Final Exam First Year Game Theory

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Do all questions. Every question has equal weight. You have three hours.

1. Prove that for a weak order $\succeq$ it is the case that if y is not weakly above x then it is strictly below it, that is that one has $(x \succ y) \Leftrightarrow \neg(y \succeq x)$.

2. Let x be a lottery (distribution on $\Re$ with finite support) owed by Bob, and suppose Anne is less risk averse than Bob. Show that Bob can sell x to Anne at a price p which makes them both better off (that is such that $p \succ_{\text{Bob}} x$ and $x \succ_{\text{Anne}} p$).

3. Consider the Battle of the Sexes game, with two players and two strategies, opera or ball-game. If both choose opera the row player gets 2 and the column player 1; if they both choose ball-game the row player gets 1 and the column player gets 2. Otherwise if they disagree both get zero. Find all the (pure and mixed) Nash equilibria when the game is simultaneous move. What is the subgame perfect equilibria if row player moves first and column plays after seeing row players move? What about the Nash equilibria in the sequential move game?

4. Consider the exit game in which row player must choose between +1 and -1 and the column player between in and out. If column player plays out both players get nothing. If column plays in the payoffs are 1,1 if row plays +1 and 2,-1 if row plays -1. If the action of the row player is observed find the Nash equilibrium, Stackelberg equilibrium (where row moves first) of the one shot game, and find a discount factor for the row player playing repeatedly against a myopic player 2 such that he get the Stackelberg payoff. Suppose now that only a noisy signal of the row player is observed: with probability $1-\epsilon$ his action is correctly reported, and with probability ϵ it is misreported. Formulate the dynamic incentive constraints and find the best equilibrium payoff for the row player when her discount factor is close to one.