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Economics 201B - Practice Final Exam

You should do three of the four questions. You have three hours. Good luck.

1. Be nice

Player 1 can be nice or mean; player 2 can opt in or out. If player 2 opts out he gets 1 and player 1 gets 0 for being nice. If player 2 opts in he gets 2 if player 1 is nice and 0 if player 1 is mean; player 1 gets 2 for being nice. Conditional on the strategy of player 2, player 1 always gets an additional payoff of 1 if he is mean instead of being nice. This is a simultaneous move game.

a. Find the extensive and normal form of this game.
b. Find the subgame perfect and Nash equilibrium of this game.
c. Are there any dominated strategies?
d. Find the pure and mixed Stackelberg equilibrium in which player 1 moves first.
e. Find the minmax for both players.

Now suppose that the game is infinitely repeated

f. Player 1 is a long-run player with discount factor δ; player 2 is a short-run player with discount factor 0. Find the set of perfect public equilibrium payoffs to the long-run player as a function of her discount factor.
g. Find strategies that support the best equilibrium from part f.
h. Player 1 and 2 are both long-run players with common discount factor δ. When δ is close to one describe the set of perfect equilibrium payoffs to both players.
i. Find a discount factor and strategies for part h such that both players receive an equilibrium payoff of 2.

2. Long Run Consumers

A short-run firm has the option of giving a single indivisible item to a long-run consumer. The consumer has the option of paying for the item or not. If the consumer pays, there is a 50% chance that the check gets lost in the mail. (Note: if the check is lost, the payment is not received by the firm, *and* the consumer is not charged for the item.) The consumer values the item at \$5.00, and the firm values the item at \$1.00. The payment is \$4.00, and both parties are risk neutral. In all that follows, <u>equilibrium</u> means perfect public equilibrium of the infinitely repeated game with public randomization.

a. Find the extensive and normal forms of the stage-game.

b. For the long-run player, find the minmax, the static Nash, maxmax, mixed precommitment and pure precommitment payoffs.

c. Find the worst equilibrium for the long-run player, and describe in general terms the set of equilibrium payoffs for the long-run player.

First assume that the firm can observe whether or not the check is lost in the mail.

d. Find the best equilibrium for the consumer as a function of the discount factor. Now assume that the firm cannot observe whether the check is lost in the mail or not, but only whether the payment is made.

e. Find the best equilibrium for the consumer as a function of the discount factor.

Bargaining

Consider the following variation on ultimatum bargaining: there is a pie worth 10 dollars. Player 1 makes a proposal to divide the pie (in dollars, not fractions of a dollar). He may not propose either \$10 for himself, or \$0 for himself, but may propose anything in between. Player 2 can accept or reject. If he accepts the pie is divided as proposed; otherwise neither player gets anything. Let m_i be the monetary payoff to player i. Suppose that player i's utility is $m_i - cm_{-i}$ where $0 \leq c < 1$.

a. Draw a sketch of the extensive form (no need to draw the entire thing).

b. Find the subgame perfect equilibrium for the different values of c. You may assume that if $c \neq 0$

c. How does the solution depend on c?

Mechanism Design

A risk neutral seller has two identical items for sale. They are worth nothing to the seller. There are two buyers: one values the item at $\overline{v}$, the other at $\underline{v}$, where $\overline{v} > \underline{v} \geq 0$. A buyer's overall utility is $\log(v^i - p)$ where p is the amount paid to the seller. The seller does not know which buyer is which. Find a mechanism that maximizes the seller's expected revenue. Is this mechanism first best?

Additional Practice Questions, not part of practice exam

1. More repeated games

Player 1 and player 2 must meet in a strange city. If they meet in the cafeteria player 1 gets 2 and player 2 gets 1. If they meet at the drive-in player 1 gets 1 and player 2 gets 2. If they fail to meet they both get zero.

j. Find the extensive and normal form of this game.

k. Find the subgame perfect and Nash equilibrium of this game.

l. Are there any dominated strategies?

m. Find the pure and mixed Stackelberg equilibrium in which player 1 moves first.

n. Find the minmax for both players.

Now suppose that the game is infinitely repeated

o. Player 1 is a long-run player with discount factor δ; player 2 is a short-run player with discount factor 0. Find the set of perfect public equilibrium payoffs to the long-run player as a function of her discount factor.

p. Find strategies that support the best equilibrium from part f.

q. Player 1 and 2 are both long-run players with common discount factor δ. When δ is close to one describe the set of perfect equilibrium payoffs to both players.

r. Find a discount factor and strategies for part h such that both players receive an equilibrium payoff of 2.

2. Long Run and Short Run Players

A long and short run player repeatedly play a stage game. The long run player chooses between U and D, and the short run player between L and R. However, the action of the long run player is not observed by the short run player: all that is observed is a signal that takes on the values u or d. Payoffs are

	L	**R**
u	**30,30**	**20,10**
l	**10,10**	**10,20**

The outcome u has probability .9 if the long run player plays U and probability .1 if he plays D. In all that follows, <u>equilibrium</u> means perfect public equilibrium of the infinitely repeated game with public randomization. Find the extensive and normal forms of the stage-game.

a. For the long-run player, find the minmax, the static Nash, maxmax, mixed precommitment and pure precommitment payoffs.
b. Find the worst equilibrium for the long-run player, and describe in general terms the set of equilibrium payoffs for the long-run player.

First suppose that U and D are directly observed.

c. Find the best equilibrium for the consumer as a function of the discount factor. Now assume that only u and d are observed.
d. Find the best equilibrium for the consumer as a function of the discount factor.