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Essays on Learning and Macroeconomics

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by

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To my mom

who prepared me for this ride

To my wife, Kathy

who joined me in this ride

To my son, Diego

who born during this ride

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ABSTRACT OF THE DISSERTATION

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This dissertation contains four essays that apply techniques of learning and implicit contracts to the analysis of macroeconomic problems. The first essay explores the role of borrowers' reputation concerns on the magnification of macroeconomic crises. The second essay investigates the importance of financial frictions in delaying the recovery of economies after crises. The third essay (coauthored with David Lagakos) develops an implicit contract model to understand the differences in wage smoothing across industries. The fourth essay studies the effects of different signalling technologies in the efficiency of organizations with career concerned managers.

CHAPTER 1

Fragility of Reputation and Clustering in Risk-Taking

I study the interplay between reputation and risk-taking in a dynamic stochastic environment where it is ex-ante efficient for firms to engage in safe projects, but ex-post preferred to invest in risky ones, appropriating surplus from lenders. By introducing fundamentals, I interpret the model as a dynamic global game in which strategic complementarities arise endogenously from reputation updating, overcoming pervasive multiple equilibria. I find that even though reputation deters opportunistic behavior, it introduces fragile incentives which may lead to large changes in aggregate risk-taking in response to small changes in aggregate fundamentals, inducing financial crises and credit crunches.

1.1 Introduction

Reputation concerns deter opportunistic behavior by creating a link between past actions and expectations about future actions. Consider, for example, an environment in which lenders provide funds to firms whose risk-taking decisions and profits are unobservable. Firms could take excessive risk, appropriating most of the benefits from large successes and imposing most of the losses from big failures on lenders. This inefficient risk-taking reduces lending and increases its cost. However, if firms generate signals correlated to decisions, lenders could use those signals to construct reputation and offer better lending conditions to firms with better reputation. Firms are then afraid of losing their reputation and are deterred from taking excessive risk. This role for reputation has been extensively discussed in the literature [1].

The point of this paper is to argue that these reputational incentives are fragile because they may suddenly collapse, inducing big changes in aggregate risk-taking in response to small changes in aggregate fundamentals. This sudden shift in behavior may have a large impact on economic outcomes such as corporate failures, credit conditions, interest rates, and returns to investors. Hence reputation may have been an unnoticed detonator of financial collapses and credit crunches characterized by confidence crises. In normal times, lenders have confidence in firms with good reputation and no confidence in firms with bad reputation. In bad times, lenders lose confidence in almost all firms and lending breaks down[2].

[1] Among the most influential papers on reputation are Milgrom and Roberts [1982], Kreps and Wilson [1982], Fudenberg and Levine [1992], Holmstrom [1999] and Mailath and Samuelson [2001]. Literature specifically relating reputation and risk-taking was pioneered by Diamond [1989]

[2] Recent examples of how a loss of confidence in ratings (a measure of reputation) can fuel crises were the financial problems experienced by many countries in August 2007. Since the implementation of Basel II regulations, under which banks holding AAA assets are allowed to keep less capital and lend more, banks around the world have been filling their vaults with AAA- rated structured products, specially CDOs. In August 2007 Central Banks were forced to inject large amounts of liquidity into the overnight money markets because banks were charging very high rates to lend to each other since they lost confidence on the meaning of AAA backed securities (The Economist. ”The game is up” and

I construct a model where incentives to take risk monotonically vary with a stochastic aggregate fundamental. All firms can invest in risky projects and some of them (strategic firms) can also invest in safer projects, with a lower probability of default and a higher probability of generating good signals. A firm's reputation is defined as the probability that lenders assign to the firm being the strategic type. Reputation is Bayesian updated by lenders from observing the signals, and firm incentives are shaped in large part by the concern for their reputation. To protect their reputation, strategic firms engage in safe projects when otherwise they would have preferred to opportunistically take risky ones.

In the absence of any equilibrium selection device, it is not possible to draw firm conclusions about the interplay between reputation and risk-taking, since this model delivers multiple equilibria. For some range of fundamentals, if lenders believe that strategic firms will play safe, then these firms will indeed have incentives to play safe to increase the probability of good signals. The reason is that good signals will be in part attributed to the firm using a safe project and then the firm being strategic. Contrarily, if lenders believe that strategic firms will undergo risky projects, then firms will indeed have incentives to take risks. In this case, good signals will be just attributed to good luck in risky projects. This strong dependence of reputation formation on lenders beliefs about firms choices is at the heart of the reputation fragility. It is irrelevant whether or not a firm has a good reputation if lenders are convinced the firm will choose the risky project.

I use techniques from global games to select a unique equilibrium that is robust to information perturbations. I assume that after the lending contract has been negotiated, but before deciding the project, firms observe a noisy signal of the fundamental. This is a key technical part of the analysis since the model is a non-standard global game.

