Long Run versus Short Run Player

a fixed simultaneous move stage game

Player 1 is long-run with discount factor δ

actions $a^1 \in A^1$ a finite set

utility $u^1(a^1, a^2)$

Player 2 is short-run with discount factor 0

actions $a^2 \in A^2$ a finite set

utility $u^2(a^1, a^2)$

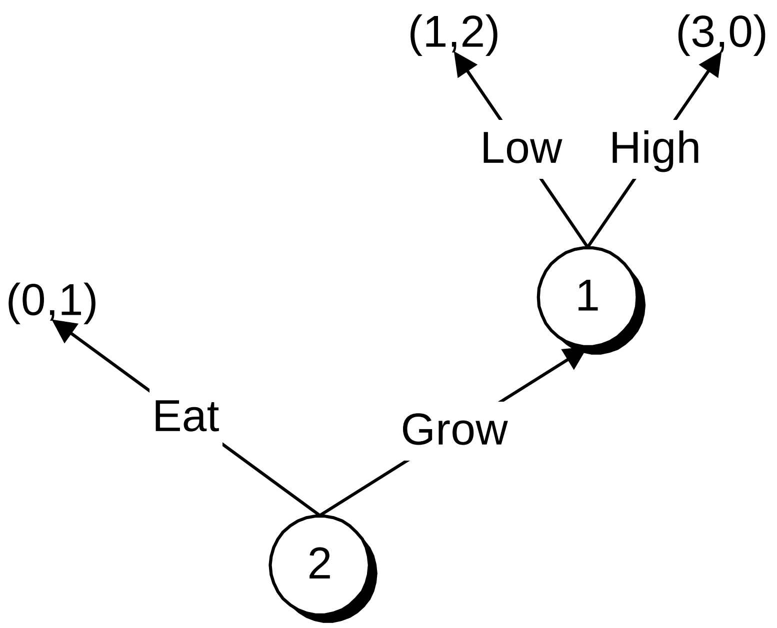
the "short-run" player may be viewed as a kind of "representative" of many "small" long-run players

♦ the “usual” case in macroeconomic/political economy models

♦ the “long run” player is the government

♦ the “short-run” player is a representative individual

Example 1: Peasant-Dictator



Example 2: Backus-Driffil

	Low	High
Low	0,0	-2,-1
High	1,-1	-1,0

Inflation Game: LR=government, SR=consumers

consumer preferences are whether or not they guess right

	Low	High
Low	0,0	0,-1
High	-1,-1	-1,0

with a hard-nosed government

Repeated Game

history $h_t = (a_1, a_2, \ldots, a_t)$

null history h_0

behavior strategies $\alpha_t^i = \sigma^i(h_{t-1})$

long run player preferences

average discounted utility

$(1-\delta)\sum_{t=1}^{T} \delta^{t-1} u^i(a_t)$

note that average present value of 1 unit of utility per period is 1

Equilibrium

Nash equilibrium: usual definition – cannot gain by deviating

Subgame perfect equilibrium: usual definition, Nash after each history

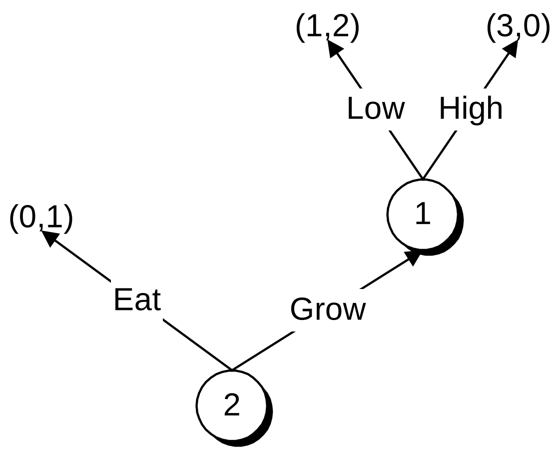
Observation: the repeated static equilibrium of the stage game is a subgame perfect equilibrium of the finitely or infinitely repeated game

