UNIVERSITY OF CALIFORNIA

Los Angeles

ESSAYS ON SOCIAL LEARNING AND OPTIMAL COMMITTEE SIZE

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Economics

by

Sawoong Kang

1996

The dissertation of Sawoong Kang is approved.

Jack Hirshleifer

Joseph M. Ostroy

Ivo Welch

David K. Levine, Committee Chair

University of California, Los Angeles

1996

To Jesus Christ Our Lord, Daniel Kang, and my parents.

Contents

List of Figures

List of Tables

ACKNOWLEDGMENTS

This dissertation is the result of guidance, assistance and encouragement from many people, and I would like to use this opportunity to thank all of them.

First of all, I would like to express my deepest appreciation to Professor David Levine, my advisor, who has guided me in the last four years of the research. Besides providing useful comments during all stages of the research, Professor Levine has been always encouraging me with his intellectual enthusiasm. Financial support through him during every Summer was very helpful.

I also want to thank my other committee members, Professor Jack Hirshleifer, Professor Joseph Ostroy, and Professor Ivo Welch, for all the comments, encouragement, and the time spent on this dissertation.

In addition to my committee members, I want to thank James Bergin for the comments on Chapter 2, which were very useful in improving the model. I am also grateful for helpful discussions with Bryan Ellickson, Keunkwan Ryu, William Zame. I received many valuable comments on Chapter 2 from the seminar participants at Carnegie Mellon University and Korea Information Society Development Institute. I am also indebted to the members of David Levine's informal seminar, Youngse Kim. Josef Perktold, and Philip Johnson for their intellectual discussions and friendship.

I would like to thank Maxwell Rhee for his proofreading and friendship. I also want to thank Changwan Kim for his assistance in using LATEX. I am also indebted to Kwanho Shin whose assistance in using GAUSS for computer simulation.

Last but not least, I would like to express my appreciation to my mother for her ceaseless prayer and love over the years. Finally, I want to thank Daniel Kang who has provided me with spiritual guidance in Jesus Christ Our Lord.

VITA

January 16, 1965	Born, Seoul, Korea
February 1987	B.A., Economics Seoul National University Seoul, Korea
February 1989	M.A., Economics Seoul National University Seoul, Korea
September 1991 - June 1994	Teaching Assistant Department of Economics University of California, Los Angeles
September-December 1994	Instructor UCLA Extension Los Angeles, California
April-June 1995, 1996	Instructor Department of Economics University of California, Los Angeles

ABSTRACT OF DISSERTATION

ESSAYS ON SOCIAL LEARNING AND OPTIMAL COMMITTEE SIZE

by

Sawoong Kang

Doctor of Philosophy in Economics

University of California, Los Angeles, 1996

Professor David K. Levine, Chair

This dissertation consists of two independent papers which address problems of social learning and optimal committee size.

Chapter 2 analyzes a heterogeneous population in which selfish players and fair players are spatially distributed, and they are randomly and repeatedly matched to play a prisoner's dilemma. Players are assumed to behave in a myopic manner. By introducing the random experimentation of fair players, it is shown that the system described by a Markov process converges to a best possible equilibrium in the long run. The long-run equilibrium is shown to be unique for a population with the distribution of players and matching rule fixed. Simulation result shows that the structure of interactions affects the long-run cooperation rate. The model is applied to collusion in a monopolistically competitive industry.

Chapter 3 provides a theory for the optimal size of committee, by examining the tradeoff between the sluggishness in arriving at a group decision versus the greater

accuracy of decision making due to the pooling of information. We consider a decision about whether to make an investment that has an uncertain value, about which each individual has an independent signal. By assuming that potential committee members are available to meet only at random times, we model the cost of delay due to scheduling problem. Within the context of individualized information and scheduling, we analyze the optimal size of a standing committee and an *ad hoc* committee. We also consider the case in which a decision maker is facing a sequence of similar decision making problems.

Chapter 1

Overview

Chapter 2 investigates how the cooperation can evolve in a heterogeneous population in which selfish players and fair players locally interact, and how the structure of interactions can affect the efficiency of the long-run equilibrium outcome. For this purpose, we consider an economy in which two types of players are spatially distributed, and they are repeatedly and randomly matched to play a prisoner's dilemma. We assume that a player possesses knowledge about his neighbors past actions but their type cannot be distinguished. Players optimize myopically with a static belief so that they choose one-shot optimal action in response to their neighbors actions in the previous period. In order to minimize the inefficiency of possibly being locked in a bad equilibrium due to their myopic behavior, fair players make random experiments in each period, in which they cooperate with a small probability when their myopic best response requires them to defect. In this setting, the dynamics of the game can be described by a Markov process. We analyze the limit behavior of the system as the probability of experimentation goes to zero. The main theorem shows that for a given distribution of players and a matching rule, there exists a unique long-run equilibrium. By performing simulation on the average long-run cooperation rate under various matching rules, we found two interesting results. First, if the population is highly selfish, or the matching rule is quite local, then the neighborhood size has a negative effect on the long-run cooperation rate. Second, the larger the population size, the lower the cooperation rate. The model is also applied to the implicit collusion in a monopolistically competitive industry, where firms compete with only a small number of rival firms.

Chapter 3 provides a theory for the optimal size of a committee. Our focus is on the tradeoff between the sluggishness in arriving at a group decision versus the greater accuracy of decision making due to the pooling of information. For this purpose, we consider a decision whether to make an investment. The investment has an uncertain value, and each individual has an independent signal about the value of the investment. We model the cost of delay as simply the time cost of forgone profit. By assuming that potential committee members are available to meet only at random times, we focus on the scheduling cost: the larger the committee, the more difficult it will be to schedule a meeting at which all members will be present. Within the context of individualized information and scheduling, we analyze the optimal size of a standing committee and an *adhoc* committee. Our model reflects our basic intuition about committees: the greater the cost of time delay and the less diverse the information, the smaller the size of the optimal committee. We also consider the case in which a decision maker is facing a sequence of similar decision making problems.

Chapter 2

Cooperation through Experimentation and Learning in a Heterogeneous Population

2.1 Introduction

The prisoner's dilemma is a well-known game in which players' mutual cooperation is desirable but difficult to enforce due to the rational behavior of selfish players. Selfishness and rationality are the two cornerstones on which conventional economic models have been established. However, it has been pointed out in the recent literature that many phenomena cannot be easily formulated as the outcome of rational choices by selfish agents. For example, in many laboratory experiments concerning the contribution to a public good—which can be characterized as a one-shot prisoner's dilemma—players contribute their resources on average nearly 40–60% of the socially optimal level. [1] Clearly, this result is inconsistent with the prediction of standard game theory that prescribes zero contribution as the dominant strategy.

Since these experimental studies seem to imply that some players make contributions while others free ride, one natural theoretical extension is to take into account the fact that in many situations which are often modeled as a prisoner's dilemma, there exist a number of players who are not as "greedy" as typical selfish players. From this cause, we would like to examine a heterogeneous population in which selfish players and fair players interact. Fair players are characterized as suffering from guilty feeling when they free ride, whereas selfish players enjoy the material gain from exploiting other players. [2] Thus, fair players seek to coordinate with other players, while selfish players always defect.

Furthermore, observe that in a very large economy, agents tend to interact with only a relatively small subset of the whole population, whom we call neighbors. Generally, the set of neighbors of a particular agent partially overlaps with one another

[1] Dawes and Thaler (1988) provide a short survey of these public good experiments. For a comprehensive survey, see Ledyard (1993).

[2] Fairness has been modeled in various ways. For example, see Hirshleifer (1985, 1993) and Rabin (1993).

so that all agents are directly or indirectly linked with each other. The size of the neighborhood depends upon the characteristics of the game being played and the level of existing technology such as transportation or communication. For example. advanced means of transportation allow people to interact with a larger number of other people.

The object of this paper is to study how the cooperation can evolve in a heterogeneous population in which selfish players and fair players locally interact, [3] and how the structure of interactions can affect the efficiency of the long-run equilibrium outcome. For this purpose. we consider an economy in which two types of players are spatially distributed, and they are repeatedly and randomly matched to play a prisoner's dilemma. The stage game is complicated with incomplete information so that the payoff matrix depends on the types of matched pair. It is assumed that a player can remember his neighbors past actions but cannot distinguish their type.

Axelrod (1984) addressed the question of how the cooperation can evolve in a population consisting only of selfish players. He set up the problem as an indefinitely repeated prisoner's dilemma, and show that if players are sufficiently patient. then cooperation can evolve from small clusters of players who use TIT FOR TAT strategy. TFT strategy cooperates on the first move and then does whatever the opponent did on the preceding move. In his computer tournaments. Axelrod found that TFT was the best strategy of all strategies submitted for a repeated prisoner's dilemma. [4] Axelrod focused on how the long-run consideration of patient selfish players can lead them to cooperate on the basis of reciprocity. In contrast, we focus on the process by which myopic fair players get to cooperate among themselves in the face of selfish players who never cooperate. In Axelrod's framework, each player is assumed to

[3] By using Ising model, Ellickson (1990) and Blume (1993) study local interactions in a homogeneous population.

[4] For a short report of this tournament result. see Axelrod and Hamilton (1981).

recognize the other player in his interactions and to remember how the two of them have interacted. Thus players can base their decision on the history of the particular interaction. In our analysis, however, a player cannot recognize his opponent except knowing that he belongs to his neighborhood. Hence, the average behavior of his neighbors in the past is taken into account by a player's strategy.

Kandori, Mailath and Rob (1993) analyzed the long-run behavior in a large population in which players are repeatedly and randomly matched to play a coordination game. They showed that evolutionary forces created by mutations and myopic behavior by players lead them to coordinate to a risk-dominant equilibrium in the long run. Since fair players in our model are analogous to players in a coordination game in the sense that they want to coordinate with other players, we can employ the modeling strategy of KMR. When players globally interact in a fairly large population as assumed in KMR, however, it will take too long for the evolutionary forces to be felt. Then the analysis of long-run equilibrium is not relevant for the prediction of a real outcome. Ellison (1993) shows that if the matching process is local rather than global, the system can quickly converge to the long-run equilibrium. In our heterogeneous population game, the structure of interactions affects not only the rate of convergence to the limit, but also the limit itself.

In principle, when a player makes a decision in each period, he has to take into account the effect of today's decision on the future payoff through the influence on the future actions of his neighbors. For simplicity, however, we assume that the complexity of the problem forces players to optimize myopically, and that they naively expect to see the same profile of neighbors' actions as one in the previous period. Then players will choose one-shot optimal action in response to their neighbors actions in the previous period. When players employ the myopic best response such as this, there exist multiple Nash equilibria, and thus a wide range of cooperation rates can be realized as outcomes. For any mix of two types, for example, the worst outcome in which everyone defects is a Nash equilibrium. There also exist other equilibrium outcomes in which some of fair players cooperate. We can call the best of the set of outcomes a *best possible equilibrium*, so that in a best possible equilibrium the maximum amount of cooperation among fair players is achieved.

In order to single out the most plausible outcome in the long run, we will focus on one desirable characteristic of the fair players. Recognizing that current bad outcome is mainly due to coordination failure among themselves, some fair players attempt to act as leaders in solving the coordination problem. The intentional efforts of leaders can be modeled by assuming that in each period all fair players make random experiments, in which they cooperate with a small probability even when doing so is not myopically optimal. These leading cooperations are allowed to be withdrawn at any time if the leaders are not satisfied with a persistent non-cooperative neighborhood. Random experimentation in our model is different from the mutations in the previous literature on two aspects. First, we introduce random experimentation to model fair players' intentional efforts to solve coordination failure amongst themselves, while mutations in KMR and Ellison are interpreted as mistakes by players. Second, experimentation in our model is only one way of moving from defection to cooperation, while KMR and Ellison's mutations allow two-way randomization between two actions. Then some of fair players attempt to cooperate even in a completely defective environment. Clusters of these leading cooperators may induce other fair players to cooperate through their learning and myopic optimization, so that they can locally succeed in fostering cooperate environment. Therefore, we can expect that in the long run the economy would eventually evolve to a best possible equilibrium.

In this setting, the dynamics of the game can be described by a Markov process. We analyze the limit behavior of the system as the probability of experimentation goes to zero. We can show that in the long run the economy converges to the best possible equilibrium, in which active- fair players cooperate and selfish and discouraged-fair players defect; active-fair (discouraged-fair) players have potentially cooperative (defective) neighborhoods. Since the distribution of players certainly affects the cooperation rate, we need to look at the average cooperation rate for a given set of other parameter values. The average cooperation rate in the long-run equilibrium varies according to the proportion of selfish players, the size of neighborhoods, the size of population and the degree of overlap in the interaction structure.

For completeness, I carried out a number of computer simulations. In these simulations, I imposed a restriction on the payoff matrix, so that a fair player's myopic best response is to follow the majority action of his neighbors in the previous period. Two interesting results were found. First, if the population is highly selfish, or the matching rule is quite local, then the neighborhood size has a negative effect on the long-run cooperation rate. Second, the larger the population size, the lower the cooperation rate. These two effects can be combined to explain why people living in a large and crowded city are relatively less cooperative.

The simulation results have an important implication for the collusion in various market structures. In the oligopoly industry, firms tend to interact globally. On the other hand, monopolistically competitive firms tend to compete with a small number of rival firms, where the number of rival firms for each firm is determined by substitutability among products. In the global interaction case, collusion among fair firms will be either perfect success or complete failure, depending only on the composition of two types in the industry. However, in the local interaction case, the degree of collusion will be in between these two extremes, where the average price level is determined by the number of rival firms as well as the proportion of each type in the industry.

The paper is organized as follows. Section 2 describes the formal model. in which payoff matrices, matching rules, myopic best response, and random experimentation are detailed. In Section 3, we prove the main theorem concerning the existence of a unique long-run equilibrium. Section 4 investigates the implication of various matching rules for the long-run equilibrium by using simulation. Finally, Section 5 applies the model to collusion in a monopolistically competitive industry and Section 6 concludes.