”Surviving the markets”, 08/16/07).

Strategic complementarities arise endogenously from reputation formation and are affected by the dynamic structure of the game. Hence standard conditions for global games to work, such as uniform limit dominance, are not assumed but obtained endogenously. Uniqueness is characterized, for each reputation level, by a threshold in fundamentals below which all firms with that reputation level take risks. This result generates a well-defined probability of risk-taking (the probability fundamentals are below the threshold) to draw conclusions about the effectiveness of reputation to deter excessive risk.

The intuition behind the collapse of reputation relies on two types of incentives to choose safe projects. First, safe projects increase the probability that firms continue operating. These "continuation incentives" increase with reputation since firms with better reputations face lower borrowing rates in the future and hence have higher expected future profits. Second, safe projects increase the probability of generating good signals, which are used for reputation formation. Because of learning, these "reputation incentives" are high for intermediate and low for extreme reputation levels. The reason is that neither firms with very high nor very low reputation can change their reputation quickly, whereas intermediate firms can. The fragility of reputation arises from combining these two types of incentives. Consider the intermediate reputation level at which reputation incentives are maximized. For reputation levels below that point, as reputation increases, both continuation and reputation incentives to increase, generating a big reduction in the fundamental threshold below which risk-taking is preferred. For reputation levels above that point, as reputation increases, continuation incentives still increase but reputation incentives decrease, compensating each other and generating small changes in thresholds. Since thresholds are less sensitive to reputation when reputation is high, risk-taking becomes attractive for firms with an increasingly larger range of reputation levels as fundamentals decline.

Finally, we relate the predictions of the model with data. First, taking credit ratings of corporate bonds as a proxy for reputation and ratings transitions as a proxy for reputation formation, we show reputation evolves gradually and changes less in bad times than in good times. Second, we discuss recent empirical evidence suggesting that both risk-taking behavior (measured by idiosyncratic risk) and corporate defaults tend to cluster "excessively" in recessions (Campbell et al. [2001] and Das et al. [2007] respectively). These empirical findings seem consistent with the model implications for reputation evolution and risk-taking clustering.

This paper combines two separate strands of literature - reputation and global games. The model is closely related to Diamond [1989] and Mailath and Samuelson [2001] who study reputation incentives in state invariant contexts. As in Diamond [1989], firms behavior affects reputation through the probability of continuation in business. As in Mailath and Samuelson [2001], firms behavior affects reputation through the generation of signals correlated to actions. I introduce these two channels in a single framework, finding that the combination is more than the sum of parts since it leads to the result that reputation incentives are fragile. Their papers also have multiple equilibria. While Diamond [1989] deals with it analyzing extreme equilibria, Mailath and Samuelson [2001] focus on the most efficient one. Since this paper is interested in understanding the time variation and state dependence properties of reputation incentives, we have explicitly tackled the multiplicity issue.

The model is also related to the literature on dynamic global games. I follow Chassang [2007] and Toxvaerd [2007] to solve for uniqueness. However, my model presents additional complications since strategic complementarities are not hard-wired into payoffs but arise endogenously from reputation updating, and hence are tied to the dynamic structure of the game. This paper also contributes to the scarce literature of learning in global games. While most papers study cases in which players learn about a policy maker or a status quo (e.g., Angeletos et al. [2006] and Angeletos et al. [2007]), my model deals with the opposite case in which the market learns about players' types, generating coordination problems. To the best of my knowledge this is the first paper that exploits fundamental-driven incentives to create a reputation global game model and select a unique equilibrium.

In the next two sections I describe a full version of the model (also considering consumers) and discuss equilibrium multiplicity when fundamentals are perfectly observed. In sections 1.4 and 1.5, I show how to select a unique equilibrium using a dynamic global games approach when fundamentals are observed with noise. In section 1.6 I discuss the fragility of reputation, characterized by big changes in risk-taking in response to small changes in fundamentals. In section 1.7, I use numerical simulations to illustrate the role of reputation in magnifying crises. In section 1.8, I show the predictions of the model are consistent with data on reputation dynamics and clustering in risk-taking. In the last section, I make some final remarks.

1.2 The Model

1.2.1 Description

The economy is comprised of a continuum of long-lived, risk neutral firms (with mass 1) that produce a good or provide a service, an infinite number of risk neutral lenders who fund firms to produce and consumers who buy the good or service from firms.

1.2.1.1 Firms

Each firm runs a unique project. The activity of all firms faces an identical market and industry risk, hence differences in results across firms are only induced by their production decisions, generating a purely idiosyncratic risk component. There are two ways to produce. Safe technologies (*s*) that have been previously proven to deliver a high probability of success, and risky technologies (*r*) that may lead to the discovery of cheaper and better production alternatives but also reduce the probability of success.