♦ strategies: play the static equilibrium strategy no matter what *“perfect equilibrium with public randomization”*

may use a public randomization device at the beginning of each period to pick an equilibrium

key implication: set of equilibrium payoffs is convex

Example: Peasant-Dictator



normal form: unique Nash equilibrium **high, eat**

	eat	grow
low	0*,1	1,2*
high	0*,1*	3*,0

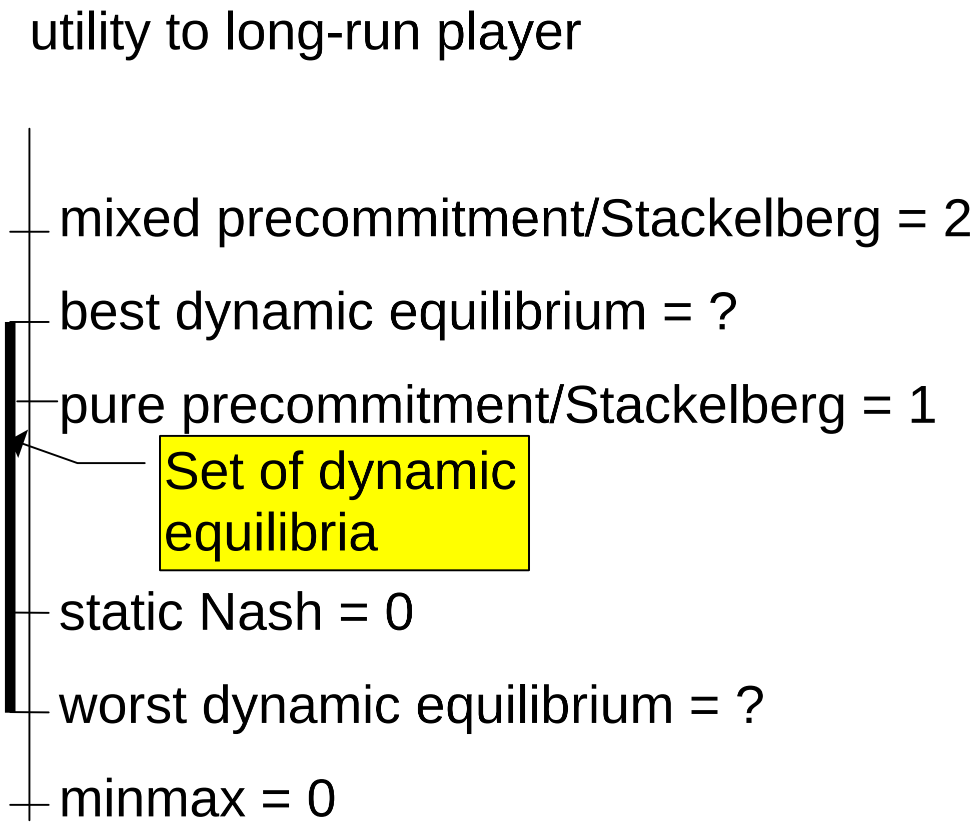
payoff at static Nash equilibrium to LR player: 0

precommitment or Stackelberg equilibrium

precommit to low get 1

mixed precommitment to 50-50 get 2

minmax payoff to LR player: 0


utility to long-run player
mixed precommitment/Stackelberg = 2
best dynamic equilibrium = ?
pure precommitment/Stackelberg = 1
Set of dynamic equilibria
static Nash = 0
worst dynamic equilibrium = ?
minmax = 0

Repeated Peasant-Dictator

finitely repeated game

final period: high, eat, so same in every period

Do you believe this??

Infinitely repeated game

begin by low, grow

if low, grow has been played in every previous period then play low, grow

otherwise play high, eat (reversion to static Nash)