2.2 The Formal Model

Consider a large population consisting of N players, letting N also represent the set of players. Each player is one of two types: selfish (S) or fair (F). S and F will also be used to denote the number of people in each type. Notice that $N \equiv S \cup F$ and $S \cap F = \emptyset$. The proportion of selfish players is denoted by $m = S/N$. Players are randomly and repeatedly matched for play in a 2-person Prisoner's Dilemma. Time is discrete, indexed by $t = 1, 2, 3, \ldots$. In each period, player i chooses one of two possible actions $a_{it} \in \{C, D\}$, where C and D denote "cooperate" and "defect" respectively. Depending on the types of the matched pair, one of three payoff matrices becomes relevant. In Table 2.1, these payoffs are tabulated, where I have assumed that $x > 0, l > 0$, and $g > y > 0$. l measures the loss from being exploited and y measures the pecuniary gain from free riding; however, fair players also suffer from "guilty" feeling of g when they cheat other people.

Players interact only with a subset of the population. To be concrete, suppose that players are uniformly distributed on a circle or torus of N sites and are not allowed to move. [5] Interactions on a circle (torus) can be thought of as one (two)-

[5] Schelling (1971) analyzes the segregation phenomena when the locally interacting agents are

dimensional. [6] Let us symbolize local interaction on a circle by C#, where # denotes the number of neighbors; for example, C4 stands for the local interaction on a circle with 4 neighbors. Likewise, T# denotes a local matching on a torus with # neighbors. Figures 2.1 and 2.2 illustrate some examples of various interaction structures. Even when the number of neighbors are equal, the degree of overlap can be very different depending on the dimension of the interaction structure. Some interactions have a highly overlapping structure in the sense that a neighbor of an agent's neighbor is likely to be also a neighbor. In other structures, the neighborhoods of two distinct agents may only slightly overlap or in fact are completely disjoint. For example, the probability of a neighbor of an agent's neighbor also being a neighbor is 1/2 in C4 but 0 in T4.

In every period, each player is randomly matched with one of his closest $2k$ neighbors with equal probability, where k is a positive integer smaller than $N/2$. Then the probability that player i meets j in a given period, π_{ij}, is defined by: [7]

$$\pi_{ij} = \begin{cases} \frac{1}{2k} & \text{if } i - j = \pm 1, \pm 2, \cdots, \pm k \pmod{N}, \\ 0 & \text{otherwise.} \end{cases}$$

If we denote the dimension of the interaction structure by d, then $M \equiv (2k, d)$ defines a matching rule.

For convenience, let us redefine the set of possible actions as $A \equiv \{1, 0\}$, where C has been replaced by 1 and D by 0. Clearly then, the expected payoff to player i in period t can be written as a function of his own action and his neighbors' actions.

allowed to move. And Ely (1995) extends the model of KMR (1993) and Ellison (1993) to the case when players can choose the neighborhoods to which they belong.

[6] A circle can be thought of as simply a line with both ends connected. Similarly, if unfolded, a torus becomes a square.

[7] When we consider two dimensional matching, we might have to use a coordinate (i, j) rather than i to locate a player. For notational simplicity, however, we focus on one dimensional matching. All of our results still apply to the two dimensional matching cases.

$u_{it} \equiv u_i(a_{it}, a_{-it})$:

$$u_{it} = \begin{cases} (1-a_{it})(\frac{x+y}{2k}\sum_{j=1}^{k} a_{i\pm jt}) + a_{it}(\frac{x+l}{2k}\sum_{j=1}^{k} a_{i\pm jt} - l) & \text{if } i \in S, \\ (1-a_{it})(\frac{x+y-g}{2k}\sum_{j=1}^{k} a_{i\pm jt}) + a_{it}(\frac{x+l}{2k}\sum_{j=1}^{k} a_{i\pm jt} - l) & \text{if } i \in F. \end{cases}$$

Let us describe the spatial distribution of players' types by an N x 1 vector, $e^0 \in \{0,1\}^N$, where 0 denotes a selfish type and 1 a fair type. Now, we can define a heterogeneous population game, $\mathcal{G}$ as follows.

Definition 2.2.1 $\mathcal{G} \equiv (E, M, \phi)$ *is a repeated game played by a heterogeneous population* $E \equiv (N, m, e^0)$, *with the matching rule* $M \equiv (2k, d)$ *and the payoffs* $\phi \equiv (x, y, g, l)$.

We assume that a player can observe his neighbors past actions but cannot distinguish their type. In principle, players wish to maximize the life-time payoff, $V(t) = \sum_{s=t}^{\infty} \beta^{s-t} u_{is}$, where β is a discount factor. In other words, when a player makes a decision in each period, he should take into account the effect of today's decision on the future payoff through the influence on the future actions of his neighbors. Since the future actions of his neighbors depend on their other neighbors' actions as well, however, it is difficult for a player to form rational belief as to how his neighbor will respond to his today's action. Therefore, we assume that players believe that their current actions do not affect their neighbors future actions, i.e., $E_i[\mathbf{a}_{-is} \mid a_{it} = 0] = E_i[\mathbf{a}_{-is} \mid a_{it} = 1]$ for all $s > t$. With this belief, players behave myopically. Then a myopic player does not care about the future and simply chooses an action to maximize his current period expected payoff, which depends on his neighbors' current period actions. [8] We also assume that players have static belief that their neighbors will stick to the same actions as in the previous period.

[8]Ellison (1995) explores the question of when the assumption of myopia can be justified in a large population. In contrast to our model, he assumes that each player knows only about the matches in which he has been involved.

i.e., $E_i[\mathbf{a}_{-it} \mid \mathbf{a}_{-it-1}] = \mathbf{a}_{-it-1}$. Then players' optimal actions can be described by the following best response function, $BR_i : \mathbf{a}_{t-1} \equiv (a_{1t-1}, a_{2t-1}, \cdots, a_{Nt-1}) \rightarrow a_{it}$:[9]

$$BR_i(\mathbf{a}_{t-1}) = \begin{cases} 0 & \text{if } i \in S, \\ 0 & \text{if } i \in F \text{ and } \frac{1}{2k}\sum_{j=1}^{k} a_{i\pm jt-1} < \frac{l}{g-y+l}, \\ 1 & \text{if } i \in F \text{ and } \frac{1}{2k}\sum_{j=1}^{k} a_{i\pm jt-1} \geq \frac{l}{g-y+l}. \end{cases}$$

In other words, selfish players always choose to defect no matter how other people behave. The optimal action of a fair player, however, depends on what proportion of his neighbors cooperated in the previous period, i.e., he will cooperate if and only if the proportion of cooperative neighbors is not less than a critical level. Now, using this best response function, we can define a Nash equilibrium of the game.

Definition 2.2.2 *For a given game, $\mathcal{G}$, an action profile $\mathbf{a} \equiv (a_1, a_2, \cdots, a_N)$ is a Nash equilibrium if and only if $BR_i(\mathbf{a}) = a_i$ for all $i \in N$.*

Since fair players' best responses depend on how other people behave, there exist multiple Nash equilibria from the worst one of complete defection, *ALL D*, to a *best possible equilibrium*, in which fair players achieve the maximum level of cooperation among themselves. Of course, the level of cooperation in the best possible equilibrium varies with the parameters of the model, such as the configuration of players in the population and the matching rule. [10]

We would like to know whether the economy can settle at the best possible equilibrium in the long run. Suppose that initially the economy is at the worst equilibrium in which both selfish players and fair players defect. Selfish players, being greedy, can be said to be deserving the low level of utility from *ALL D*. For fair players, however, the Pareto inferior outcome should be attributed to the lack of coordination

[9] Here, we assume that when they are indifferent, fair players cooperate.

[10] From now on, by the configuration of players we mean the spatial distribution of players, ϵ^0.

among themselves as well as to the fear of being exploited by selfish players. Once coordination failure is recognized as the main reason for a bad outcome, some fair players are motivated to play a role in mobilizing incentives to improve the current bad state. Then they will attempt to cooperate even in a non-cooperative environment. Clusters of these leaders may induce other fair players to cooperate through their myopic best response and learning about the changing environment. In some neighborhoods, these coordination efforts may be discouraged due to a large fraction of selfish players relative to fair leaders. In the long run, however, the economy may eventually move from a non- cooperative state to the best possible equilibrium.

To capture this idea, let us assume that in each period fair players, in the hope of minimizing the inefficiency of possibly being locked in a bad equilibrium due to their myopic behavior, choose to cooperate with probability ϵ, even when their best response calls for defection. [11] On the other hand, selfish players are not allowed to make any experimentation because here we introduce random experimentation to model intentional efforts rather than mistakes. Then the best response function should be replaced by a *behavior rule*, $B_i : \mathbf{a}_{t-1} \rightarrow a_{it}$, which is given by:

$$B_i(\mathbf{a}_{t-1}) = \begin{cases} 0 & \text{if } i \in S, \\ 0 & \text{with probabilty } (1-\epsilon) \text{ if } i \in F \text{ and } \frac{1}{2k}\sum_{j=1}^{k} a_{i\pm jt-1} < \frac{l}{g-y+l}, \\ 1 & \text{with probabilty } \epsilon \text{ if } i \in F \text{ and } \frac{1}{2k}\sum_{j=1}^{k} a_{i\pm jt-1} < \frac{l}{g-y+l}, \\ 1 & \text{if } i \in F \text{ and } \frac{1}{2k}\sum_{j=1}^{k} a_{i\pm jt-1} \geq \frac{l}{g-y+l}. \end{cases}$$

Although all fair players make random efforts to improve the outcome, not all of them will be successful. Intuitively, we expect that those who interact with a relatively fair neighborhood will be able to succeed in fostering a cooperative neighborhood, while those who interact with a relatively selfish neighborhood are unable to succeed.

[11] We assume that this experimentation is reversible in that players are allowed to go back to defection whenever their experimental cooperation is not immediately matched by sufficient number of neighbors.

In order to make a clear distinction between these two types of fair players according to the quality of their neighborhoods, we introduce the notion of a *discouragement operator* $\mathcal{D}$.

Definition 2.2.3 *Discouragement operator,* $\mathcal{D} \equiv (\mathcal{D}_1, \mathcal{D}_2, \cdots, \mathcal{D}_N)$ *is defined by:*

$$\mathcal{D}_i(e_i, e_{-i}) = \begin{cases} 0 & \text{if } e_i = 0 \\ 0 & \text{if } e_i = 1 \text{ and } \frac{1}{2k}\sum_{j=1}^{k} e_{i\pm j} < \frac{l}{g-y+l}. \\ 1 & \text{if } e_i = 1 \text{ and } \frac{1}{2k}\sum_{j=1}^{k} e_{i\pm j} \geq \frac{l}{g-y+l}. \end{cases}$$

Obviously, there exists a unique limit

$$e^*(e^0) = \lim_{n \to \infty} \mathcal{D}^n(e^0).$$

Using this limit, we can divide fair players into two categories.

Definition 2.2.4 *A fair player i is called an active-fair player if $e_i^* = 1$ and a discouraged- fair player if $e_i^* = 0$.*

In other words, fair players become discouraged when they are surrounded by fairly many selfish players or discouraged-fair players. Notice that whether a fair player is discouraged depends not only on the types of their neighbors but the types of their neighbors' neighbors and their neighbors and so on.

2.3 Characterization of the Long-Run Equilibrium

In order to analyze the dynamics of the system, let us define a state of an economy as a profile of actions of players. Then the state space can be represented by $A^N \equiv \{0, 1\}^N$. Although the total number of possible states is 2^N, along the equilibrium path at most 2^F states will be observed because selfish players have a dominant strategy and are not allowed to make any mistakes or experimentation. In order to give a systematic order to these 2^F states, we introduce a *lexicographic ordering.*

Definition 2.3.1 *For any two states* (a, b)*, in which all selfish players defect, we define a lexicographic ordering,* $>$ *as follows.*

$a > b$ *if and only if one of the following conditions holds.*

$(i) \sum_{i=1}^{N} a_i e_i^* > \sum_{i=1}^{N} b_i e_i^*$

$(ii) \sum_{i=1}^{N} a_i e_i^* = \sum_{i=1}^{N} b_i e_i^*$*, and* $\sum_{i=1}^{N} a_i > \sum_{i=1}^{N} b_i$

$(iii) \sum_{i=1}^{N} a_i e_i^* = \sum_{i=1}^{N} b_i e_i^*$, $\sum_{i=1}^{N} a_i = \sum_{i=1}^{N} b_i$*, and* $\sum_{i=1}^{N} a_i 2^i > \sum_{i=1}^{N} b_i 2^i$

For each state, we first count how many active-fair players cooperate and then count how many discouraged-fair players cooperate. For those states which cannot be distinguished by these two numbers, we give a particular order by using the player numbers. Based on this ordering, we can assign a number to each state like 1 to the state $(0, 0, \cdots, 0)$ and $n \equiv 2^F$ to the state in which all fair players cooperate. Then the state space can be redefined by $Z = \{1, 2, \cdots, n\}$. In the theorem below, we will show that active-fair players cooperate and discouraged-fair players and selfish players defect in the long-run equilibrium. For convenience, let n^* be a number assigned by the lexicographic ordering to a state in which all active-fair players cooperate and all discouraged-fair and selfish players defect. Clearly, the value of n^* depends on the parameters of the game, as ϵ^* does.

On the modified state space, $Z = \{1, 2, \cdots, n\}$, we can define transition probabilities as follows:

$$p_{ij}(\epsilon) = Prob[z_{t+1} = j \mid z_t = i] \text{ for any } z \in Z \text{ and } \epsilon > 0.$$

By using the lexicographic ordering in counting the states, we get some nice properties of $p_{ij}(\epsilon)$.

Lemma 1 $(a) p_{ij}(\epsilon) = 0$ *for* $j < n^* \leq i$.

$(b) \sum_{j=n^*}^{n} p_{ij}(\epsilon) > 0$ *for* $i < n^*$.

Proof

(a) In states $i \geq n^*$, all active-fair players cooperate, but in states $j < n^*$, some active-fair players defect. As long as all active-fair players cooperate, their behavior rule tells them to cooperate, regardless of the value of ϵ.

(b) For a given state $i < n^*$, we can always find a state $j (n^* \leq j \leq n)$ in which all active-fair players cooperate, and all discouraged-fair players choose myopic best responses to the state i. Then for these i and $j, p_{ij}(\epsilon) > 0$ due to the fair players' random experiments. ■

Lemma 2 $(a) lim_{\epsilon \to 0} p_{ii}(\epsilon) = 1$ *if* $i = n^*$.

