdot z_j^i(p) = 0$ (Walras's law ).

Because aggregate excess demand is simply the sum of individual excess demands aggregate excess demand must have the same two properties.

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## Implications for Theory of Competitive Equilibrium

Homogeneity and Walras's law have important implications for the theory of competitive equilibrium. First, consider the fact that according to our definition of competitive equilibrium, excess demand can actually be negative at equilibrium, that is, demand may be less than supply. However, according to Walras's law, the aggregate value of excess demand must be zero. Writing out the sum at equilibrium prices $\hat{p}$

$$\hat{p}_1 z_1(\hat{p}) + \hat{p}_2 z_2(\hat{p}) + \ldots + \hat{p}_m z_m(\hat{p}) = 0.$$

However, in equilibrium each term $\hat{p}_j z_j(\hat{p})$ must be greater than or equal zero. It follows this and Walras's law that each term must actually equal zero: $\hat{p}_j z_j(\hat{p}) = 0$. This leaves two possibilities: in market *j* either supply **equals** demand $(z_j(\hat{p}) = 0)$, or the price $\hat{p}_j = 0$. The latter is not typically the case, although it is possible if individuals have saturated (or satiated) preferences. Suppose that good *j* is air. There is a limit to how much air you would like to consume (breath), and indeed if you were forced to consume too much (forced pumping?) you would probably be quite unhappy. Moreover there more than enough air for everyone to breath as much as they want (leaving aside issues such as pollution). In this "market" demand is less than supply and the price is zero (no one has to pay for the air they breathe).

Taking the case where prices are not zero, we can calculate competitive equilibrium prices by solving the system of *m* equations in *m* unknowns:

$$z_j(\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_m) = 0 \quad \text{for} \quad j = 1, 2, \ldots, m\ .$$

However, since $z_j(p)$ is homogeneous of degree zero, if $\hat{p}$ is an equilibrium price vector, then so is $\lambda\hat{p}$ for any number $\lambda > 0$. Another way to say this is that the absolute value of prices does not matter, only the ratios between prices matter. This means that we may arbitrarily choose one good to be numeraire and set its price to one. This leaves $m - 1$ prices to solve for using the *m* different market clearing conditions. Fortunately one of the equations is redundant: If excess demand is zero for $j = 1, 2, \ldots, m - 1$, then from Walras's law $\hat{p}_m z_m(\hat{p}) = 0$. Provided that $\hat{p}_m \neq 0$ it must be that $z_m(\hat{p}) = 0$. In words, if $m - 1$ markets clear, then the *m*th market must clear as well. This means that we can solve **any** $m - 1$ of the equations to find the $m - 1$ prices.