e201b: practice final exam—suggested answers

[Since we did the first two questions during section,
I'll be brief in answering questions one and two.]

be nice

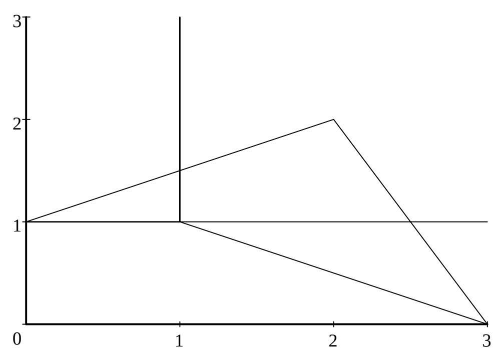
The normal form of this game looks as follows.

	in	*out*
N	2,**2**	0,1
M	**3**,0	**1**,**1**

From the point of view of player 1 (the long-run player) the Nash-equilibrium payoff is 1, and the minmax is also 1. Hence, $\underline{v} = 1$, the worst dynamic equilibrium payoff. Since M dominates N, the only equilibrium of the stage game is (M, out). The pure precommittment Stackelberg payoff is 2, and since N is strictly dominated by M, the mixed precommittment Stackelberg payoff is also 2. Therefore, $\overline{v} = 2$. For what values of δ are these extreme equilbrium payoffs attainable in an infinitely repeated game? For $\overline{v}$ to be attained, we have the following conditions.

$$\begin{aligned}
\overline{v} &= (1-\delta)2 + \delta w(N) \\
\overline{v} &\geq (1-\delta)3 + \delta w(M) \geq (1-\delta)3 + \delta \\
2 &\geq (1-\delta)3 + \delta \Rightarrow \delta \geq 1/3.
\end{aligned}$$

So we need $\delta \geq 1/3$ to sustain a dynamic equilibrium paying 2 to player 1. When players 1 and 2 have the same discount factor, δ, the set of perfect equilibrium payoffs is the set of socially feasible, individually rational payoffs.



FOLK THEOREM

So for δ suffiently close to unity, payoffs in the intersection of the L and the polyhaedron are attainable in perfect equilibrium.

long run consumers

The normal form of this game looks as follows.

	send	*withhold*
pay	3,**2**	**0**,1
cheat	**5**,0	**0**,**1**

Minmax equals Static Nash equals 0 equals $\underline{v}$ for player 1. Pure precommittment Stackelberg is 3, and mixed precommittment is also 3, since *pay* is weakly dominated by *cheat*, so $\overline{v}$ is also 3. First suppose that firms can condition on player 1's actions. Then

$$\begin{aligned} \overline{v} &= (1-\delta)3 + \delta w(\mathit{pay}) \\ \overline{v} &\geq (1-\delta)5 + \delta w(\mathit{cheat}) \geq (1-\delta)5 \\ 3 &\geq (1-\delta)5 \Rightarrow \delta \geq 2/5. \end{aligned}$$

So $\overline{v} = 3$ is attainable for $\delta \geq 2/5$. Now assume that firms cannot condition on the consumer's actions. They are only able to react to whether or not the check arrived. Denote by ✓ the event that the check arrived, and ? the event that it didn't. Then

$$\begin{aligned} \overline{v} &= (1-\delta)3 + \delta(w(\checkmark)/2 + w(?)/2) \\ \overline{v} &\geq (1-\delta)5 + \delta w(?) \end{aligned}$$

We want to make $\overline{v}$ as large as possible, so we'll make $w(\checkmark) = \overline{v}$, and we'll make $w(?)$ as possible by making the IC constraint bind with equality. Hence,

$$\begin{aligned} \overline{v} &= (1-\delta)3 + \delta(\overline{v}/2 + w(?)/2) \\ \overline{v} &= (1-\delta)5 + \delta w(?) \\ &\Rightarrow \overline{v} = 1. \end{aligned}$$

The values of δ for which this payoff is supportable by perfect public equilibrium strategies can be found by using the constraint that $\overline{v} \geq (1-\delta)5 + \delta w(?) \geq (1-\delta)5$, since the worst possible punishment that can be inflicted here is zero.

$$1 \geq (1-\delta)5 \Rightarrow \delta \geq 4/5.$$

bargaining

Here the issue is that it will be credible for player two to refuse certain offers. Player two will reject an offer, m_2 if $m_2 - c(10 - m_2) < 0$, that is, if $m_2 < 10c/(1+c)$. So player one wants to solve the following problem.