$(b) lim_{\epsilon \to 0} p_{ii}(\epsilon) = 0$ *if* $i > n^*$.

Proof

Since n^* is the best response to itself, (a) holds. In states $i > n^*$, some discouraged-fair players are cooperating. Then, some of them will change their actions to defection in the next period. Therefore, the same state cannot be repeated in the following period, which implies (b). ■

Due to the behavior rule derived from myopic best response and random experimentation, the dynamics of the game can be described by a Markov process. Now, we introduce the notion of stationary distribution on a simplex $\triangle \equiv \{\mu \in R_+^n \mid \sum_{i=1}^{n} \mu_i = 1\}$.

Definition 2.3.2 *$\mu(\epsilon)$ is a stationary distribution if and only if $\mu(\epsilon) = \mu(\epsilon)P(\epsilon)$, where $P(\epsilon)$ is an n x n transition probability matrix.*

It can be easily shown that there is a unique stationary distribution for our $P(\epsilon)$, which satisfies Lemma 1. We are interested in the long-run behavior of the game when the probability of random experimentation is small. To be precise, let us define the limit distribution and the set of long run equilibria.

Definition 2.3.3 *The limit distribution* μ^* *is defined by*
$\mu^* = lim_{\epsilon \to 0} \mu(\epsilon)$.

Definition 2.3.4 *The set of long run equilibria is defined as*
$LRE(\mu^*) = \{i \in Z \mid \mu_i^* > 0\}$.

Now, we can show the following theorem.

Theorem 1 *For a given heterogeneous population game, $\mathcal{G}$, there exists a unique long-run equilibrium in which all active-fair players cooperate and all discouraged-fair and selfish players defect, i.e.,*

$$lim_{\epsilon \to 0} \mu_{n^*}(\epsilon) = 1.$$

Before we prove Theorem 1, it is useful to introduce two kinds of graphs defined on a state space $Z = \{1, 2, \ldots, n\}$. [12] First, a z-tree h is defined as a directed graph defined on Z such that each state except z has a unique successor, and that there are no closed loops. The set of z-trees are denoted by H_z. Now, for each z-tree, calculate the product of transition probabilities along the tree, and then take the summation of the products for all z-trees. Then this sum will be denoted by $q_z \equiv \sum_{h \in H_z} \prod_{(i \to j) \in h} p_{ij}$. Second, for a given state z, define a digraph on Z such that each state i has a unique successor, $j \neq i$, and that there is a unique closed loop, containing state z. The set of these graphs is denoted by G_z. For example, Figure

[12] In characterizing the limit distribution, I simply follow KMR(1993)'s approach.

2.3 and 2.4 illustrate the digraphs in H_z and G_z for $Z = \{1, 2, 3\}$. For each $g \in G_z$, calculate the product of transition probabilities along g, and then take the summation of the products over $g \in G_z$. Let us denote this by $Q_z \equiv \sum_{g \in G_z} \prod_{(i \to j) \in g} p_{ij}$.

Lemma 3 *$q \equiv (q_1, q_2, \cdots, q_n)$ is proportional to μ.*

Proof

There are two ways to generate G_z from H_z. First, for each $h \in H_{z'}, z' \neq z$, by adding a branch from z' to z, we can generate G_z. Second, for each $h \in H_z$, by adding a branch from z to $z' \neq z$, we can generate the same G_z. Thus, Q_z can be expressed in two equivalent ways, so that $Q_z = \sum_{k \neq z} q_k p_{kz} = \sum_{l \neq z} q_z p_{zl}$. The right hand side of this equation is equal to $(1 - p_{zz}) q_z$. Thus, the equation above can be written as $\sum_k q_k p_{kz} = q_z$, which implies $qP = q$. Therefore, q is proportional to μ. ■

Since q is proportional to μ, we get $\mu_z^* = lim \frac{q_z(\epsilon)}{\sum_i q_i(\epsilon)} = \frac{q_z^*}{\sum_i q_i^*}$, where q^* is defined using $p_{ij}^* \equiv lim_{\epsilon \to 0} p_{ij}(\epsilon)$ instead of $p_{ij}(\epsilon)$.

Lemma 4 *$(1 - p_{zz}^*) q_z^* = 0$ for any $z \in Z = \{1, 2, \cdots, n\}$.*

Proof

If $z = n^*$, then $(1 - p_{n^*n^*}^*) q_z^* = 0 \cdot q_z^* = 0$. If $z \neq n^*$, for any tree $h \in H_z$, there is an arrow from n^* to some state $j \neq n^*$. But the probability of this transition, $p_{n^*j}^* = 0$ for all $j \neq n^*$. Hence, for any $h \in H_z, \prod_{(i \to j) \in h} p_{ij}^* = 0$. Thus, $q_z^* = \sum_{h \in H_z} \prod_{(i \to j) \in h} p_{ij}^* = 0$. This establishes the lemma. ■

Proof of Theorem 1

Lemma 1(a) simplifies the transition probability matrix $P(\epsilon)$ as follows.

$$P(\epsilon) = \begin{pmatrix} p_{11}(\epsilon) & p_{12}(\epsilon) & \cdots & p_{1n^*-1}(\epsilon) & p_{1n^*}(\epsilon) & \cdots & p_{1n}(\epsilon) \\ p_{21}(\epsilon) & p_{22}(\epsilon) & \cdots & p_{2n^*-1}(\epsilon) & p_{2n^*}(\epsilon) & \cdots & p_{2n}(\epsilon) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n^*-11}(\epsilon) & p_{n^*-12}(\epsilon) & \cdots & p_{n^*-1n^*-1}(\epsilon) & p_{n^*-1n^*}(\epsilon) & \cdots & p_{n^*-1n}(\epsilon) \\ 0 & 0 & \cdots & 0 & p_{n^*n^*}(\epsilon) & \cdots & p_{n^*n}(\epsilon) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & p_{nn^*}(\epsilon) & \cdots & p_{nn}(\epsilon) \end{pmatrix}$$

From $\mu(\epsilon)P(\epsilon) = \mu(\epsilon)$, we have [13]

$$\begin{aligned} \mu_1 p_{1n^*} + \mu_2 p_{2n^*} + \cdots + \mu_n p_{nn^*} &= \mu_{n^*} \\ \mu_1 p_{1n^*+1} + \mu_2 p_{2n^*+1} + \cdots + \mu_n p_{nn^*+1} &= \mu_{n^*+1} \\ &\vdots \\ \mu_1 p_{1n} + \mu_2 p_{2n} + \cdots + \mu_n p_{nn} &= \mu_n \end{aligned}$$

Summing both sides of these equations gives rise to

$$\sum_{i=1}^{n} \mu_i \sum_{j=n^*}^{n} p_{ij} = \sum_{j=n^*}^{n} \mu_i \tag{2.1}$$

This equation can be written as follows.

$$\sum_{i=1}^{n^*-1} \mu_i \sum_{j=n^*}^{n} p_{ij} + \sum_{i=n^*}^{n} \mu_i \sum_{j=1}^{n^*-1} (1 - p_{ij}) = \sum_{j=n^*}^{n} \mu_i \tag{2.2}$$

[13]From now on, for notational simplicity, we replace $p_{ij}(\epsilon)$ and $\mu_i(\epsilon)$ by p_{ij} and μ_i, respectively.

Since $p_{ij} = 0$ for $j < n^* \leq i$, the second term of the left-hand side of this equation is reduced to $\sum_{j=n^*}^{n} \mu_i$. Then from the equation (2.1), we get $\sum_{i=1}^{n^*-1} \mu_i \sum_{j=n^*}^{n} p_{ij} = 0$. Now notice that $\sum_{j=n^*}^{n} p_{ij} > 0$ for $i < n^*$ from Lemma 1(b). From this it follows that $\mu_1 = \mu_2 = \cdots = \mu_{n^*-1} = 0$, and thus $\mu_1^* = \mu_2^* = \cdots = \mu_{n^*-1}^* = 0$. Since we are interested in the limit sets, we can focus on the reduced state space. $\bar{Z} = \{n^*, n^*+1, \cdots, n\}$. Since the argument used in the proof of Lemma 4 does not depend on the specification of state space, we get $(1-p_{zz}^*)q_z^* = 0$ for any $z \in \bar{Z}$, where q_z^* is defined using only the states contained in $\bar{Z}$. Then for $z > n^*, (1-p_{zz}^*) = 1$ implies $q_z^* = 0$. Then $\mu_z^* = 0$ for $z > n^*$, because $\mu_z^* = \frac{q_z^*}{\sum_i q_i^*}$. Therefore, $\mu_{n^*} = 1$ from $\sum_{i=1}^{n} \mu_i^* = 1$. ■

2.4 Simulation Results

As shown in the previous section, in the long-run equilibrium, active-fair players cooperate while selfish and discouraged-fair players defect. However, how many fair players are discouraged depends on the initial configuration of players and the matching rule. If the same population is spatially distributed in a different way, the long-run cooperation rate will also be different. For example, for a sufficiently high m, the cooperation rate will be smaller if two types of players are integrated rather than segregated. We would like to see how the "average" cooperation rate in the long-run equilibrium changes as we vary matching rules. For this purpose, I performed the following simulation analysis. [14]

The behavior rule specified in Section 3 can be used to generate various rules for fair players depending on the values of g, y, and l. For example, if $1 < \frac{g-y}{l} < \frac{k+1}{k-1}$,

[14]There are many simulation analyses on the evolution of cooperation in various settings. For example, see Glance and Huberman (1993), Nowak and May (1992).

fair players will employ *follow-the-majority rule* in deciding their choices, which is described by [15] [16]

$$B_i(\mathbf{a}_{t-1}) = \begin{cases} 0 & \text{if } i \in S, \\ 0 & \text{with probabilty } (1-\epsilon) \text{ if } i \in F \text{ and } \sum_{j=1}^{k} a_{i\pm jt-1} < k, \\ 1 & \text{with probabilty } \epsilon \text{ if } i \in F \text{ and } \sum_{j=1}^{k} a_{i\pm jt-1} < k, \\ 1 & \text{if } i \in F \text{ and } \sum_{j=1}^{k} a_{i\pm jt-1} \geq k. \end{cases}$$

According to follow-the-majority rule, fair players cooperate unless more than half of their neighbors defect. Follow-the-majority rule is useful in that it is consistent with commonly observed behavior rule of fair players, and that it also simplifies the simulation analysis.

2.4.1 Long-Run Cooperation Rate

For a given fraction of selfish players, the long-run cooperation rate is determined by the initial configuration of each type, the size of neighborhood, the size of population, and the degree of overlap or dimension effect. Table 2.2 presents the average long-run cooperation rate for various matching rules, where $N = 100$ and number of repetitions $r = 100$. As expected, the long-run cooperation rate is decreasing in the fraction of selfish players m. [17]

[15]This condition says that the loss from deviating from mutual cooperation should be slightly greater than that from mutual defection. As the number of neighbors increases, the set of parameter values satisfying this condition becomes smaller.

[16]Due to $\frac{g-y}{l} > 1$, this follow-the-majority rule is biased toward cooperation when there is a tie in the number of cooperators and defectors. If $\frac{g-y}{l} = 1$, fair players become indifferent between two actions in the case of tie.

[17]From now on, the long-run cooperation rate means the *average* long-run cooperation rate.

Number of Neighbors Effect

Figures 2.5 and 2.6 show how the long-run cooperation rate changes as the number of neighbors increases. There are two effects behind this number-of-neighbors effect. First, we can expect a globalization effect. As the number of neighbors approaches to the size of population, fair players' action will be mainly determined by m; In the case of global interactions, all fair players defect (cooperate) when $m \geq 50\%$ ($m < 50\%$). Thus for $m < 50\%$, as the number of neighbors approaches N, the long-run cooperation rate will increase. This effect is shown in Figure 2.7, which reports the simulation results for wide range of neighborhood sizes. Second, there is a bias effect. As the number of neighbors increases, follow-the-majority rule of fair players makes them more likely to defect because it favors cooperation when there are equal number of cooperators and defectors. [18] However, if the population is highly selfish, or matching rule is quite *local*, the neighborhood size has a negative effect on the long-run cooperation rate. Therefore, the neighborhood size effect can be summarized as follows. First, if $m > 0.5$, then as the number of neighbors increases, the long-run cooperation rate decreases. Second, if $m < 0.5$, then the long-run cooperation rate initially falls and, beyond some point, rises again.

Population Size Effect

Figure 2.8 shows how the long-run cooperation rate is affected by the size of total population. Striking is the fact that the larger the population, the lower the minimum cooperation rate. This tells us that large society is more likely to suffer from extremely low cooperation even when the population is not seriously selfish.

The combination of a number-of-neighbors effect and a population size effect can

[18] If $\frac{k-1}{k+1} < \frac{g-y}{l} < 1$, we obtain another follow-the-majority rule that favors defection. In this case, this second effect is in the opposite direction.

explain the commonly observed fact that as people interact with more people, they become less cooperative. For example, people living in a large and crowded city like New York or Seoul are relatively non-cooperative. As social interaction becomes broader with the spread of modern civilization, social cooperation becomes harder to realize.

The Rate of Convergence to Long-Run Equilibrium

For low m, the number of neighbors effect is negligible. However, it takes more time with larger number of neighbors case to converge to the long-run equilibrium. Therefore, the number of neighbors effect matters even for low m during the transition period. This point is clear in Figure 2.9.

Dimension Effect

Figures 2.10 and 2.11 depict the effect of the degree of overlap on the long-run cooperation rate. Beyond some level of m, the long-run cooperation rate is lower in two-dimensional matching. There are two factors behind this dimension effect. First, a high degree of overlap implies that discouraged-fair players have many neighbors who have discouraged them, and with whom they can discourage other neighbors. Second, a high degree of overlap also implies that discouraged-fair players have few neighbors whom they can discourage. As m increases, the second factor becomes more important. Therefore, when the fraction of selfish people is relatively high, one-dimensional matching is desirable and vice-versa.