There are two types of firms, defined by their access to these production technologies. Strategic firms $\mathscr{S}$ can decide whether to follow safe or risky technologies[3]. Risky firms $\mathscr{R}$ do not have access to safe technologies, so they do not have a choice but to follow risky technologies. Reputation is defined by $\phi = Pr(\mathscr{S})$, the probability of being a strategic firm. The introduction of these two types are based on my (maybe pessimistic) belief that all firms can play risky but not all of them can play safe. While all firms can perform trial-error procedures, not all of them have access to well-designed procedures[4].

Firms cannot accumulate assets. At the beginning of the period, the firm negotiates a loan (normalized to 1) to produce. Then, strategic firms should decide whether to use safe or risky technologies[5]. At the end of the period, after production has taken place, the firm may continue (*c*) or die (*d*), with probabilities depending on the technology used. If the firm continues, two possible signals (good *g* or bad *b*), correlated with the technology chosen, are generated[6].

[3]I use interchangeably play safe or take safe actions (*s*) and play risky or take risky actions (*r*)

[4]Another possible assumption is that non-strategic firms only have access to safe technologies. In this case the characterization of equilibrium is different but the main result of large changes in aggregate behavior in response to small changes in fundamentals is the same. However, I believe a better description of reality is that some firms are restricted to use superior technologies rather restricted to use inferior ones.

[5]Since in this section I focus on a given period *t* reputational problem, I am not using any subscript to refer to time. In the next section, when introducing dynamic considerations, I will explicitly denote periods by subscripts.

[6]These two signals can be interpreted as results from production. Good signals are the firm growth or high-quality production. Bad signals are the production of defective products or the provision of a low-quality service.

Probabilities are,

$$Pr(c|s) = p_s > Pr(c|r) = p_r \tag{1.1}$$

$$Pr(g|c,s) = \alpha_s > Pr(g|c,r) = \alpha_r \tag{1.2}$$

Hence, the unconditional probability of good signals is higher using safe technologies than using risky ones (i.e., $p_s\alpha_s > p_r\alpha_r$). Additionally, assume that the unconditional probability of bad signals is higher playing risky than playing safe (i.e., $p_r(1-\alpha_r) > p_s(1-\alpha_s)$)[7].

1.2.1.2 Lenders and Consumers

Lenders and consumers cannot observe the technology used by the firm nor its profits but can observe whether the firm continues or not and, in case of continuation, whether it generates good signals (g) or bad signals (b). To give room for reputation to introduce incentives, signals are observable but unverifiable on court, which means interest rates charged by lenders and prices paid by consumers cannot be conditioned on observed signals.

Lenders provide funds to firms. There is an infinite number of risk neutral lenders whose outside option is a risk free interest rate $\overline{R} > 1$[8]. Repayment is characterized by a costly state verification with a bankrupt procedure that destroys the value of the output. This is a natural way to introduce truth-telling by firms. When profits are greater than the value of interest rates, it is always optimal for firms to repay and get the positive differential rather than default and file for bankruptcy. I assume that, conditional on continuation, firms can always pay back their loans, hence default occurs only in case

[7] This assumption is not particularly relevant but introduces monotonicity in learning and spare us from dealing with awkward expressions and extra conditions.

[8] Since lenders are the long side of the market, there is no competition for funds. The introduction of such competition makes reputation effects more important and magnifies the results

of firm's death.[9].

Consumers buy production from firms. Consumers' utility depends on whether the signals are good or bad. If the firm generates good signals (for example the production of high-quality products) the utility for consumers is $u(g) = 1$. If the firm generates bad signals, consumers' utility is $u(b) = 0$[10]. I assume consumers pay up front for the good or service and the market interaction is given by perfect price discrimination. Each consumer buys one unit of the good in each period and pays a price P, which is a function of their expectations about the probabilities of receiving good signals. This price does not depend on the firm's actions (which are not observable) or the signals (which are known only after the good is purchased). However, as will be shown later, P depends on the firm's reputation and on consumers' expectations about the probabilities the firm played risky.

1.2.1.3 Cash Flows

If the firms dies, present and future cash flows are zero. If the firm continues, cash flows depend on the technology used. Expected instantaneous cash flows from playing safe are $\Pi_s(\theta) = \alpha_s \pi_{s,g}(\theta) + (1-\alpha_s)\pi_{s,b}(\theta)$ and expected instantaneous cash flows from playing risky are $\Pi_r(\theta) = \alpha_r \pi_{r,g}(\theta) + (1-\alpha_r)\pi_{r,b}(\theta)$. Cash flows also depend positively on a single-dimensional variable $\theta \in \mathbb{R}$, which represents the aggregate economic fundamentals that affect the profitability of the project[11]. Fundamentals θ are

[9] Nothing fundamental changes with this assumption but it simplifies the notation and eases the exposition. The analysis relaxing it, such that default also exists in case of continuation, reinforces results and is available upon request

[10] Without loss of generality we also assume consumers' utility if the firm dies is just 0, as in the presence of bad signals. A better assumption may be a negative utility in the case of firm death. However, the conclusions of the model are identical at the cost of complicating the exposition.

