Equity Premium Puzzle

Present Value vs. Average Present Value

infinite discounted utility

$\sum_{t=1}^{\infty} \delta^{t-1} u_t$

average discounted utility

$(1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} u_t$

note that average present value of 1 unit of utility per period is 1

- macro and finance use present value
- game theory uses average present value
- why? Common units across different discount factors

Risk Aversion for Wealth and Consumption

relative risk aversion for wealth versus consumption at steady state

wealth the present value of consumption

$$W = c/(1-\delta)$$

utility of wealth the present value of the utility of consumption

$$U(W) = u(c)/(1-\delta) = U((1-\delta)W)/(1-\delta)$$

calculate coefficient of relative risk aversion

$$\rho_W = -U''(W)W/U'(W) = -(1-\delta)^2 u''(c)W/((1-\delta)u'(c))$$

$$= u''(c)c/u'(c) = \rho_c$$

Simple Portfolio Choice Model

have initial wealth W

invest a fraction $1-\alpha$ in safe bonds with certain return r_b

α in risky stock with risky return

$r_s = \overline{r}_s + \sigma y$ where $Ey = 0, Ey^2 = 1$

equity premium is defined as $\lambda = \overline{r}_s - r_b$

final wealth is

$$W + W(\alpha r_s + (1-\alpha) r_b) = W + W(r_b + \alpha(\lambda + \sigma y))$$

Fundamental Risk Equation

$U(W + W(r_b + \alpha\lambda + \alpha\sigma y))$

derivative with respect to α

$U'(W + Wr_b + W\alpha\lambda + W\alpha\sigma y)(W\lambda + W\sigma y)$

set $W' = W + Wr_b + W\alpha\lambda$

linear approximation to the derivative

$U'(W')(W\lambda + W\sigma y) + U''(W')W\alpha\sigma y(W\lambda + W\sigma y)$

take the expectation and equate to zero

$U'(W')W\lambda + U''(W')W^2\alpha\sigma^2 = 0$ gives

$$\rho = -U''W/U' = \lambda/(\alpha\sigma^2)$$

Equity Premium and Relative Risk Aversion

Mehra and Prescott [1985]; Shiller [1989] data annual 1871-1984

Mean real return on bonds $r_b = 1.9\%$;

Mean real return on S&P $\overline{r}_s = 7.5\%$

Equity premium $\lambda = 0.056$

Standard error of real stock return $\sigma = 0.181$

$\rho = \lambda/(\alpha\sigma^2) = 1.81\alpha^{-1}$

that is, at least 1.81

What is the portfolio?

Assume consumption proportional to wealth $c = \phi W$

recall final wealth

$$W_1 = W + W(r_b + \alpha(\lambda + \sigma y))$$

define

$$s^2 = \mathrm{var} c/\mathrm{E} c = \mathrm{var} W_1/\mathrm{E} W_1 = \alpha^2\sigma^2 W/W' \approx \alpha^2\sigma^2$$

(wealth does not change much in a single period)

in the data $s = .035$

hence $\alpha^{-1} = \sigma/s = 5.17$ so $\rho = 8.84$

The real equity premium puzzle

suppose CRRA $u(c) = c^{1-\rho}/(1-\rho)$

$u'(c) = c^{\rho}$

consumption grows at a constant rate $c_t = \gamma^t$

interest rate determined by indifference condition

$$\frac{1}{1+r} = \frac{\delta u'(x_{t+1})}{u'(x_t)} = \frac{\delta \gamma^{-\rho(t+1)}}{\gamma^{-\rho t}} = \delta \gamma^{-\rho}$$

average real US per capita consumption growth rate 1.8%

with $\delta = 1$ and $\rho = 8.84$ this gives $r = 17\%$

rather hard to reconcile with mean real return on bonds 1.9%; Mean real return on S&P 7.5%

How does the market react to good news?

Value of claims to future consumption relative to current consumption

$c_1 = 1$

$$\frac{\sum_{t=2}^{\infty} \delta^{t-1} u'(c_t) c_t}{u'(1)}$$

$$\sum_{t=2}^{\infty} \delta^{t-1} \gamma^{-(t-1)\rho} \gamma^{t-1} = \sum_{t=1}^{\infty} \left[\delta \gamma^{1-\rho} \right]^t = \frac{\delta \gamma^{1-\rho}}{1 - \delta \gamma^{1-\rho}}$$

to be finite we need $\delta \gamma^{-\rho} < 1$

$$\frac{\partial}{\partial \gamma} \frac{\delta \gamma^{1-\rho}}{1 - \delta \gamma^{1-\rho}} = \frac{\delta (1-\rho) \gamma^{-\rho}}{\left(1 - \left[\delta \gamma^{-\rho} \right] \right)^2}$$

$\rho > 1$ this is negative

Separability

we can’t have both separability between states (expected utility) and separability between periods

we have a strong reason for expected utility and none at all for intertemporal separability

various theories of non-separable time preferences

Risky Drinking

suppose that all consumption takes place in a nightclub

at the beginning of the year before you see your stock return you choose the quality of nightclub you will attend c^q

if are anticipating low income you choose the cheap beer place

if you are anticipating high income you choose the expensive champagne place

utility $u(c|c^q)$

we are going to assume $u(c|c) = \log(\mathrm{c})$

$$u(c|c^q) = \log c^q - \frac{(c/c^q)^{1-\rho} - 1}{\rho - 1}$$

conditional on c^q you have relative risk aversion ρ

but with growth you have intertemporal separability 1

Types of Models

- habit formation
- indivisibility (houses)