2.4.2 Payoff Difference

In order to compare the average payoff of players of each type, we consider a numerical payoff matrix. The payoff matrix from play between selfish and fair type is presented in Table 3. If $0 < \delta < \frac{2}{k-1}$, fair players will use follow-the-majority strategy. By assuming that δ is close to 0, however, we ignore δ in calculating the average payoff of players because this does not make any significant difference. Figure 2.12 provides the level of utility for each type. For any m, a fair player's average payoff is decreasing as the number of neighbors increases. Here we consider only the quite local matching rules such as C2, C4, and C8. But the effect of the number of neighbors on the average payoff of selfish players depends upon the level of m. For low m, the long-run cooperation rate is increasing in the number of neighbors, so that a selfish player's benefit from cheating becomes larger. For high m, however, as more fair players defect, the chance of taking advantage of other people becomes smaller. Figure 2.13 depicts the payoff difference of two types of players. Since high m makes agents less cooperative, the payoff difference becomes smaller as m increases. The payoff difference becomes smaller (larger) as the number of neighbors increases in a relatively selfish (fair) population.

Until now, we assumed that player's types are fixed. Now let us suppose that if the payoff difference between two types is greater than a critical value, players are allowed to change their types. Consider a population with a small fraction of selfish players. As the number of neighbors increases, the payoff difference becomes larger. Then some fair players will be changed to selfish players. This will lower the overall cooperation rate while the payoff difference becomes smaller. By the same argument, if we start with a highly selfish population, the payoff difference becomes smaller as the number of neighbors increases. Then there will be more fair players than in the beginning. Therefore we can conclude that if players can change their types in response to the change in payoff difference, the fraction of selfish players tends to approach a medium level as the number of neighbors increase. [19]

2.5 An Application to Implicit Collusion in the Monopolistically Competitive Market

2.5.1 The Economic Framework

Consider a monopolistically competitive market in which a large number of firms produce a homogeneous product. Each firm does not compete with all other firms. but with a subset of these firms. For a simple description of the local interactions among firms, let us suppose that N firms are geographically distributed on a circle or a torus as illustrated in Figure 2.14. [20] In both cases. a group of consumers (C) is located between two adjacent firms(F). Each consumer wishes to purchase one unit of good, but because of transportation costs. he can only buy from one of the two closest firms. Therefore, each firm on a circle interacts with two rival firms. and each firm operating on a torus interacts with four rivals. [21]

To simplify computations, we assume that production cost is zero. The demand of each consumer group is normalized at 2. Suppose that initially all firms are selling the product at a low price, P_L, and market share is equally distributed across firms.

[19] Frank, Gilovich, and Regan (1993) derived a similar result for one-shot prisoner's dilemma. They considered two types of players, those who always cooperate and those who always defect. And players are assumed to be able to tell the type of other players with some effort and choose their own opponent. In this framework, they show that the population settles at a stable mix of cooperators and defectors, one in which everyone receives the same average payoff and therefore equally likely to survive.

[20] We may replace the assumption of geographically distributed homogeneous firms by the assumption that firms produce differentiated products and that each consumer group has demand for only two perfect substitutes without much loss of generality.

[21] For this particular example, only two matching rules apply, C2 and T4.

Now suppose that for some reason, consumers' reservation price has risen to P_H, and this becomes a common knowledge among firms. This makes a room for firms to collude and raise the price to $P = P_H$. Since there is a large number of firms in this market, however, any collusive behavior tends to be implicit and spontaneous. If all firms charge the high price, the profit of each firm will rise to P_H from P_L. If one of the two firms competing for the same group of consumers charges a high price and the other firm charges a low price, then the high price firm will get 0 and the low price firm will get $2P_L$. We assume that firms are utility maximizers and that the utility function depends on the firm's type. For a selfish firm, the level of profit represents the level of utility. But a fair firm is characterized as feeling guilty if it breaks the collusion by undercutting the rival firm, so that it gets the utility of $2P_L - g$ in this case. Table 2.4 describes the payoff in terms of utility for the two firms competing for the same group of consumers.

When a firm meets a consumer, it cannot know with which firm it is competing for the particular customer. When deciding the selling price, therefore, each firm considers the prices of all of its rival firms and charges the same price for all of its customers. Selfish firms will stick to the low price, which is dominant strategy for them. On the other hand, if $3P_L - P_H < g < \frac{11}{3}P_L - P_H$ is satisfied, fair firms in the model of C2 or T4 will use the follow-the-majority rule. And by assuming $P_H > 2P_L$, we can guarantee any fair firms incentive to make random experimentation.

2.5.2 Relationship between Structure of Interactions and Degree of Collusion

In the long run, only and all active-fair firms are expected to collude by the dynamics described in Section 3. Figure 2.15 compares the long-run cooperation rate of C2 and

T4. For $m > 0.30$, C2 yields the higher cooperation rate. Therefore, firms will make a larger profit in C2 and consumers will face a lower price in T4. The significance of this difference would be large during the transition period if the probability of random experimentation is small.

From the simulation result in Section 4, we can get the following implication for the relationship between the number of rival firms and the degree of implicit collusion. First, when $m > 0.5$, the more rival firms each firm competes with, i.e., the more variety of substitute good each consumer has, the lower the average price. Second, when $m < 0.5$, as the number of rival firms increases, the degree of collusion initially declines and, beyond some point, rises. This result is in contrast with a common knowledge that the more firms in the industry, the harder it is to keep the collusion. It depends on what proportion of firms in the industry are fair type.

The literature on cartel among selfish oligopolistic firms tells us that with more firms, it becomes harder to sustain the cartel because of the increased difficulty in enforcing the explicit agreement. In a monopolistically competitive industry, any collusive behavior tends to be implicit because of the network structure of interactions across the whole industry. When fair firms coexist with selfish firms in the industry, the structure of interactions plays an important role in determining the performance of collusion. In the oligopoly industry, firms tend to interact globally. On the contrary, monopolistically competitive firms tend to compete with a small number of rival firms, where the number of rival firms for each firm is determined by substitutability among products. In the global interaction case, collusion among fair firms will be either perfect success or complete failure, depending only on the composition of two types in the industry. However, in the local interaction case, the degree of collusion will be in between these two extremes, where the average price level is determined by the number of rival firms as well as the proportion of each type in the industry.

2.6 Conclusion

In this paper I have discussed how fair players will cooperate in the face of selfish players who never cooperate. The main conclusion of this analysis is that the structure of interactions plays an important role in determining the level of cooperation in a heterogeneous population. This implies that when we study the collusion of firms, we need to take into account the market structure. In the oligopoly industry, for example, firms tend to interact globally. On the other hand, monopolistically competitive firms tend to interact locally. Therefore, the performance of collusive behavior can be different in these two industries.

It will be an interesting project to verify the simulation results by using empirical data about the performance of collusive behavior of monopolistically competitive industries with different number of rival firms. This model can also be used to explain sociological issues such as the effect of the interaction structure on the rate of drug usage among the youth, the rate of firearms possession, the diffusion of fashion, and student behavior in school.

Table 2.1: Three Possible Games

(* $x > 0, l > 0$, and $0 < y < g$.)

S:F	C	D
C	x, x	$-l, x + y - g$
D	$x + y, -l$	$0, 0$

S:S	C	D
C	x, x	$-l, x + y$
D	$x + y, -l$	$0, 0$

F:F	C	D
C	x, x	$-l, x + y - g$
D	$x + y - g, -l$	$0, 0$

Table 2.2: Long-Run Cooperation Rate

m (%)	C2	C4	C8	C12	T4	T8	T12
10	89.15	89.59	89.95	90.00	89.71	89.98	90.00
20	77.10	76.30	77.19	77.75	77.69	78.89	79.83
30	63.82	59.10	52.21	46.68	62.79	54.46	50.19
40	50.31	41.22	22.85	9.94	43.77	5.79	1.06
50	36.95	25.87	8.02	3.23	23.24	0	0
60	25.40	13.43	2.89	0.49	9.71	0	0
70	15.09	5.84	1.03	0	2.46	0	0
80	6.96	1.68	0.05	0	0.20	0	0

Table 2.3: Payoff to Players of Different Type

(*$0 < \delta < \frac{2}{k-1}$.)

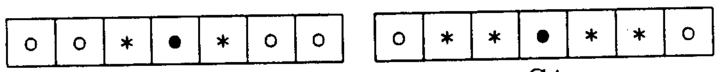
S:F	C	D
C	$2+\delta$, $2+\delta$	0, 1
D	3, 0	1, 1

Table 2.4: Payoff to Firms of Different Type

(*$3P_L - P_H < g < \frac{11}{3}P_L - P_H$ and $P_H > 2P_L$).

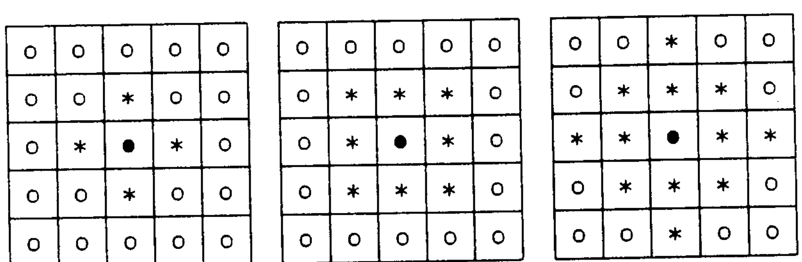
S:F	High Price (P=P_H)	Low Price (P=P_L)
High Price (P=P_H)	P_H, P_H	0, $2P_L - g$
Low Price (P=P_L)	$2P_L$, 0	P_L, P_L

Figure 2.1: One dimensional matchings



C2 C4

Figure 2.2: Two dimensional matchings



T4 T8 T12

Figure 2.3: H_3

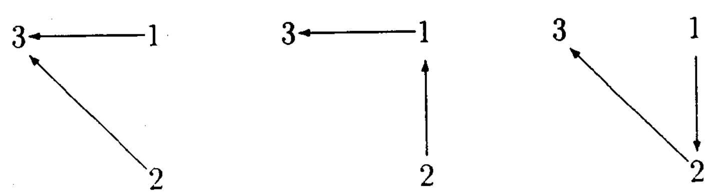


Figure 2.4: G_3

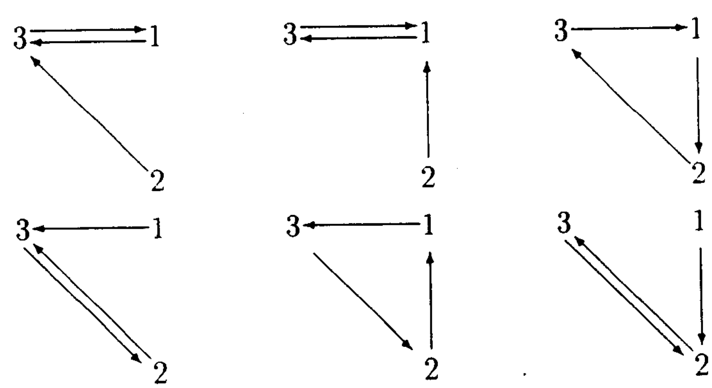


Figure 2.5: Number-of-Neighbors Effect (1): One Dimensional Case

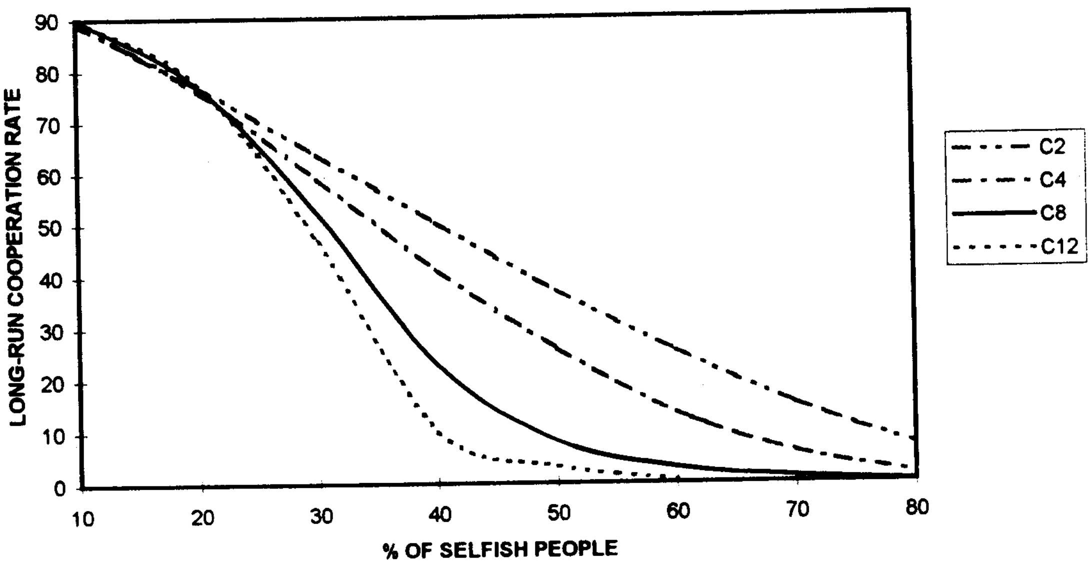


Figure 2.6: Number-of-Neighbors Effect (2): Two Dimensional Case

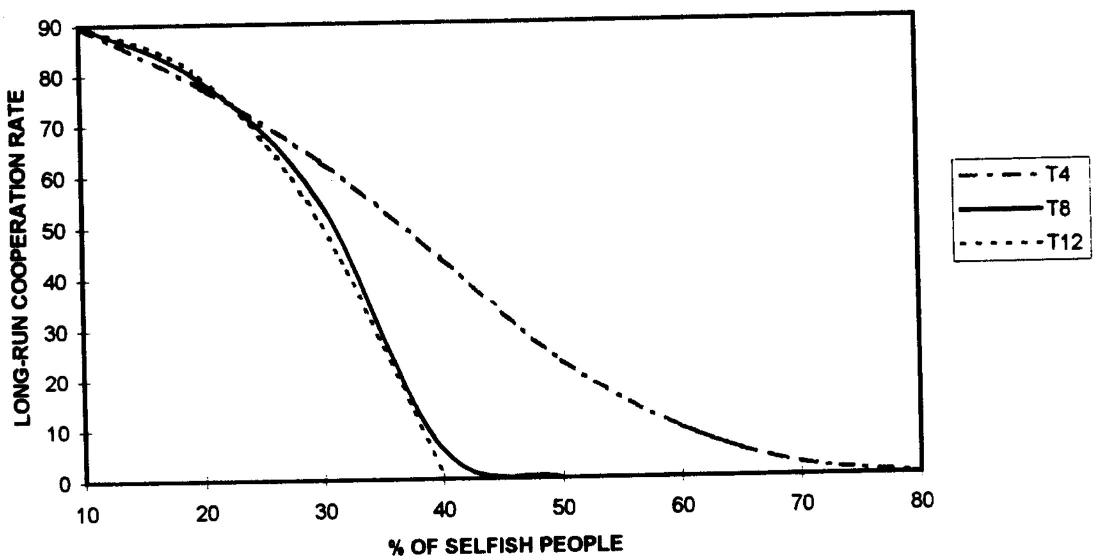


Figure 2.7: Number-of-Neighbors Effect (3): ($N = 100, r = 500$)

