E201B: Midterm Exam—Suggested Answers

David Rahman, UCLA[*,†]

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Risk Aversion

We have the utility function $u(c) = -e^{-c}$. The coefficients of absolute risk aversion, $A(c)$, and relative risk aversion, $R(c)$ are obtained from the following formulae:

$$A(c) = -\frac{u''(c)}{u'(c)}, \qquad R(c) = -\frac{u''(c)c}{u'(c)}. \tag{1}$$

For this particular choice of utility, the coefficients are:

$$A(c) = -\frac{-e^{-c}}{e^{-c}} = 1, \qquad R(c) = -\frac{-e^{-c}c}{e^{-c}} = c. \tag{2}$$

Taking derivatives, it follows that $A'(c) = 0$, showing that absolute risk aversion is *constant*, and that $R'(c) = 1$, showing that relative risk aversion is *increasing*.

[*]Please send any comments to dmr@ucla.edu.

Nash Equilibrium

We are asked to consider the following game. (Bold-face numbers denote best responses.)

	H	G
H	0,0	**2**,**1**
G	**1**,**2**	0,0

There are two pure-strategy Nash equilibria to this game: (H, G) and (G, H), with respective payoffs $(2, 1)$ and $(1, 2)$. Neither of these Nash equilibria is symmetric. There is also a mixed-strategy Nash equilibrium. It can be computed as follows. Assuming that both players are mixing between H and G, it follows that they must each be indifferent between either strategy. Hence, if player II's probability of playing H is β, then

$$E_\beta[u_I(H)] = E_\beta[u_I(G)] \quad \Rightarrow \quad 0\beta + 2(1-\beta) = \beta + 0(1-\beta) \quad \Rightarrow \quad \beta = 2/3. \tag{3}$$

By symmetry of the payoff bimatrix, it follows that α, player I's probability of playing H, is also $2/3$. Hence, the mixed-strategy Nash equilibrium $(2[H]/3 + [G]/3, 2[H]/3 + [G]/3)$ is symmetric. The vector of payoffs associated with this equilibrium is $(2/3, 2/3)$.

The following correlated strategy (call it μ) is a symmetric correlated equilibrium that Pareto-dominates the symmetric Nash.

	H	G
H	0	1/2
G	1/2	0

Clearly this is symmetric. The vector of payoffs associated with this strategy is $(1.5, 1.5)$, which clearly Pareto-dominates the symmetric Nash payoffs. To show that μ is indeed a correlated equilibrium, first notice that—by symmetry—if the incentive constraints are satisfied for player I then they are also satisfied for player II, so it suffices to check the constraints for player I only. If the mediator recommends player I to play H then player I will obey the recommendation if

$$E_\mu[u_I(H)|H] \geq E_\mu[u_I(G)|H]. \tag{4}$$

If the mediator recommends player I to play H, then player I knows that the mediator must have recommended player II to play G, i.e., player I's posterior probability over his opponent's actions given H is that G will be played with probability 1. Hence, $E_\mu[u_I(H)|H] = 2$, which is clearly greater than $0 = E_\mu[u_I(G)|H]$.

If the mediator recommends player I to play G then player I will obey the recommendation if

$$E_\mu[u_I(G)|G] \geq E_\mu[u_I(H)|G]. \tag{5}$$

If the mediator recommends player I to play G, then player I knows that the mediator must have recommended player II to play H, i.e., player I's posterior probability over his opponent's actions given G is that H will be played with probability 1. Hence, $E_\mu[u_I(G)|G] = 1$, which is clearly greater than $0 = E_\mu[u_I(H)|G]$. It now follows that μ is a correlated equilibrium.

Trembling Hand Perfection

Definition 1 *A strategy profile σ is* trembling hand perfect *if there exists a sequence of strategy profiles $\{\sigma^n\}$ with $\sigma^n \to \sigma$ such that $\sigma_i^n(a_i) > 0$ for every $a_i \in A_i$ and $\sigma_i(a_i) > 0$ implies that $a_i \in B_i(\sigma_{-i}^n)$, where $B_i(\cdot)$ is player i's* best-response correspondence.

Claim 2 *Every trembling hand perfect equilibrium is a Nash equilibrium.*

Proof—Let σ be a trembling hand perfect equilibrium. Then there is a sequence $\{\sigma^n\}$ that satisfies the conditions of Definition 1. Fix any player i, and fix any action $a_i \in A_i$ such that $\sigma_i(a_i) > 0$. By hypothesis,

$$u_i(a_i, \sigma_{-i}^n) \geq u_i(b_i, \sigma_{-i}^n), \qquad \forall b_i \in A_i. \tag{6}$$

Since u_i is continuous, $\sigma_{-i}^n \to \sigma_{-i}$ implies that $u_i(\cdot, \sigma_{-i}^n) \to u_i(\cdot, \sigma_{-i})$. Therefore, taking limits on each side of (6), we may conclude that

$$u_i(a_i, \sigma_{-i}) \geq u_i(b_i, \sigma_{-i}), \qquad \forall b_i \in A_i. \tag{7}$$

Notice finally that this condition applies to any action a_i with $\sigma_i(a_i) > 0$. Therefore for any two a_i and a_i' such that $\sigma_i(a_i) > 0$ and $\sigma(a_i') > 0$, we must have both $u_i(a_i, \sigma_{-i}) \geq u_i(a_i', \sigma_{-i})$ and $u_i(a_i', \sigma_{-i}) \geq u_i(a_i, \sigma_{-i})$, so $u_i(a_i, \sigma_{-i}) = u_i(a_i', \sigma_{-i})$. This, together with (7), implies that σ be a Nash equilibrium. ■

Subgame Perfection

To find a subgame perfect equilibrium we must show that it's Nash in every subgame. We will find it by backward induction. In the subgame below, there is only one Nash: (U, L).

	L	R
U	$\mathbf{6}, \mathbf{6}$	$\mathbf{3}, 0$
D	$0, 0$	$2, \mathbf{2}$

This is apparent from the fact that D is *strictly* dominated by U.

Proceeding up the game tree, player I has the option of ending the game (playing E) or participating in the subgame above (playing P). Payoffs for each player assuming that the unique Nash equilibrium will be played in the subgame above are as follows.

E	$4, 4$
P	$\mathbf{6}, 6$

Therefore the unique subgame perfect equilibrium is $((P, U), L)$.

The strategic-form representation of this game looks like this.

	L	R
(P, U)	$\mathbf{6}, \mathbf{6}$	$3, 0$
(P, D)	$0, 0$	$2, \mathbf{2}$
(E, U)	$4, \mathbf{4}$	$\mathbf{4}, \mathbf{4}$
(E, D)	$4, \mathbf{4}$	$\mathbf{4}, \mathbf{4}$

Applying iterated weak dominance, we eliminate (P, D) since it's strictly dominated by (P, U). But then R is weakly dominated by L, and finally, restricted to L, (P, U) dominates the rest of player I's remaining strategies, leaving only one profile surviving, $((P, U), L)$. This is the subgame perfect equilibrium profile.