$$\begin{aligned} \max \quad & 10 - m_2 - cm_2 \\ \text{s.t.} \quad & m_2 \geq \frac{10c}{1+c} \end{aligned}$$

where m_2 is allowed to vary between 1 and 9. Clearly player one wants to minimize m_2, so the optimum is $m_2^* = \lceil 10c/(1+c) \rceil$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

mechanism design

We have that $v_1 > v_2 > 0$ and $u_i(p) = \ln(v_i - p)$. I'm going to further suppose that $v_1 > v_2 > 1$. The seller's optimization problem looks like this.

$$\begin{aligned} \max \quad & \pi_1 p_1 + \pi_2 p_2 \\ \text{s.t.} \quad & \pi_i \ln(v_i - p_i) \geq \pi_{\neg i} \ln(v_i - p_{\neg i}) \\ & \pi_i \ln(v_i - p_i) \geq 0 \\ & \pi_i \in [0,1]. \end{aligned}$$

Only the IC constraint for the high type and the IR constraint for the low type will bind. In this case, we already have the following restrictions: $p_2 = v_2 - 1$ and $\ln(v_1 - p_1) = \ln(v_1 - (v_2 - 1))^{\pi_2/\pi_1}$, so $p_1 = v_1 - (v_1 - (v_2 - 1))^{\pi_2/\pi_1}$. Now let's calculate the first-order conditions. Let λ denote the Lagrange multiplier associated with the high type's IC constraint, μ the multiplier for the low type's IR constraint, and η_i the multiplier for the constraint that $\pi_i \leq 1$. Then first-order conditions look like

$$
\begin{aligned}
p_1 &: \quad \pi_1 = \frac{\lambda \pi_1}{v_1 - p_1} \Rightarrow \lambda = v_1 - p_1 \\
\pi_1 &: \quad p_1 + \lambda \ln(v_1 - p_1) = \eta_1 \qquad (*) \\
p_2 &: \quad \pi_2 + \frac{\lambda \pi_2}{v_1 - p_2} = \frac{\mu}{v_2 - p_2} \\
\pi_2 &: \quad p_2 - \lambda \ln(v_1 - p_2) = \eta_2
\end{aligned}
$$

Look at (*). I claim that $\eta_1 > 0$, so that $\pi_1 = 1$. If not, then $\eta_1 = 0$ would make (*) look like $p_1 = -\lambda \ln(v_1 - p_1) = -(v_1 - p_1)\ln(v_1 - p_1) = -(v_1 - p_1)\ln(v_1 - (v_2 - 1))^{\pi_2/\pi_1}$ by replacing our expression for p_1 derived from the IC constraint above. Now, $v_1 - (v_2 - 1) = 1 + v_1 - v_2 > 1$, so certainly $(v_1 - (v_2 - 1))^{\pi_2/\pi_1} > 1$ and it follows that $p_1 = -(v_1 - p_1)\ln(v_1 - (v_2 - 1))^{\pi_2/\pi_1} < 0$, which is a contradiction. Therefore, $\pi_1 = 1$, and so $p_1 = v_1 - (v_1 - (v_2 - 1))^{\pi_2}$. Now, notice that if $\pi_2 = 1$, then it better be that $p_2 = p_1$, since otherwise the IC constraint for the high type will be violated. But with $\pi_2 < 1$, it follows from the first-order conditions for π_2 that $p_2 = \lambda \ln(v_1 - p_2)$. Substituting for λ, p_1, and p_2 we get

$$
\begin{aligned}
v_2 - 1 &= (v_1 - p_1)\ln(v_1 - (v_2 - 1)) \\
\Rightarrow v_2 - 1 &= (v_1 - (v_2 - 1))^{\pi_2} \ln(v_1 - (v_2 - 1)) \\
\Rightarrow \ln(v_2 - 1) &= \pi_2 \ln(v_1 - (v_2 - 1)) + \ln(\ln(v_1 - (v_2 - 1))) \\
\Rightarrow \pi_2 &= \frac{\ln(v_2 - 1) - \ln(\ln(v_1 - (v_2 - 1)))}{\ln(v_1 - (v_2 - 1))}.
\end{aligned}
$$

Please tell me if you disagree with this.