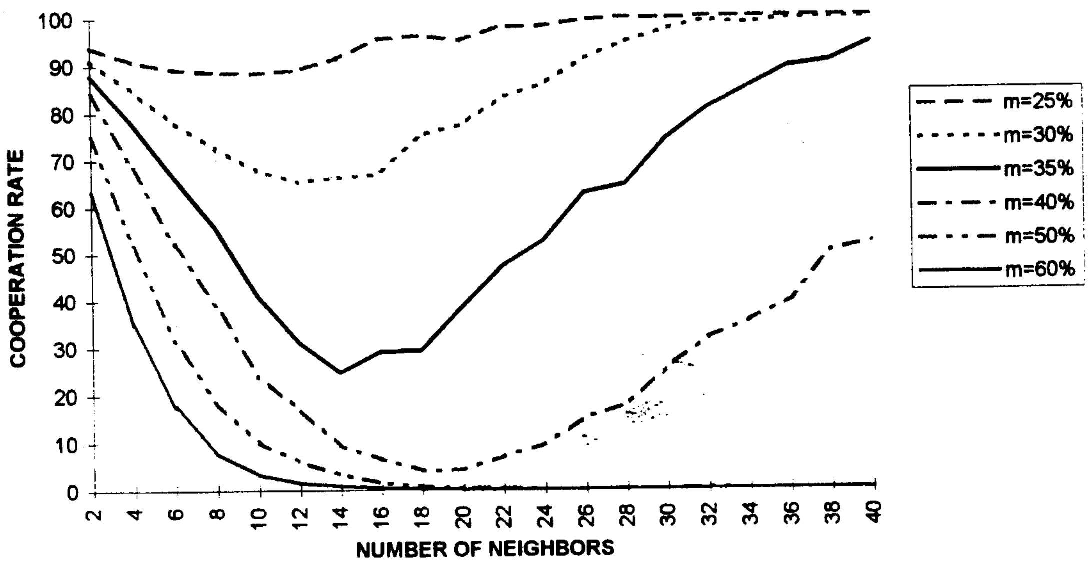


Figure 2.8: Population Size Effect: $(m = 35\%, r = 500)$

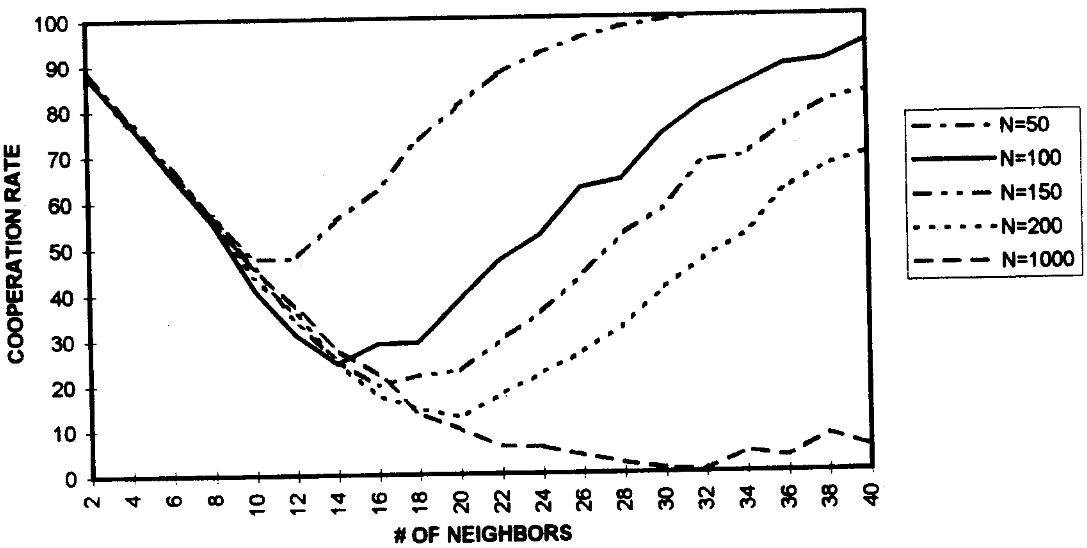


Figure 2.9: Cooperation Rate over Time ($m = 10\%, p = 0.1$)

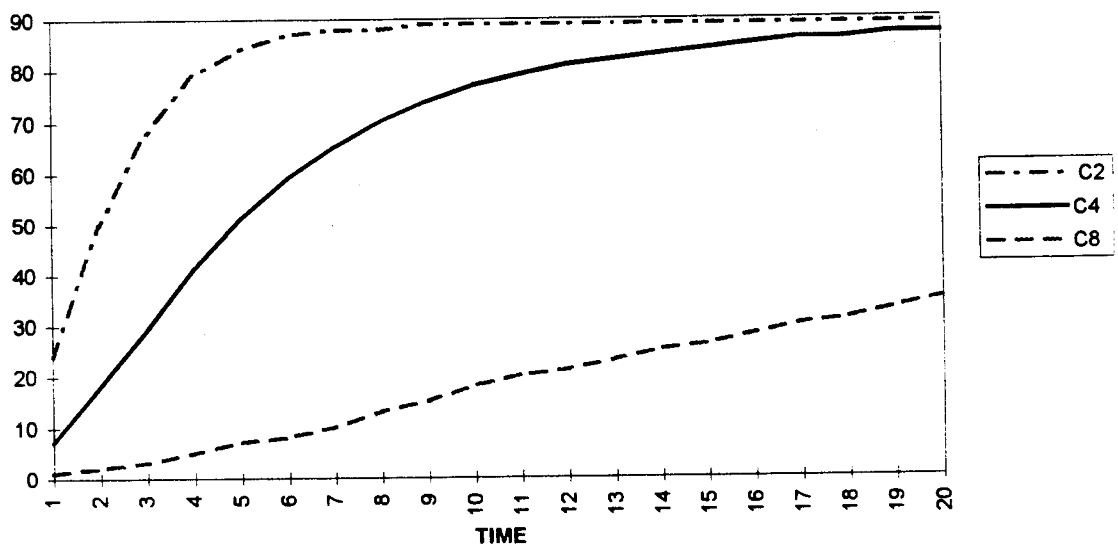


Figure 2.10: Dimension Effect (1)

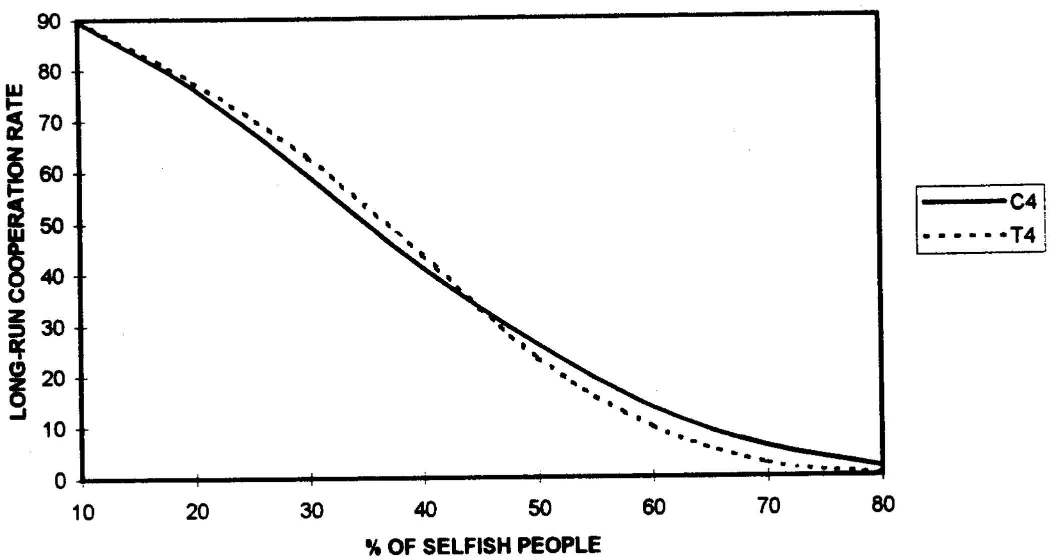


Figure 2.11: Dimension Effect (2)

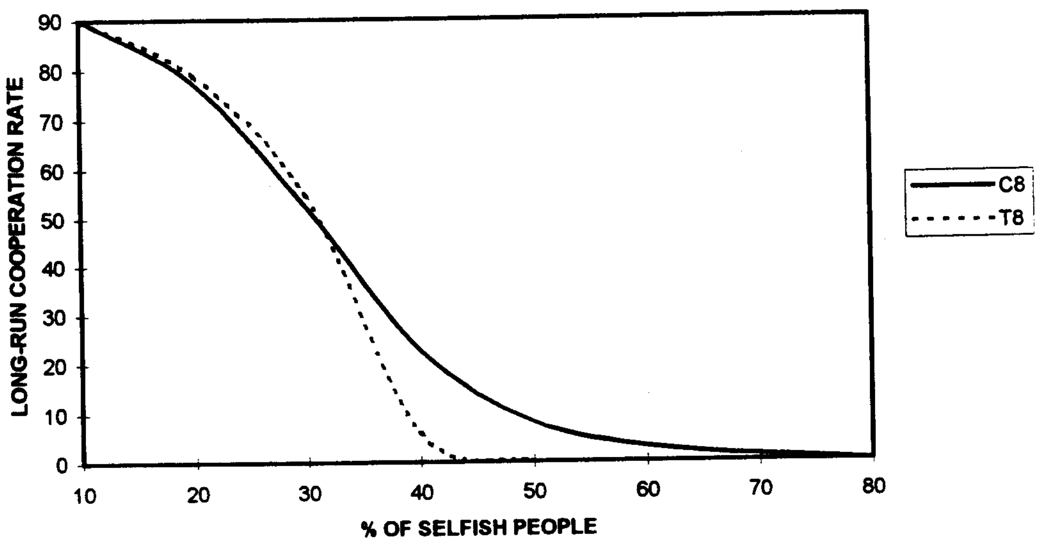


Figure 2.12: Average Payoff of Each Type (top:selfish)

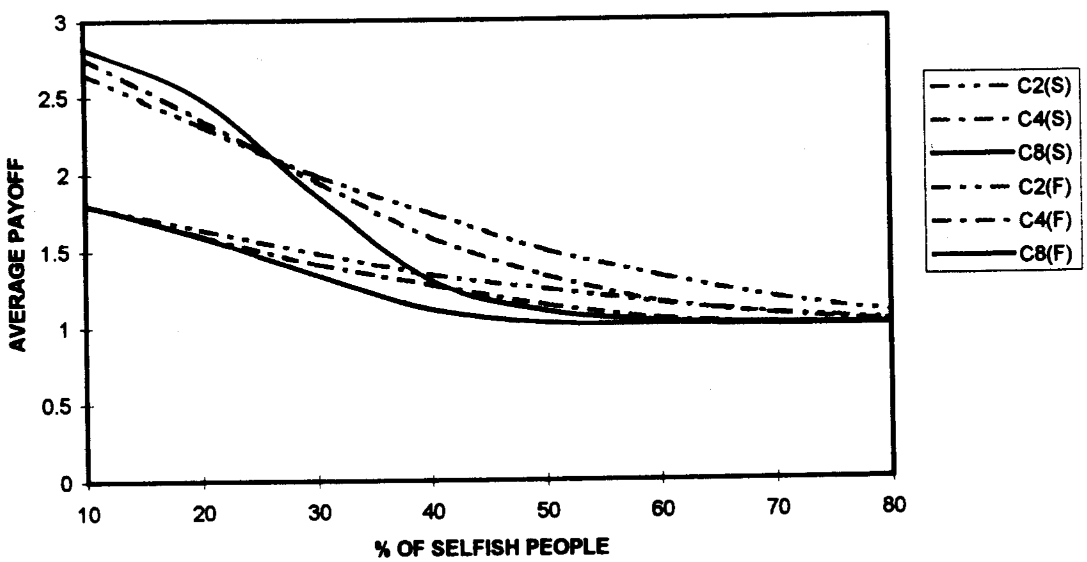


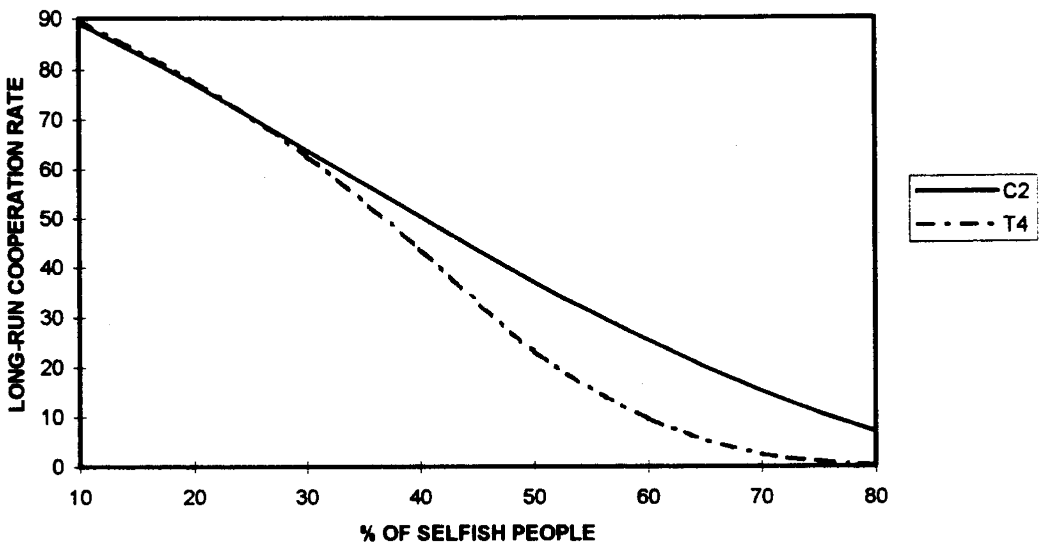
Figure 2.13: Payoff Difference between Types

Figure 2.14: Geographical Distribution of Firms and Consumers: C2 and T4.

