

Consider this 2 player, 5 x 5 game. There are 5 pure equilibria, highlighted in yellow. Since there is not a single pure equilibria, theory tell the players to use mixed strategy in order to make an optimal decision.

	p C1	q C2	r C3	s C4	t C5
F1	4 ; 1	0 ; 3	8 ; 7	0 ; 6	6 ; 6
F2	7 ; 1	9 ; 8	4 ; 5	4 ; 7	3 ; 0
F3	10 ; 9	8 ; 3	7 ; 5	4 ; 0	0 ; 0
F4	3 ; 1	3 ; 3	4 ; 0	2 ; 0	8 ; 4
F5	7 ; 1	3 ; 0	1 ; 0	10 ; 2	1 ; 0

Let's consider player 2, who has to choose columns. Which probability is every pure strategy (C1 ... C5) to be assigned, so that the expected utility for player 1 yields the same value, no matter which strategy (F1 ... F5) he chooses?

Let p be the probability for player 2 to choose C1, q the probability for C2, r the probability for C3, s the probability for C4 and t the probability for C5. Player 2 has to solve the following system of equations:

$$\begin{aligned}
 4 \cdot p + 0 \cdot q + 8 \cdot r + 0 \cdot s + 6 \cdot t &= u \\
 7 \cdot p + 9 \cdot q + 4 \cdot r + 4 \cdot s + 3 \cdot t &= u \\
 10 \cdot p + 8 \cdot q + 7 \cdot r + 4 \cdot s + 0 \cdot t &= u \\
 3 \cdot p + 3 \cdot q + 4 \cdot r + 2 \cdot s + 8 \cdot t &= u \\
 7 \cdot p + 3 \cdot q + 1 \cdot r + 10 \cdot s + 1 \cdot t &= u
 \end{aligned}$$

Where u is the expected utility for player 1. Moreover, we have to define that $p + q + r + s + t = 1$, and now we have a system of 6 equations with 6 unknown. We can solve this system by means of the Gauss-Jordan method, where the matrix for this system is:

1	1	1	1	1	0	1
4	0	8	0	6	-1	0
7	9	4	4	3	-1	0
10	8	7	4	0	-1	0
3	3	4	2	8	-1	0
7	3	1	10	1	-1	0

The first line represents:

$$1 \cdot p + 1 \cdot q + 1 \cdot r + 1 \cdot s + 1 \cdot t + 0 \cdot u = 1$$

And the others, for instance the second line:

$$4 \cdot p + 0 \cdot q + 8 \cdot r + 0 \cdot s + 6 \cdot t = 1 \cdot u$$

Hence:

$$4 \cdot p + 0 \cdot q + 8 \cdot r + 0 \cdot s + 6 \cdot t - 1 \cdot u = 0$$

I've solved this system with three different computer programs that uses the Gauss-Jordan procedure. I wrote one of these programs, and the other two can be found here:

http://ww2.unime.it/weblab/ita/Gauss/gauss_auto_es.htm

http://people.hofstra.edu/Stefan_waner/RealWorld/tutorialsf1/scriptpivot2.html

In all these three tests, the results I get are:

$$p = -0,32$$

$$q = 0,17$$

$$r = 0,52$$

$$s = 0,48$$

$$t = 0,14$$

$$u = 3,76$$

Remember that u is the expected utility for player 1.

The problem, obviously, is that negative value the system yields for the probability p .

I've used the same procedure for other games, from 2 x 2 to 5 x 5, and *most* of the times the probability values are ok (that is, no negative probability).

So, what is wrong with the system that formalizes the game? I don't think it is a calculation mistake, since the three programs work properly when solving system of equations.

I've also tried to formalize the game with others system of equations, without using u , and the values I get are exactly the same. (Were it useful/helpful, in a next draft I can write down the whole transformations).

I'd be very grateful for any answer or clue to solve this.