[11] For tractability reasons we assume fundamentals only affect the profitability of projects and not the probabilities of success. However, assuming for example that higher fundamentals increase probabilities of success from playing safe respect than from playing risky (i.e., p_s in relation to p_r) we would obtain the same results but with the complication that reputation updating varies in different states of the economy.

i.i.d. over time and distributed with a density $v(\theta)$, a mean $E(\theta)$, and a variance γ_θ.

To be more specific about the structure of these cash flows, $\pi_{a,j}(\theta) = A(\theta)[P - c_{a,j}(\theta)]$, for $a \in \{s,r\}$ and $j \in \{g,b\}$. $A(\theta)$ is the level of demand for the firm's product, which depends positively on the aggregate state of the economy (or fundamentals θ), P is the unit price and $c_{a,j}(\theta)$ is the average cost of production, which depends on the aggregate state of the economy, the technology used and the signal generated. I assume that for any fundamental θ, costs from playing risky are more volatile than costs from playing safe.

Assumption 1 $c_{r,g}(\theta) < c_{s,g}(\theta) < c_{s,b}(\theta) < c_{r,b}(\theta)$ *for all* θ

This assumption arises naturally from the definition of risky technology. For example, taking risks by cutting costs beyond safe procedures may be highly beneficial if the results obtained are good but can be very costly if the results are bad since it may be necessary to pay fines, face demands, cover guarantees, etc. Since the price P charged for the product does not depend on the technology used or the generated signals, this assumption immediately implies $\pi_{r,g}(\theta) > \pi_{s,g}(\theta) > \pi_{s,b}(\theta) > \pi_{r,b}(\theta)$ for all θ. Hence expected profits from risky actions are more volatile and centered around expected profits from safe actions.

With respect to the cost structure, I also assume both expected average costs from playing risky (i.e., $C_r(\theta) = \alpha_r c_{r,g}(\theta) + (1-\alpha_r)c_{r,b}(\theta)$) and from playing safe (i.e., $C_s(\theta) = \alpha_s c_{s,g}(\theta) + (1-\alpha_s)c_{s,b}(\theta)$) depend negatively on fundamentals. As will be discussed later P will decrease as fundamentals weaken, which means profits per unit of production decrease in bad times[12].

Hence, in bad times there is a reduction in total expected instantaneous cash flows from two channels. On the one hand, demand decreases from a reduction in $A(\theta)$.

[12]Note this always happens with fixed costs of production when facing a demand reduction

On the other, average profits also decrease from an increase in expected average costs. Finally, I assume changes in C_r are less sensitive to fundamentals than changes in C_s.

Assumption 2 $\frac{\partial C_s}{\partial \theta} < \frac{\partial C_r}{\partial \theta} \leq 0$

What is really important about the assumption is not the direction of the inequality but rather the monotonic change in incentives as fundamentals vary. Unlike other reputation models, we allow incentives behind moral hazard and adverse selection to differ in different stages of the cycle. In this particular case, the assumption means that playing risky is more attractive in bad times than in good times. When fundamentals weaken, expected instantaneous cash flows decrease more when using safe technologies than risky ones, for example if in the latter case, firms adjust easily to a changing environment.

The particular direction of the assumption can be justified in two ways. First, under limited liability, there will be a maximum cost $\overline{c} = P$ above which it is not possible for the firm to cover the consequences of its actions (such as fines, demands, etc.). Under this situation, even if $\frac{\partial C_s}{\partial \theta} = \frac{\partial C_r}{\partial \theta}$, since $c_{r,b}$ is the highest possible cost, it binds first with $\overline{c}$. Hence, in expectation, average costs from safe actions effectively increase faster than average costs from risky actions in bad times. Since potential losses are bounded while potential gains are not, it is more attractive to take risk in bad times than in good times. In other words, the highest variance distribution gets truncated faster in its left tail. Second, in bad times experimentation in new production procedures is cheaper. This idea has been extensively discussed since Schumpeter, who believed recessions are good opportunities for firms to innovate and try new ways to produce.

1.2.2 Timing

The timing of the model is:

- Firms, lenders and consumers observe the reputation level ϕ of all firms. Firms acquire a loan of 1 from lenders. Lenders' outside option is a risk free interest rate $\overline{R} > 1$
- Fundamentals θ (that only affect profits) are realized by everybody in the economy[13].
- The firm decides between following safe (s) or risky (r) technologies.
- Production occurs and the firm either continues (c) or dies (d).
- If the firm dies, it defaults on the loan.
- If the firm continues, it pays to lenders the negotiated interest rate $R > 1$ and sell the product to consumers at a price P.
- After the sale, the firm generates good (g) or bad (b) signals of its actions. Lenders and consumers observe those unverifiable signals and use them to update the reputation from ϕ to ϕ'

This is the timing in each period. Since reputation only makes sense in a dynamic context, the game will consist in many repeated periods with this sequence of interactions and decisions. We will discuss the results of the model in both finite and infinite horizon versions of the repeated game.