C	**F**	C	*F*	C	**F**	C

C	F	C	**F**	C	F	C
	C		C		C	
C	**F**	C	*F*	C	**F**	C
	C		C		C	
C	F	C	**F**	C	F	C

Figure 2.15: Comparison of C2 and T4



Bibliography

[1] Axelrod, Robert (1984): *The Evolution of Cooperation*, New York: Basic Books.

[2] Axelrod, Robert and William Hamilton (1981): "The Evolution of Cooperation." *Science*, 211, 1390-1396.

[3] Blume, Lawrence E. (1993): "The Statistical Mechanics of Strategic interaction." *Games and Economic Behavior*, 5, 387-424.

[4] Dawes, Robyn M. and Richard H. Thaler (1988): "Anomalies: Cooperation." *Journal of Economic Perspectives*, 2, 187-198.

[5] Ellickson, Bryan (1990): "Nearest Neighbor Externalities," mimeo. UCLA

[6] Ellison, Glenn (1993): "Learning, Local Interaction, and Coordination," *Econometrica*. 61(5), 1047-71.

[7] Ellison, Glenn (1995): "Learning from Personal Experience: On the Large Population Justification of Myopic Learning Models," mimeo, MIT.

[8] Ely, Jeffrey (1995): "Local Conventions," mimeo, University of California at Berkeley.

[9] Frank Robert H., Thomas Gilovich and Dennis T. Regan (1993): "The Evolution of One-Shot Cooperation: An Experiment," *Ethology and Sociobiology*. 14, 247-256.

[10] Glance, Natalie S. and Bernado A. Huberman(1993): "The Outbreak of Cooperation," *Journal of Mathematical Sociology*, 17(4), 281-302.

[11] —— (1993): "Organizational Fluidity and Sustainable Cooperation," in *Computational Organizational Theory*, ed. by K. Carley and M. Prietula. Lawrence Elrbaum Associates.

[12] —— (1993): "Evolutionary Games and Computer Simulations." *Proc. Natl. Acad. Sci*. 90. 7716-7718.

[13] Hirshleifer, Jack (1985): "The Expanding Domain of Economics," *American Economic Review*. 75(6), 53-68.

[14] Hirshleifer, Jack (1993): "The Affections and the Passions: Their Economic Logic," *Rationality and Society*, 51(2), 185-202.

[15] Kandori, Michihiro, George J. Mailath and Rafael Rob (1993): "Learning, Mutation, and Long Run Equilibria in Games." *Econometrica*. 61(1). 29-56.

[16] Ledyard, John O. (1993): "Public Goods: A Survey of Experimental Research." forthcoming in *The Handbook of Experimental Economics.*

[17] Nowak, Martin A. and Robert M. May (1992): "Evolutionary Games and Spatial Chaos," *Nature*, 359, 826-829.

[18] Rabin, Mattew (1993): "Incorporating Fairness into Game Theory and Economics." *American Economic Review*, 83(5), 1281-1302.

[19] Schelling, Thomas C. (1971): "Dynamic Models of Segregation." *Journal of Mathematical Sociology,* 1, 143-186.

[20] Smith, John Maynard (1982): *Evolution and the Theory of Games,* Cambridge: Cambridge University Press.

Chapter 3

The Optimal Size of Committee

3.1 Introduction

In this chapter, we want to provide a theory of optimal size of decision making body, be it a single person or a committee of several. We abstract from political and strategic issues by assuming that all potential committee members have the same preferences. Our focus is on the element of slowness in a group decision, versus the greater accuracy of decision making due to the pooling of information from several sources.

The pooling of information is straightforward to model. We consider a decision whether or not to make an investment (or some other single binary decision). The investment has an uncertain value that may be either positive or negative. Each individual has an independent signal about the value of the investment. Consequently, the highest value will be realized if all these signals are averaged prior to making a decision.

The issue of delay is more complex. We model the cost of delay as simply the time cost of forgone profit. This implicitly includes various possibilities that can be modeled through discounting, such as a fixed chance each period that the investment opportunity disappears. There are several possible reasons why a large committee may have a greater propensity to delay. First, there may be more political maneuvering, but this is ruled out in our team setting. Second, there is the problem of communicating information. It may take some time for each member of the committee to fully communicate his signal to the entire committee, so that information can be pooled. Finally, there is the scheduling problem: the larger the committee, the more difficult it will be to schedule a meeting at which all members will be present. Implicitly this third model subsumes the second: the length of the meeting is due to the length of time required to communicate, and the scheduling problem occurs because all members must be present for the information to be pooled. Consequently, we focus on a model in which potential committee members are available to meet only at random times. The larger the committee, the less likely a meeting of all members can be scheduled quickly.

We also consider two types of committees: standing committees and *ad hoc* committees. Standing committees have a fixed set of members all of whom must be available simultaneously for a meeting to take place. This is a sensible form of organization when experience is an important consideration in decision making. In *ad hoc* meeting, scheduling is easier. There is a large set of equally capable potential committee members, and a meeting takes place as soon as the first group of n members can be arranged. Exactly which people form the committee is irrelevant. This is sensible when no special expertise is involved in making the decision.

Within the context of individualized information and scheduling, our goal is to analyze optimal committee size, whether one or greater. Our goal is to establish that the model reflects our basic intuition about committees; the greater the cost of time delay and the less diverse the information, the smaller the size of the optimal committee. We are apt to expect optimal *ad hoc* committee sizes to be larger than standing committees, as it is less costly to form a meeting. However, we will show that it is possible for the optimal size of standing committee to be larger than that of ad hoc committee.

Finally we will consider a situation in which experience can improve the accuracy of decision making. By repeatedly participating in a sequence of decision makings, standing committee members can accumulate expertise in the form of knowledge that cannot be transferred to the other people, or ability to obtain more precise signal. By modeling the effectiveness of standing committees in terms of the ability to receive more signals at the same cost, we will show that if the decision maker is facing a larger number of decisions in the future, he will form a standing committee consisting of fewer people.

This chapter is organized as follows. In Section 2, we model the decision maker's uncertainties and the distribution of signal, and derive the posterior distribution of the value of investment. And we also derive the expected return from the committee decision making. Section 3 examines the optimal size of committee by considering the cost of forming a committee. Section 4 investigates the ad hoc committee case. In Section 5, we compare the optimal sizes in the standing committee and the ad hoc committee. Finally, Section 6 extends the model to the relaxed decision making problem and Section 7 concludes.

3.2 The Pooling of Individualized Information

3.2.1 Uncertainties and Signals

We consider a decision whether or not to make an investment. The investment has an uncertain value, μ, which is distributed according to a normal distribution with the mean $\hat{\mu}$ and the variance s^2. The values of $\hat{\mu}$ and s^2 are known to the decision maker. where $\hat{\mu}$ can be interpreted as a prior about the value of the investment project. Then we have

$$\mu \sim N(\hat{\mu}, s^2)$$
$$f(\mu) = \tfrac{1}{s}\phi(\tfrac{\mu-\hat{\mu}}{s}),$$

where $f(\mu)$ denotes the probability density function of μ. And $\phi(u)$ denotes the probability density function of standard normal distribution, so we have

$$\phi(u) = \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}u^2).$$

Then the cumulative distribution function is given by

$$\Phi(x) = \int_{-\infty}^{x} \phi(u)du.$$

Each individual i receives a signal x_i about the value of the investment project. Signals are independently drawn from a normal distribution with the mean μ and the variance σ^2, where σ^2 is known to the decision maker. Then we have

$$x_i \sim N(\mu, \sigma^2)$$
$$g(x_i \mid \mu) = \frac{1}{\sigma}\phi(\frac{x_i - \mu}{\sigma}),$$

where $g(x_i \mid \mu)$ is the conditional probability density function of x_i given that the value of investment is μ.

Now consider a committee consisting of n individuals. By pooling n signals of this committee, we can compute the sample mean, $\bar{x}_n$. [1] Then we have

$$\bar{x}_n \sim N(\mu, \frac{\sigma^2}{n})$$
$$h(\bar{x}_n \mid \mu) = \frac{\sqrt{n}}{\sigma}\phi(\frac{\sqrt{n}(\bar{x}_n - \mu)}{\sigma}),$$

where $h(\bar{x}_n \mid \mu)$ denotes the conditional probability density function of $\bar{x}_n$ given that the value of investment is μ. Notice here that the larger committee, the smaller the variance of the sample mean. Thus, the decision maker can get a more accurate estimate of the unknown value of the investment.

[1]Here we implicitly assume that individuals within any committee can fully communicate their signals to others. By contrast, Sah and Stiglitz (1984, 1985) consider the case in which communication is very limited such that individuals can convey to one another only whether their signal is positive or negative. Koh (1992, 1994) and Sah (1991) also adopt this assumption of human fallibility.

By using signals from the committee members, the decision maker will update his belief about the distribution of μ according to Bayes' rule.

Proposition 1 *Suppose that the prior distribution of μ is a normal distribution with known values of the mean $\hat{\mu}$ and the variance s^2. Suppose also that $(x_1, \cdots, x_n)$ is a random sample from a normal distribution for which the value of the mean μ is unknown and the value of the variance σ^2 is known. Then the posterior distribution of μ is a normal distribution for which the mean μ_1 and the variance s_1^2 are as follows:*

$\mu_1 = \lambda \bar{x}_n + (1 - \lambda)\hat{\mu}$

and

$s_1^2 = (1 - \lambda)s^2$, *where* $\lambda = \frac{ns^2}{\sigma^2 + ns^2}$.

Proof

See Theorem 3 in DeGroot(1989) p.324. ■

3.2.2 Expected Return from the Committee Decision Making

Once having pooled its members' information, the committee will undertake the investment project if $\lambda \bar{x}_n + (1 - \lambda)\hat{\mu} \geq 0$. Then the conditional probability that the committee will decide to make the investment given the value of μ can be expressed as

$$\begin{aligned}
Prob[\lambda\bar{x}_n + (1-\lambda)\hat{\mu} \geq 0 \mid \mu] &= Prob[-\bar{x}_n \leq \frac{1-\lambda}{\lambda}\hat{\mu} \mid \mu] \\
&= Prob[\frac{-\bar{x}_n + \mu}{\sigma/\sqrt{n}} \leq \frac{(1-\lambda)\hat{\mu}/\lambda + \mu}{\sigma/\sqrt{n}} \mid \mu] \\
&= \Phi(\frac{(1-\lambda)\hat{\mu}/\lambda + \mu}{\sigma/\sqrt{n}}) \\
&= \Phi(\frac{\sigma^2\hat{\mu}/ns^2 + \mu}{\sigma/\sqrt{n}}).
\end{aligned}$$

Now the expected return from the committee decision making can be written as follows.

$$\begin{aligned}
V(n;\sigma^2,s^2,\hat{\mu}) &= \int_{-\infty}^{\infty} \mu Prob[\lambda\bar{x}_n + (1-\lambda)\hat{\mu} \geq 0 \mid \mu] f(\mu) d\mu \\
&= \int_{-\infty}^{\infty} \mu\Phi(\frac{\sigma^2\hat{\mu}/ns^2 + \mu}{\sigma/\sqrt{n}})\frac{1}{s}\phi(\frac{\mu-\hat{\mu}}{s})d\mu.
\end{aligned}$$

The above form of $V(n)$ is difficult to integrate without making further assumption. In order to simplify the computation, we assume that $\hat{\mu} = 0$, i.e., the decision maker is indifferent to making investment before he collects any information.

Proposition 2 *If $\hat{\mu} = 0$, then the expected return from the committee decision making is given by*

$$V(n;\sigma^2,s^2) = \frac{1}{\sqrt{2\pi}}\frac{s^2\sqrt{n}}{\sqrt{ns^2+\sigma^2}}.$$

Proof

If $\hat{\mu} = 0$, then $V(n)$ is reduced to

$$V(n) = \int_{-\infty}^{\infty} \mu \Phi(\frac{\mu}{\sigma/\sqrt{n}}) \frac{1}{s} \phi(\frac{\mu}{s}) d\mu.$$

Now $V(n)$ can be integrated by parts. For this, let us change the variables as follows. [2]

$$\begin{aligned} u &= \Phi(\frac{\mu}{\sigma/\sqrt{n}}) \\ du &= \frac{\sqrt{n}}{\sigma} \phi(\frac{\mu}{\sigma/\sqrt{n}}) d\mu \\ dv &= \frac{\mu}{s} \phi(\frac{\mu}{s}) d\mu \\ v &= -s\phi(\frac{\mu}{s}) \end{aligned}$$

Then we have

$$\begin{aligned} V(n) &= \int_{-\infty}^{\infty} u dv \\ &= uv]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} v du \\ &= -s\Phi(\frac{\mu}{\sigma/\sqrt{n}})\phi(\frac{\mu}{s})]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{s\sqrt{n}}{\sigma} \phi(\frac{\mu}{\sigma/\sqrt{n}}) \phi(\frac{\mu}{s}) d\mu \\ &= \int_{-\infty}^{\infty} \frac{s\sqrt{n}}{\sigma} \phi(\frac{\mu}{\sigma/\sqrt{n}}) \phi(\frac{\mu}{s}) d\mu. \end{aligned}$$

Now we can rewrite the integrand as follows.

$$\begin{aligned} \phi(\frac{\mu}{\sigma/\sqrt{n}})\phi(\frac{\mu}{s}) &= \frac{1}{2\pi} exp - \frac{1}{2}(\frac{\mu^2}{(\sigma/\sqrt{n})^2} + \frac{\mu^2}{s^2}) \\ &= \frac{1}{2\pi} exp - \frac{1}{2}(\frac{\mu\sqrt{ns^2+\sigma^2}}{\sigma s})^2 \end{aligned}$$

[2]Note that $\frac{d\phi(u)}{du} = -u\phi(u)$.