claim: this is subgame perfect

clearly a Nash equilibrium following a history with high or eat

SR play is clearly optimal

for LR player

may high and get $(1-\delta)3+\delta 0$

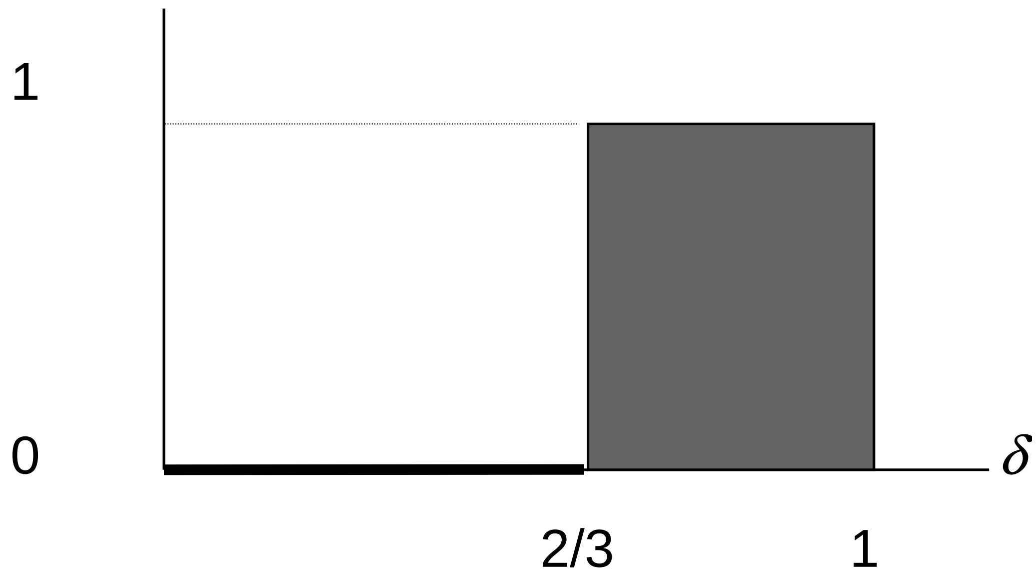
or low and get 1

so condition for subgame perfection

$$(1-\delta)3 \leq 1$$

$$\delta \geq 2/3$$

equilibrium utility for LR



General Deterministic Case *(Fudenberg, Kreps and Maskin)*

- max $u^1(a)$
- mixed precommitment/Stackelberg
- $\bar{v}^1$ best dynamic equilibrium
- pure precommitment/Stackelberg

Set of dynamic equilibria

- static Nash
- $\underline{v}^1$ worst dynamic equilibrium
- minmax
- min $u^1(a)$

Characterization of Equilibrium Payoff

$\alpha = (\alpha^1, \alpha^2)$ where α^2 is a b.r. to α^1

α represent play in the first period of the equilibrium

$w^1(a^1)$ represents the equilibrium payoff beginning in the next period

$$v^1 \geq (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1)$$

$$v^1 = (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1), \alpha^1(a^1) > 0$$

$$\underline{v}^1 \leq w^1(a^1) \leq \overline{v}^1$$

strategy: impose stronger constraint using n static Nash payoff

for best equilibrium $n \leq w^1(a^1) \leq \overline{v}^1$

for worst equilibrium $\underline{v}^1 \leq w^1(a^1) \leq n$

avoids problem of best depending on worst remark: if we have static Nash = minmax then no computation is needed for the worst, and the best calculation is exact.

max problem

fix $\alpha = (\alpha^1, \alpha^2)$ where α^2 is a b.r. to α^1

$$\bar{v}^1 \geq (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1)$$
$$\bar{v}^1 = (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1), \alpha^1(a^1) > 0$$
$$n^1 \leq w^1(a^1) \leq \bar{v}^1$$

how big can $w^1(a^1)$ be in = case?

Biggest when $u^1(a^1, \alpha^1)$ is smallest, in which case

$$w^1(a^1) = \bar{v}^1$$

$$\bar{v}^1 = (1-\delta)u^1(a^1, \alpha^2) + \delta \bar{v}^1$$

conclusion for fixed α

$$\min_{a^1|\alpha(a^1)>0} u^1(a^1,\alpha^2)$$

i.e. worst in support

$$\bar{v}^1 = \max_{\alpha^2 \in BR^2(\alpha^1)} \min_{a^1|\alpha(a^1)>0} u^1(a^1,\alpha^2)$$

observe:

mixed precommitment $\geq \bar{v}^1 \geq$ pure precommitment

Peasant-Dictator Example

	eat	grow
low	0*,1	1,2*
high	0*,1*	3*,0

p(low)	BR	worst in support
1	grow	1
$\frac{1}{2}<p<1$	grow	1
p=1/2	any mixture	≤ 1 (low)
$0<p<\frac{1}{2}$	eat	0
p=0	eat	0

check: $w^1(a^1) = \dfrac{\bar{v}^1 - (1-\delta)u^1(a^1,\alpha^2)}{\delta} \geq n^1$

as $\delta \to 1$ then $w^1(a^1) \to \bar{v}^1 \geq n^1$

min problem

fix $\alpha = (\alpha^1, \alpha^2)$ where α^2 is a b.r. to α^1

$$\underline{v}^1 \geq (1-\delta)u^1(a^1, \alpha^2) + \delta w^1(a^1)$$
$$\underline{v}^1 \leq w^1(a^1) \leq n^1$$