1.2.3 Definition of Equilibrium

Before formally defining the equilibrium, we discuss some preliminaries concerning the properties of reputation updating by lenders and consumers and the definition of the value function that firms maximize.

[13] The timing in which fundamentals are observed will be relevant later to select a unique equilibrium. An alternative, and possibly more realistic, assumption is that a subset of fundamentals is observed before the loan is negotiated while another subset is observed before production but after lending

1.2.3.1 Reputation Updating

When updating a firm's reputation, lenders and consumers have a prior about the firm's reputation and have beliefs about whether the firm plays risky. These two ingredients deserve a detailed explanation.

The model assumes lenders and consumers receive a public signal g or b about the firm's actions[14]. Unfortunately, a model with common, public realizations has many equilibria where reputation does not have the asset characteristics we are focusing on and where an implausible degree of coordination between firm behavior and the market belief about the firm behavior is required[15]. I eliminate these equilibria by requiring behavior to be Markov. However, even when restricting attention to Markovian strategies, reputation formation still depends on beliefs about the probabilities the firm plays risky.

Since we focus on Markovian strategies, the sufficient statistic about the firm's type is the reputation level ϕ. Let $x(\phi,\theta)$ be the probability a strategic firm with reputation ϕ plays risky when the fundamental is θ. Additionally, let $\widehat{x}(\phi,\theta)$ be lenders and consumers' beliefs about the probability a strategic firm with reputation ϕ plays risky when the fundamental is θ. By Bayesian updating, after observing a continuing firm generating good signals,

$$Pr(\mathscr{S}|c,g) = \phi_g(\phi,\widehat{x}) = \frac{[p_r\alpha_r\widehat{x} + p_s\alpha_s(1-\widehat{x})]\phi}{[p_r\alpha_r\widehat{x} + p_s\alpha_s(1-\widehat{x})]\phi + p_r\alpha_r(1-\phi)} \tag{1.3}$$

[14]The obvious and natural alternative is that each agent in the market receives an idiosyncratic signal. However the idiosyncrasy of signals present the same problems to analyze than model of private monitoring. Obstructing the ability to coordinate continuation play, it imposes serious constraints on the ability to construct equilibria. See a complete discussion in Mailath and Samuelson [2006], Ch. 18

[15]One of these equilibria can be, for example, to play safe for certain fundamentals until the first bad result happens and then play risky afterward. In this particular equilibrium reputation does not exist as we interpret it, and beliefs about firms' behavior requires implausible degrees of complexity and coordination. See discussion in Mailath and Samuelson [2001]

and, after observing a continuing firm generating bad signals,

$$Pr(\mathscr{S}|c,b) = \phi_b(\phi,\widehat{x}) = \frac{[p_r(1-\alpha_r)\widehat{x}+p_s(1-\alpha_s)(1-\widehat{x})]\phi}{[p_r(1-\alpha_r)\widehat{x}+p_s(1-\alpha_s)(1-\widehat{x})]\phi+p_r(1-\alpha_r)(1-\phi)} \tag{1.4}$$

where ϕ_g is the posterior after the observation the firm continued with good signals and ϕ_b the posterior after the observation the firm continued with bad signals, given a prior ϕ.

1.2.3.2 Profits

For the moment, we will focus on the static problem that firms have to solve just assuming a fixed stream of continuation values for different ϕ in the future. I impose three restrictions on continuation values $V(\phi)$. First, they are well-defined. That this is indeed the case will be shown in Section 1.5, where a fully fledged dynamic model is considered. Second, they are positive, which is clear since profits are bounded below by zero. Finally, they are monotonically increasing in the reputation level ϕ. Even when this seems a natural assumption because reputation is a valuable asset, it is also a useful assumption for expositional purposes. In Section 1.5, I discuss the conditions for this assumption to hold and why it is convenient to discuss the results but not critical to obtain them.