Then we have

$$V(n) = \frac{1}{\sqrt{2\pi}} \frac{s\sqrt{n}}{\sigma} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp - \frac{1}{2} (\frac{\mu\sqrt{ns^2+\sigma^2}}{\sigma s})^2 d\mu.$$

Now let us change the variables as follows.

$$\begin{aligned} x &= \frac{\mu\sqrt{ns^2+\sigma^2}}{\sigma s} \\ d\mu &= \frac{\sigma s}{\sqrt{ns^2+\sigma^2}} dx \end{aligned}$$

Then we have

$$\begin{aligned} V(n) &= \frac{1}{\sqrt{2\pi}} \frac{s\sqrt{n}}{\sigma} \frac{\sigma s}{\sqrt{ns^2+\sigma^2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp - \frac{x^2}{2} dx \\ &= \frac{1}{\sqrt{2\pi}} \frac{s^2\sqrt{n}}{\sqrt{ns^2+\sigma^2}}. \qquad \blacksquare \end{aligned}$$

It is interesting to see how the expected return depends on n, σ^2, and s^2.

Lemma 1 *$(a) V(n;\sigma^2, s^2)$ is increasing in n.*

$(b) V(n;\sigma^2, s^2)$ is increasing in s^2.

$(c) V(n;\sigma^2, s^2)$ is decreasing in σ^2.

Proof

From the proposition 2, we know

$$V(n;\sigma^2, s^2) = \frac{1}{\sqrt{2\pi}} n^{\frac{1}{2}} s^2 (ns^2+\sigma^2)^{-\frac{1}{2}}.$$

Now we have only to check the signs of partial derivatives of $V(n;\sigma^2, s^2)$ with respect to n, s^2, and σ^2.

$$\begin{aligned}
\frac{\partial V}{\partial n} &= \frac{s^2}{\sqrt{2\pi}}[\frac{1}{2}n^{-\frac{1}{2}}(ns^2+\sigma^2)^{-\frac{1}{2}} - \frac{1}{2}n^{\frac{1}{2}}s^2(ns^2+\sigma^2)^{-\frac{3}{2}}] \\
&= \frac{\sigma^2 s^2}{2\sqrt{2\pi}}n^{-\frac{1}{2}}(ns^2+\sigma^2)^{-\frac{3}{2}} > 0 \\
\frac{\partial V}{\partial s^2} &= \frac{1}{\sqrt{2\pi}}n^{\frac{1}{2}}[(ns^2+\sigma^2)^{-\frac{1}{2}} - \frac{1}{2}ns^2(ns^2+\sigma^2)^{-\frac{3}{2}}] \\
&= \frac{1}{2\sqrt{2\pi}}n^{\frac{1}{2}}(ns^2+\sigma^2)^{-\frac{3}{2}}(2\sigma^2+ns^2) > 0 \\
\frac{\partial V}{\partial \sigma^2} &= -\frac{1}{2\sqrt{2\pi}}n^{\frac{1}{2}}s^2(ns^2+\sigma^2)^{-\frac{3}{2}} < 0 \qquad \blacksquare
\end{aligned}$$

Lemma 1 verifies our intuition. First, a bigger committee can make more accurate decision by pooling more information. Second, a high value of s^2 implies that there is a high chance that the absolute value of the investment is very large. Since the investment will be undertaken if and only if μ is judged to be negative, the expected return increases as s^2 increases. Third, if signals are less precise, the probability of making an incorrect decision is high, so the expected return becomes smaller.

3.3 The Optimal Size of Committee

By using more signals, the decision maker can improve the chance of making the right decision, i.e., undertaking only the profitable investment. However, we should also consider the cost involved in increasing the size of committee. There are many possible sources of this cost. It can be a compensation for hiring committee members, or it can be a time cost of possible delay in making a decision. Here we want to focus on the cost of scheduling a meeting in terms of delay. We model the cost of delay as simply the time cost of forgone profit. This implicitly includes various possibilities that can be modeled through discounting, such as a fixed chance each period that the investment opportunity disappears. Now we want to model the cost of delay in forming a committee of n people. Suppose that each individual is available to meet with probability p. We will consider two types of committees; standing committees and *ad hoc* committees. In this section, we focus only on standing committees. Standing committees have a fixed set of members all of whom must be available simultaneously for a meeting to take place. This is a sensible form of organization when experience is an important consideration in decision making.

Let $\pi^S(n;p)$ denote the probability that a committee of n individuals can be formed. Then we have

$$\pi^S(n;p) = p^n. \tag{3.1}$$

Then the probability that a meeting of n people is possible for the first time at period t is $(1-\pi(n))^{t-1}\pi(n)$. [3] And let δ denote the probability that the investment opportunity is still available next period.

Now the decision maker's optimization problem can be expressed as follows.

$$Max_n \quad \sum_{t=1}^{\infty}(1-\pi(n))^{t-1}\pi(n)\delta^{t-1}V(n) = \frac{\pi(n)V(n)}{1-\delta+\delta\pi(n)}.$$

which is equivalent to

$$Max_n \quad log\frac{\pi(n)}{1-\delta+\delta\pi(n)} + logV(n). \tag{3.2}$$

Let us define the total benefit and total cost of committee decision making as follows.

$$\begin{aligned} TB(n) &= logV(n) \\ TC(n) &= log(\frac{1-\delta}{\pi(n)}+\delta) \end{aligned}$$

[3] $\pi(n)$ is a simplifying notation for $\pi^S(n;p)$ until we introduce *ad hoc* committee.

Since the choice variable (n) is discrete, the optimal committee size, n^* satisfies

$$TB(n^*-1) - TC(n^*-1) < TB(n^*) - TC(n^*) < TB(n^*+1) - TC(n^*+1). \tag{3.3}$$

For simplicity, however, we want to approximate the discrete optimization problem by the continuous optimization problem. [4] Then the first order condition is

$$F(n) \equiv \frac{1-\delta}{1-\delta+\delta\pi}\frac{1}{\pi}\frac{\partial\pi}{\partial n} + \frac{1}{V}\frac{\partial V}{\partial n} = 0. \tag{3.4}$$

Let us define marginal benefit and marginal cost as follows.

$$\begin{aligned} MB(n) &= \frac{1}{V}\frac{\partial V}{\partial n} = \frac{1}{2n(ns^2/\sigma^2+1)} \\ MC^S(n) &= \frac{\delta-1}{1-\delta+\delta\pi^S}\frac{1}{\pi^S}\frac{\partial\pi^S}{\partial n} = \frac{(\delta-1)logp}{1-\delta+\delta p^n} \end{aligned}$$

Then the first order condition simply says that marginal cost and marginal benefit should be equal at the optimal size of committee, n_S^*.

Lemma 2 *(a)MB is decreasing in n.*

(b)MC^S is increasing in n.

Proof

$$\begin{aligned} \frac{\partial MB}{\partial n} &= -\frac{2ns^2/\sigma^2+1}{2n^2(ns^2/\sigma^2+1)^2} < 0. \\ \frac{\partial MC}{\partial n} &= \frac{1-\delta}{(1-\delta+\delta\pi)^2}\frac{\delta}{\pi}\left(\frac{\partial\pi}{\partial n}\right)^2 + \frac{1-\delta}{1-\delta+\delta\pi}\frac{1}{\pi^2}\left(\frac{\partial\pi}{\partial n}\right)^2 - \frac{1-\delta}{1-\delta+\delta\pi}\frac{1}{\pi}\frac{\partial^2\pi}{\partial n^2} \\ &> 0 \quad \text{if and only if} \quad \frac{\partial^2\pi}{\partial n^2}/\left(\frac{\partial\pi}{\partial n}\right)^2 < \frac{1}{\pi} + \frac{\delta}{1-\delta+\delta\pi}. \end{aligned}$$

[4] Since we have only one choice variable, this can be a good approximation.

Since we know

$$\begin{aligned}\frac{\partial \pi^S}{\partial n} &= p^n logp < 0, \\ \text{and} \quad \frac{\partial^2 \pi^S}{\partial n^2} &= p^n (logp)^2 > 0,\end{aligned}$$

we have

$$\frac{\partial^2 \pi^S}{\partial n^2} / (\frac{\partial \pi^S}{\partial n})^2 = \frac{1}{p^n} = \frac{1}{\pi^S}.$$

Therefore, MC^S increases as the committee size increases. ■

Corollary 1 *In case of standing committees, the second order condition is always satisfied for any n.*

Lemma 3 *MC^S is decreasing in δ.*

Proof

$$\frac{\partial MC^S}{\partial \delta} = \frac{1}{(1-\delta+\delta\pi^S)^2}\frac{\partial \pi^S}{\partial n} < 0. \qquad ■$$

Lemma 4 *MC^S is decreasing in p.*

Proof

$$\frac{\partial MC^S}{\partial p} = \frac{\delta - 1}{(1-\delta+\delta p^n)^2}(1-\delta+\delta p^n + (1-p)\delta n p^{n-1}) < 0. \qquad ■$$

Lemma 5 *MB is decreasing in $\frac{s^2}{\sigma^2}$.*

Proof

Since $MB(n) = \frac{1}{2n(ns^2/\sigma^2+1)}$, it is obvious that marginal benefit is decreasing in $\frac{s^2}{\sigma^2}$.

■

Theorem 2 *n^*_S is increasing in p, δ, and $\frac{\sigma^2}{s^2}$.*

Proof

It is following from Lemma 2, 3, 4, 5. ■

Theorem 1 verifies our intuition. First, the higher the probability with which potential committee members are available to meet, the easier it is to schedule a meeting. Therefore, the optimal size of committee is bigger. Second, if the investment opportunity is less likely to disappear, the cost of delay becomes smaller. Thus the optimal size of committee increases in δ. Third, as s^2 increases, the value of additional signal increases because of the higher probability of making big money, but as σ^2 increases, the value of additional signal decreases because the signal becomes less precise. Since the effects of s^2 and σ^2 are in the opposite direction, n^*_S is increasing in $\frac{\sigma^2}{s^2}$.

Corollary 2 *The set of parameter values for which the optimal committee is composed of a single individual is decreasing in p, δ, and $\frac{\sigma^2}{s^2}$.*

We can say that $n^*_S \leq 1$ if $MC^S(n = 1) \geq MB(n = 1)$, i.e., $\frac{s^2}{\sigma^2} \geq \frac{1-\delta+\delta p}{2(\delta-1)logp} - 1$. In Figure 3.1, the area above the curve shows the range of the parameter values for which the optimal committee size is equal to one, where we focus on the cases in which $\delta = 0.9$.

3.4 ad hoc Committees

In this section, we will consider *ad hoc* committees. There is a large set of N equally capable potential committee members, and a meeting takes place as soon as the first group of n members can be arranged. Exactly which people form the committee is irrelevant. This is sensible when no special expertise is involved in making the decision. Then the probability that an *ad hoc* committee of n members is formed in each period is given by

$$\pi^A(n; N, p) = \sum_{k=n}^{N} \binom{N}{k} p^k (1-p)^{N-k}. \tag{3.5}$$

Since this probability is defined only for integer n, the derivatives of this function can be replaced by the differences.

$$\begin{aligned}
\frac{\partial \pi}{\partial n} &\equiv \pi(n+1) - \pi(n) \\
&= -\binom{N}{n} p^n (1-p)^{N-n} < 0 \\
\frac{\partial^2 \pi}{\partial n^2} &\equiv \frac{\partial \pi(n+1)}{\partial n} - \frac{\partial \pi(n)}{\partial n} \\
&= -\binom{N}{n+1} p^{n+1} (1-p)^{N-n-1} + \binom{N}{n} p^n (1-p)^{N-n} \\
&< 0 \quad \text{if and only if} \quad n < p(N+1) - 1.
\end{aligned}$$

We know that $\frac{\partial MC}{\partial n} > 0$ if and only if $\frac{\partial^2 \pi}{\partial n^2} / (\frac{\partial \pi}{\partial n})^2 < \frac{1}{\pi} + \frac{\delta}{1-\delta+\delta\pi}$. Therefore, if $n_A^* < p(N+1) - 1$, and thus $\frac{\partial^2 \pi}{\partial n^2} < 0$, then the second order condition of optimization problem is satisfied. In contrast to the standing committee case, we have to check the second order condition in the *ad hoc* committee case.