Biggest $u^1(a^1, \alpha^1)$ must have smallest $w^1(a^1) = \underline{v}^1$

$$\underline{v}^1 = (1-\delta)u^1(a^1, \alpha^2) + \delta \underline{v}^1$$

conclusion

$$\underline{v}^1 = \max u^1(a^1, \alpha^2)$$

or

$$\underline{v}^1 = \min_{\alpha^2 \in BR^2(\alpha^1)} \max u^1(a^1, \alpha^2)$$

that is, constrained minmax

Example

	L	M	R
U	0,-3	1,2	0,3
D	0,3*	2,2	0,0

static Nash gives 0

minmax gives 0

worst payoff in fact is 0

pure precommitment also 0

mixed precommitment

p is probability of up

to get more than 0 must get SR to play M

$-3p+(1-p)3 \le 2$ and $3p \le 2$

first one

$-3p+(1-p)3 \le 2$

$-3p-3p \le -1$

$p \ge 1/6$

second one

$3p \leq 2$

$p \leq 2/3$

want to play D so take $p = 1/6$

get $1/6 + 10/6 = 11/6$

utility to long-run player

max $u^1(a)$=2

mixed precommitment/Stackelberg=11/16

$\bar{v}^1$ best dynamic equilibrium=1

pure precommitment/Stackelberg=0

Set of dynamic equilibria

static Nash=0

$\underline{v}^1$ worst dynamic equilibrium=0

minmax=0

min $u^1(a)$=0

calculation of best dynamic equilibrium payoff

p is probability of up

p	BR^2	worst in support
<1/6	L	0
1/6<*p*<5/6	M	1
p>5/6	R	0

so best dynamic payoff is 1

Moral Hazard

choose $a^i \in A$

observe $y \in Y$

$\rho(y|a)$ probability of outcome given action profile

private history: $h^i = (a_1^i, a_2^i, \ldots)$

public history: $h = (y_1, y_2, \ldots)$

strategy $\sigma^i(h^i, h) \in \Delta(A^i)$

"public strategies" , *perfect public equilibrium*

Moral Hazard Example

mechanism design problem

each player is endowed with one unit of income

players independently draw marginal utilities of income $\eta \in \{\overline{\eta}, \underline{\eta}\}$

player 2 (SR) has observed marginal utility of income

player 1 (LR) has unobserved marginal utility of income

player 2 decides whether or not to participate in an insurance scheme player 1 must either announce his true marginal utility or he may announce $\overline{\eta}$ independent of his true marginal utility

non-participation: both players get $\gamma = \dfrac{\overline{\eta} + \underline{\eta}}{2}$

participation: the player with the higher marginal utility of income gets both units of income normal form

	non-participation	participate
truth	γ, γ	$\frac{\overline{\eta}+\gamma}{2}, \frac{\overline{\eta}+\gamma}{2}$
lie	γ, γ	$\frac{3\gamma}{2}, \frac{\overline{\eta}}{2}$

$p^* = \frac{\underline{\eta}}{\gamma}$ makes player 2 indifferent max $u^1(a)=\frac{3\gamma}{2}$

mixed precommitment/Stackelberg$=\frac{\overline{\eta}+\gamma}{2}+(1-\frac{\underline{\eta}}{\gamma})\frac{\underline{\eta}}{2}$

$\overline{v}^1$ best dynamic equilibrium$=\frac{\overline{\eta}+\gamma}{2}$

pure precommitment/Stackelberg$=\frac{\overline{\eta}+\gamma}{2}$

Set of dynamic equilibria

static Nash$=\gamma$

$\underline{v}^1$ worst dynamic equilibrium$=\gamma$

min $u^1(a)=\gamma$, minmax$=\gamma$

moral hazard case

player 1 plays "truth" with probability p^* or greater

player 2 plays "participate"

$$\bar{v} = (1-\delta)\frac{\bar{\eta}+\gamma}{2} + \delta\left(\frac{1}{2}w(\underline{\eta}) + \frac{1}{2}w(\bar{\eta})\right)$$

$$\bar{v} \geq (1-\delta)\frac{3\gamma}{2} + \delta w(\bar{\eta})$$

$$\bar{v} \geq w(\underline{\eta}), w(\bar{\eta})$$

$w(\bar{\eta})$ must be as large as possible, so inequality must bind; $w(\underline{\eta}) = \bar{v}$

$$\bar{v} = (1-\delta)\frac{3\gamma}{2} + \delta w(\bar{\eta})$$

solve two equations

$$\bar{v} = \bar{\eta} - \frac{\gamma}{2}$$

$$w(\bar{\eta}) = \frac{\bar{v} - (1-\delta)3\gamma / 2}{\delta}$$

check that $w(\overline{\eta}) \geq \gamma$

leads to $\delta \geq 2\left(2 - \frac{\overline{\eta}}{\gamma}\right)$

from $\delta < 1$ this implies

$\overline{\eta} > 3\underline{\eta}$