Total discounted profits for a given reputation level ϕ and observed fundamental θ, conditional on the probability of risk-taking x and on market's beliefs $\widehat{x}$ about that probability of risk-taking, are:

$$\begin{aligned}\widehat{V}(\phi,\theta|x,\widehat{x}) = \quad & x \quad [p_r[\Pi_r(\theta)-R(\phi)]+\beta p_r\left[\alpha_r\mathbf{V}(\phi_{g_{(\phi,\widehat{x})}})+(1-\alpha_r)\mathbf{V}(\phi_{b_{(\phi,\widehat{x})}})\right]] \\ & +(1-x)[p_s[\Pi_s(\theta)-R(\phi)]+\beta p_s\left[\alpha_s\mathbf{V}(\phi_{g_{(\phi,\widehat{x})}})+(1-\alpha_s)\mathbf{V}(\phi_{b_{(\phi,\widehat{x})}})\right]]\end{aligned} \tag{1.5}$$

where

$$V(\phi,\theta|\widehat{x}) = \max_{x\in[0,1]} \widehat{V}(\phi,\theta|x,\widehat{x})$$

and $\mathbf{V}(\phi'_{(\phi,\widehat{x})}) = \int_{-\infty}^{\infty} V(\phi',\theta'|\widehat{x}')v(\theta')d\theta'$ are elements of a given stream of continuation values $\Upsilon' = \{\mathbf{V}(\phi')\}_{\phi'=0}^{1}$

Note the value function depends on the reputation level (ϕ), fundamentals (θ), and beliefs the market assigns to the firm playing risky ($\widehat{x}$). Naturally, in equilibrium a strategy for firms uniquely determines the equilibrium updating rule the market must use if their beliefs are to be correct (i.e., $x = \widehat{x}$).

In what follows I focus on cutoff strategies in which the firm will decide to play risky if it observes a fundamental below a certain threshold and safe if it observes a fundamental above that threshold, such that

$$x(\phi,\theta) = \begin{cases} 0 & if \quad \theta > k^*(\phi) \\ 1 & if \quad \theta < k^*(\phi) \end{cases} \tag{1.6}$$

In Section 1.3 we show that even restricting attention to this type of strategies, we have a multiplicity of equilibria when fundamentals are common knowledge. In Section 1.4 we show that introducing noise in the observation of fundamentals, the unique equilibrium surviving iterated elimination of dominated strategies as the precision of signals goes to infinity, is a threshold strategy of this type. Intuitively this result arises from the monotonicity assumption of the relation between incentives to play safe and fundamentals.

1.2.3.3 Equilibrium

Definition 1 *A Markov perfect equilibrium in cutoff strategies is: cutoffs $k^*(\phi)$, interest rates $R(\phi)$ and posteriors ϕ_g and ϕ_b, such that*

- *Each firm with reputation ϕ observes θ and chooses $x^*(\phi,\theta)$ to maximize $\widehat{V}(\phi,\theta|x,\widehat{x})$*

(given by equation 1.10) following a cutoff strategy such that

$$x^*(\phi,\theta) = \begin{cases} 0 & if \quad \theta > k^*(\phi) \\ 1 & if \quad \theta < k^*(\phi) \end{cases}$$

- *Lenders charge $R(\phi)$ such that they obtain $\overline{R}$ in expectation.*
- *Posteriors ϕ_g and ϕ_b are updated using Bayes' Rule (equations 1.3 and 1.4).*
- *A strategy for ϕ firms uniquely determines the equilibrium interest rates and updating rule the market must use if their beliefs are to be correct (i.e., $\widehat{x}(\phi,\theta) = x^*(\phi,\theta)$).*

1.3 Multiplicity with Complete Information about Fundamentals

In this section we show there is a continuum of Markovian perfect equilibria in monotone cutoff strategies when firms perfectly observe fundamentals. This result arises from the impossibility of pinning down a unique belief for lenders and consumers to use in updating firms' reputation.

To achieve this result, we first discuss the dependence of the value and formation of reputation on lenders and consumers' beliefs about firms' actions. Then we discuss properties of the differential gains from playing safe rather than risky that are used to determine firms' optimal actions. Finally, we show equilibrium multiplicity in each period for a given stream of continuation values and discuss how this multiplicity problem becomes more serious as the horizon of the game grows.

1.3.1 Reputation and Beliefs

The next Proposition shows the role of reputation and beliefs as a source of multiplicity.

Proposition 2 *For a given reputation level ϕ and a fundamental θ, the reputation formation (measured by $\phi_g - \phi_b$) and the reputation value (measured by $R(\phi)$ and $P(\phi)$) decrease as lenders and consumers assign a greater probability the firm plays risky (i.e., greater $\widehat{x}(\phi, \theta)$).*

In the next subsections, we show this proposition by parts, first focusing on the formation of reputation and then in the value of reputation. Finally, we discuss the place of this result within the reputation literature.

Intuitively, when lenders and consumers assign a low probability of the firm playing risky, good signals are also signals that a firm had played safe with high probability and then it is more likely the firm is strategic. In this case, learning is easier and playing safe is a good way to increase probabilities of having good results and to increase reputation. On the contrary, when the market assigns a high probability of the firm playing risky, good signals are attributed to good luck rather than the use of safe procedures. In this case, since learning is difficult, firms do not have incentives to increase the probability of generating good signals by playing safe.