Lemma 6 *MC^A is decreasing in δ.*

Proof

$$\frac{MC^A}{\partial \delta} = \frac{1}{(1-\delta+\delta\pi^A)^2} \frac{\partial \pi^A}{\partial n} < 0. \qquad \blacksquare$$

Lemma 7 *MC^A is decreasing in p.*

Proof

Since we know $\frac{\partial MC^A}{\partial p} = \frac{1-\delta}{(1-\delta+\delta\pi)^2} \cdot \frac{\delta}{\pi} \cdot \frac{\partial \pi}{\partial p}\frac{\partial \pi}{\partial n} + \frac{1-\delta}{1-\delta+\delta\pi}\frac{1}{\pi^2}\frac{\partial \pi}{\partial p}\frac{\partial \pi}{\partial n} - \frac{1-\delta}{1-\delta+\delta\pi} \cdot \frac{1}{\pi}\frac{\partial^2}{\partial p \partial n} > 0$ if and only if $\frac{\partial^2}{\partial p \partial n}/(\frac{\partial \pi}{\partial p} \cdot \frac{\partial \pi}{\partial n}) < \frac{1}{\pi} + \frac{\delta}{1-\delta+\delta\pi}$. In order to prove that marginal cost is decreasing in p, we have only to show

$$\frac{\partial^2 \pi}{\partial p \partial n} / \frac{\partial \pi}{\partial p} \cdot \frac{\partial \pi}{\partial n} \leq \frac{1}{\pi}. \tag{3.6}$$

Since we have

$$\begin{aligned}
\frac{\partial \pi^A}{\partial p} &= \sum_{k=n}^{N} \binom{N}{k} p^{k-1}(1-p)^{N-k-1}(k-Np) > 0 \quad \text{for} \quad 0 < n < N, \\
\frac{\partial^2 \pi^A}{\partial p \partial n} &= -\binom{N}{n} p(1-p)^{N-n-1}(n-Np) = \frac{n-Np}{p(1-p)} \cdot \frac{\partial \pi^A}{\partial n},
\end{aligned}$$

the equation (2) can be written as

$$\frac{n-Np}{\sum_{k=n}^{N}\binom{N}{k}p^k(1-p)^{N-k}(k-Np)} < \frac{1}{\sum_{k=n}^{N}\binom{N}{k}p^k(1-p)^{N-k}}.$$

And this inequality holds because we have

$$(n-Np)\sum_{k=n}^{N}\binom{N}{k}p^k(1-p)^{N-k} < \sum_{k=n}^{N}\binom{N}{k}p^k(1-p)^{N-k}(k-Np). \qquad \blacksquare$$

Theorem 3 *n_A^* is increasing in p, δ, and $\frac{\sigma^2}{s^2}$.*

Proof

The effect of an increase in p, or δ on n_A^* follows from Lemma 2, 6, and 7. And since the proof of Lemma 5 does not depend on cost condition, the effect of $\frac{\sigma^2}{s^2}$ on n^* is independent of the type of committee. $\blacksquare$

3.5 Comparison of the Optimal Sizes in the Standing Committee and the ad hoc Committee

Because of different cost conditions, the optimal size of standing committee may differ from the optimal size of ad hoc committee. We know that the total cost is always greater in the standing committee, since $\pi^A(n) > \pi^S(n)$ for $0 < n < N$. However, marginal cost can be greater in the *ad hoc* committee for n sufficiently close to N. Therefore, if n_A^* is sufficiently close to N, then n_S^* may be greater than n_A^*. In order to see this, we consider some numerical examples, which are depicted in Figure 3.2. [5] Here we consider the cases in which $\delta = p = 0.9$, and $\frac{s^2}{\sigma^2} = 0.1$. And (a), (b), and (c) are corresponding to the case in which $N = 15$, $N = 12$ and $N = 10$, respectively. As N decreases, we can see that the difference between the optimal committee sizes, $(n_A^* - n_S^*)$, is decreasing; when $N = 12$, n_A^* is almost the same as n_S^*. And when $N = 10$, the optimal size is smaller in the ad hoc committee case. This result reflects our intuition that the less the potential committee members, the sooner disappears the cost advantage of ad hoc committee in scheduling a meeting. Figure 3.3 shows how the optimal size of committee is changing as the values of $\frac{s^2}{\sigma^2}$, p, and δ change.

3.6 Repeated Decision Making

Until now, we have considered only one-shot decision making problem. In the real world, however, we may face a sequence of decision making problems of the same kind. In this case, standing committees may be more efficient than *ad hoc* committees. By repeatedly participating in a sequence of decision makings, standing committee mem-

[5]When we draw marginal cost curves, we are using $\pi(n+1) - \pi(n)$ instead of $\frac{\partial \pi(n)}{\partial n}$, because $\pi^A(n)$ is defined only for integer values. By doing this, we can compare two types of committees with ease.

bers can accumulate expertise in the form of knowledge that cannot be transferred to other people, or ability to obtain more precise signal than other unexperienced or less experienced people. In this section, we want to model the effectiveness of standing committees in obtaining signals. Since having more signals is equivalent to having better signals in improving the accuracy of decision making, in order to simplify the analysis, we assume that the number of signals that a standing committee member receives is proportional to the number of times that he or she has been participating in the decision makings of the committee.

Let K denote the number of decisions that a committee is expecting to make in the future. Then the expected return of k-th decision making can be written as $V(kn)$, for $k = 1, 2, \cdots, K$, since each individual receives k signals in the k-th stage. And suppose that the decision maker discounts the value of future investment by using the discount factor β. Then the optimal size of standing committee facing K decisions, n_S^*, solves

$$Max_n \quad (\frac{\pi(n)V(n)}{1-\delta+\delta\pi(n)}) + \beta(\frac{\pi(n)V(2n)}{1-\delta+\delta\pi(n)}) + \cdots + \beta^{K-1}(\frac{\pi(n)V(Kn)}{1-\delta+\delta\pi(n)}),$$

which is equivalent to

$$Max_n \quad \frac{\pi(n)}{1-\delta+\delta\pi(n)}\sum_{k=1}^{K}\beta^{k-1}V(kn),$$

which can be rewritten as

$$Max_n \quad log\frac{\pi(n)}{1-\delta+\delta\pi(n)} + log\sum_{k=1}^{K}\beta^{k-1}V(kn). \tag{3.7}$$

Let us define the total benefit and total cost of committee decision making as follows.

$$TB(n;K) = log\sum_{k=1}^{K}\beta^{k-1}V(kn)$$
$$TC(n) = log(\frac{1-\delta}{\pi(n)}+\delta)$$

Then the marginal benefit can be written as

$$MB(n;K) = \frac{\sum_{k=1}^{K}\beta^{k-1}kV'(kn)}{\sum_{k=1}^{K}\beta^{k-1}V(kn)}.$$

Then the first order condition is given by

$$MC^S(n_S^*(K)) = MB(n_S^*(K),K).$$

Lemma 8 *$MB(n;K)$ is decreasing in K.*

Proof

It is enough to show

$$\frac{(k+1)V'((k+1)n)}{V((k+1)n)} < \frac{kV'(kn)}{V(kn)},$$

which is equivalent to

$$\frac{(k+1)}{k}\frac{V'((k+1)n)}{V'(kn)} < \frac{V((k+1)n)}{V(kn)}. \tag{3.8}$$

Since we know

$$V(n) = \frac{s^2}{\sqrt{2\pi}}n^{\frac{1}{2}}(ns^2+\sigma^2)^{-\frac{1}{2}}$$
$$V'(n) = \frac{\sigma^2 s^2}{2\sqrt{2\pi}}n^{-\frac{1}{2}}(ns^2+\sigma^2)^{-\frac{3}{2}}$$

the right hand side of the inequality (3.8) is given by

$$\frac{V((k+1)n)}{V(kn)} = (\frac{k+1}{k})^{\frac{1}{2}}(\frac{kns^2+\sigma^2}{(k+1)ns^2+\sigma^2})^{\frac{1}{2}}.$$

And the left hand side of the inequality (3.8) is given by

$$\frac{k+1}{k}\frac{V'((k+1)n)}{V'(kn)} = (\frac{k+1}{k})(\frac{k}{k+1})^{\frac{1}{2}}(\frac{kns^2+\sigma^2}{(k+1)ns^2+\sigma^2})^{\frac{3}{2}}.$$

Then the difference between the right hand side and the left hand side of the inequality (3.8) is given by

$$\frac{V((k+1)n)}{V(kn)}(1 - \frac{kns^2+\sigma^2}{(k+1)ns^2+\sigma^2}) > 0. \qquad \blacksquare$$

Theorem 4 *$n_S^*(K)$ is decreasing in K.*

Proof

It is straightforward from Lemma 8. ■

If a committee is expecting to make a large number of similar decisions. the optimal size of committee is likely to be small, because as the committee members accumulate expertise, the marginal benefit from additional signal is decreasing. By the same logic, we can say that as the rate with which the expertise is accumulated increases, the optimal committee size is decreasing.

3.7 Conclusion

In this chapter, we verified our basic intuition about the optimal size of committee: First, the greater the cost of time delay and the less diverse the information, the smaller the size of the optimal committee. Second, for a wide range of parameter values, the optimal sizes are larger in the ad hoc committee than the standing committee. However, standing committee can be larger than ad hoc committee under some conditions. Third, when a committee is expecting to make a large number of similar decisions, the optimal committee size is likely to be small.

It is obvious that, for one-shot decision making, standing committees are definitely inferior to ad hoc committees because of larger cost and no chance to utilize experience. However, in the repeated decision making case, standing committees may can be better than ad hoc committees because of the benefit from experience. In the future research, I want to consider the relative performance of a standing committee and an ad hoc committee by examining the tradeoff between larger scheduling cost and better signals from expertise accumulation.

Figure 3.1: Set of Parameter Values for which $n_S^* = 1$

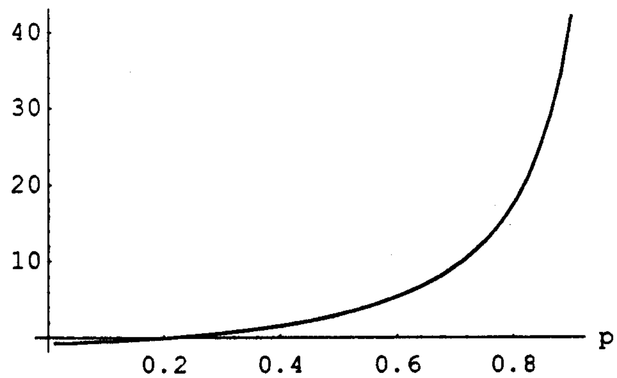
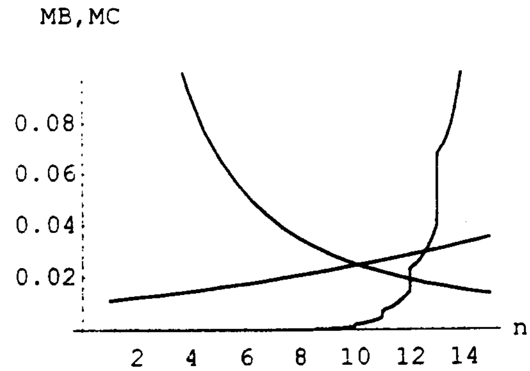
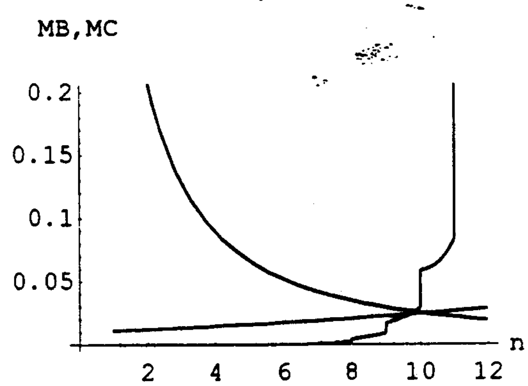


Figure 3.2: Comparison of n_A^* and n_S^*

(a): $p = \delta = 0.9, \frac{s^2}{\sigma^2} = 0.1, N = 15$



(b): $p = \delta = 0.9, \frac{s^2}{\sigma^2} = 0.1, N = 12$



(c): $p = \delta = 0.9, \frac{s^2}{\sigma^2} = 0.1, N = 10$

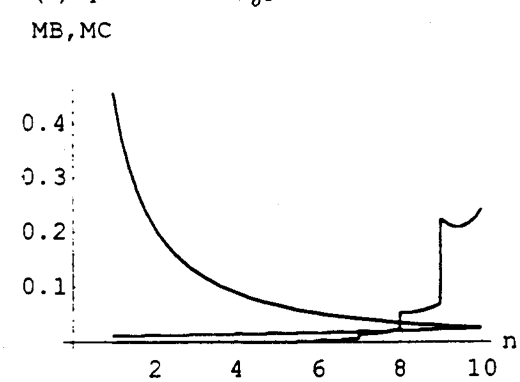
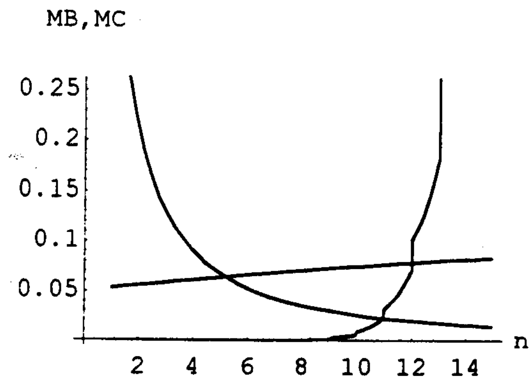
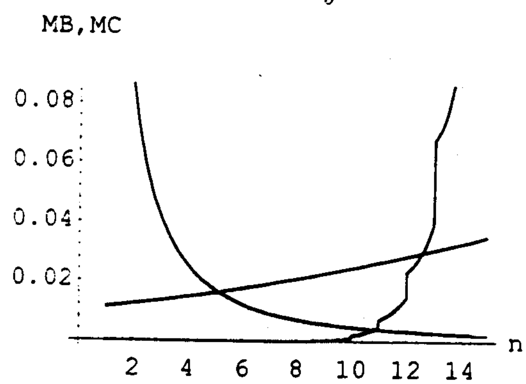


Figure 3.3: Comparison of n_A^* and n_S^*

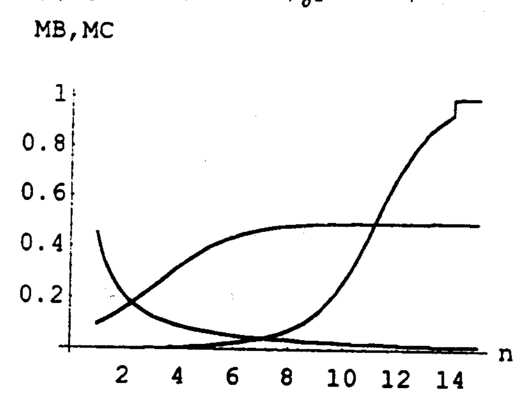
(a): $p = 0.9, \delta = 0.5, \frac{s^2}{\sigma^2} = 0.1, N = 15$



(b): $p = 0.9, \delta = 0.9, \frac{s^2}{\sigma^2} = 1, N = 15$



(c): $p = 0.5, \delta = 0.9, \frac{s^2}{\sigma^2} = 0.1, N = 15$



Bibliography

[1] DeGroot, Morris H. (1989): *Probability and Statistics*, Second Edition, Addison-Wesley.

[2] Koh, Winston T.H. (1992): "Human Fallibility and Sequential Decision Making: Hierarchy versus Polyarchy," *Journal of Economic Behavior and Organization*, 18, 317-345.

[3] —— (1994): "Making Decisions in Committees: A Human Fallibility Approach," *Journal of Economic Behavior and Organization*, 23, 195-214.

[4] Sah, Raag Kumar and Joseph E. Stiglitz (1984): "Economics of Committees," mimeo, Princeton University.

[5] Sah, Raag Kumar and Joseph E. Stiglitz (1985): "The Theory of Economic Organizations: Human Fallibility and Economic Organization," *American Economic Review*, 75(2), 292-297.

[6] Sah, Raag Kumar (1991): "Fallibility in Human Organizations and Political Systems," *Journal of Economic Perspectives*, 5(2), 67-88.