The value of reputation also depends on beliefs about risk-taking. If lenders believe it is very likely strategic firms play risky, they will charge high interest rates since it is less likely in expectation to recover the loan. If they believe firms play safe, they will charge low interest rates. Similarly, if consumers believe strategic firms played risky, the willingness to pay for the product is low because it is less likely to be a good product. However, if they believe strategic firms played safe, the willingness to pay for the product is higher for high reputation levels since it is more likely to enjoy good products.

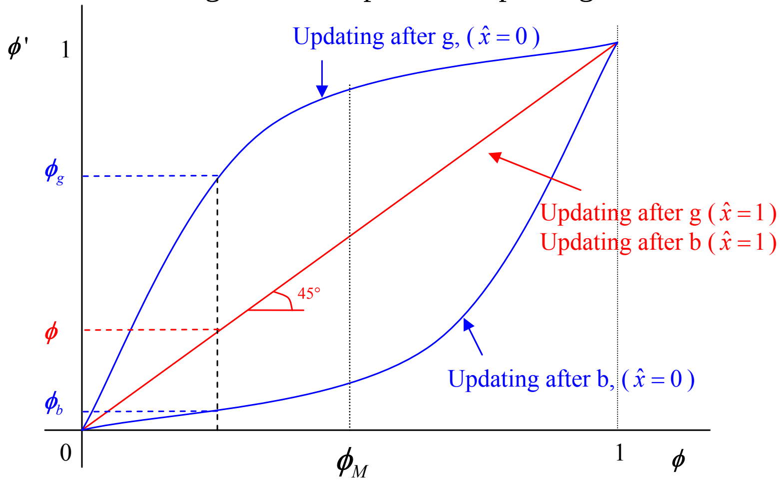
Hence, reputation is a valuable asset, not because it represents an assumed intrinsic valuable characteristic, but because it increases instantaneous cash flows and reduces expected future interest rates by having access to safe actions. However, the magnitude of these effects in a given period depends heavily on the beliefs about the firm playing risky in that period. This property is the main source of multiplicity. If lenders and consumers believe the firm plays risky, not only do not update the reputation but also the reputation does not have any effect on increasing instantaneous cash flows or in reducing interest rates. This eliminates the deterring effects of reputation on risk-taking by making firms more prone to take risks. Contrarily, if lenders and consumers believe firms plays safe, both value and formation of reputation is important, preventing risk-taking by making firms more likely to play safe.

1.3.1.1 Reputation Formation

In this setting, the formation of reputation depends heavily on the beliefs of lenders and consumers about firm's actions. This is because reputation is not understood as the possession of an intrinsically valuable characteristic but the possession of a characteristic that only has value if it is really used.

Note from equations 1.3 and 1.4 that $\phi_g = \phi_b = \phi$ when $\widehat{x} = 1$ and $\phi_g \geq \phi \geq \phi_b$ when $\widehat{x} \leq 1$, with the gap $\phi_g - \phi_b$ increasing as $\widehat{x}$ goes to 0. Graphically, reputation evolves as in Figure 1.1. Reputation priors ϕ are represented in the horizontal axis and reputation posteriors ϕ' are represented in the vertical axis. Take, for example, the case in which lenders and consumers believe strategic firms play safe for sure (i.e., $\widehat{x} = 0$). In this case, given a current reputation level ϕ, the gain in terms of reputation of generating good signals rather than bad signals is determined by the gap $\phi_g - \phi_b$. Contrarily, when lenders and consumers believe strategic firms play risky for sure (i.e., $\widehat{x} = 1$), there is no gain in terms of reputation from generating good signals rather than bad ones. Recall also that when $\phi = 0$ or $\phi = 1$ there is no updating, no matter the signals nor the beliefs about the firm's actions. Contrarily, the maximum updating gap $(\phi_g - \phi_b)$ is obtained at an intermediate value ϕ_M for any value of $\widehat{x} < 1$.

Figure 1.1: Reputation Updating



1.3.1.2 Reputation Value

Now we will discuss the value of reputation in increasing expected profits by reducing interest rates and instantaneous cash flows and how this value decreases as beliefs of risk-taking increase.

First, interest rates decrease as reputation levels ϕ increase. Since I assume loans are negotiated before knowing fundamentals, interest rates are defined by the risk free interest rate $\overline{R}$ divided by the expected probability of continuation. Hence,

$$R(\phi) = \frac{\overline{R}}{Pr(c|\phi)} \tag{1.7}$$

where

$$Pr(c|\phi) = \int_{-\infty}^{\infty} [(p_r \widehat{x}(\phi,\theta) + p_s(1-\widehat{x}(\phi,\theta)))\phi + p_r(1-\phi)]v(\theta)d\theta$$

Note that $R(\phi) \in [\frac{\overline{R}}{p_s}, \frac{\overline{R}}{p_r}]$ and that $\frac{\partial R(\phi)}{\partial \phi} < 0$ for a fixed $\widehat{x}$. This is important because it is the first reason why firms would like to build and maintain reputation. For a given $\widehat{x}$, high reputation levels imply lenders charge lower interest rates to firms.

Since we are focusing on cutoff strategies, from equation (1.6) beliefs $\widehat{x}(\phi,\theta)$ are a function of the cutoff $\widehat{k}(\phi)$ that lenders and consumers believe a firm with reputation ϕ use. Hence, interest rates can be expressed as,

$$R(\phi|\widehat{k}) = \frac{\overline{R}}{Pr(c|\phi,\widehat{k})} \tag{1.8}$$

where

$$Pr(c|\phi,\widehat{k}) = (1-\phi)p_r + \phi\left[p_r\mathscr{V}(\widehat{k}(\phi)) + p_s(1-\mathscr{V}(\widehat{k}(\phi)))\right]$$

where $\mathscr{V}(\widehat{k}(\phi))$ is the cumulative distribution of θ up to $\widehat{k}(\phi)$. The sufficient condition for interest rates to decrease with reputation is that cutoff beliefs $\widehat{k}(\phi)$ are non-increasing in ϕ. As we will show, this is the case in equilibrium, in which beliefs are correct (i.e., $\widehat{k}(\phi) = k^*(\phi)$).

Second, instantaneous cash flows increase as reputation levels ϕ increase, since consumers are willing to pay a higher price P for the same product. This is common in most models in which firms care about having a reputation of producing high-quality products. Rather than just assuming this relation, we obtain it from our perfect price discrimination setup. Since consumers get a per period utility of 1 from the consumption of products under good signals and 0 from the consumption of products under bad signals, they are willing to pay for the product up to their reservation value.

$$P(\phi,\theta) = (\alpha_r\widehat{x}(\phi,\theta) + \alpha_s(1-\widehat{x}(\phi,\theta)))\phi + \alpha_r(1-\phi) \tag{1.9}$$

Note that $P(\phi,\theta) \in [\alpha_r,\alpha_s]$ and that $\frac{\partial P(\phi,\theta)}{\partial\phi} < 0$ for a fixed $\widehat{x}$. This is an additional reason why firms care about reputation. For a given $\widehat{x}(\phi,\theta)$, high reputation imply firms can charge higher price for their products. Again, since we are focusing on cutoff strategies, we can express prices also as,

$$P(\phi,\theta|\widehat{k}) = \begin{cases} (1-\phi)\alpha_r + \phi\alpha_s & if \quad \theta > \widehat{k}(\phi) \\ \alpha_r & if \quad \theta < \widehat{k}(\phi) \end{cases}$$

Similarly to the interest rates case, the sufficient condition for expected prices to increase with reputation is that cutoff beliefs $\widehat{k}(\phi)$ are non-increasing in ϕ. As we will show, this is the case in equilibrium, in which beliefs are correct (i.e., $\widehat{k}(\phi) = k^*(\phi)$).

Hence interest rates R and instantaneous cash flows Π_s and Π_r can be written fully explicitly as functions of fundamentals and reputation levels, $R(\phi,\overline{R})$, $\Pi_s(\phi,\theta)$ and $\Pi_r(\phi,\theta)$

1.3.1.3 Relation with the Literature

In this model, reputation is not intrinsically valuable, as would be the case of talents, quality or skills but it is defined by access to actions. To take advantage of a reputation of having access to a safe technology, lenders and consumers must also believe that the firm will in fact decide to play safe in that period. It is worthless to be seen as a firm that can choose if at the same time lenders and consumers believe the choice will be to play risky, the same action taken by firms that do not have a choice.

Since risk-taking is the product of a certain action rather than an intrinsic characteristic, reputation should be defined both using adverse selection (lenders and consumers do not know if the firm has the possibility to choose or not) and moral hazard (actions have value in themselves to lenders and consumers other than being just signals). A more general setting should allow reputation to be also a signal of the possession of an intrinsic value. We may think, for example, that a firm that knows how to play safe is also a firm that can produce better products simply because its managers are talented people. Assuming this extra effect would reinforce the monotonicity of the continuation values on ϕ, sustaining the main results. However, it also adds unnecessary elements to the exposition of the main conclusions[16].

This model differs importantly from other models relating reputation and risk-taking. Here we will discuss the main differences with the two most relevant related papers, Diamond [1989] and Mailath and Samuelson [2001][17].

Diamond [1989] considers three types of firms - naturally risky, naturally safe, and strategic firms that can choose between risky or safe projects. He shows that strategic firms may choose safe projects and forego profits to enjoy lower interest rates in the future. A difference with our model is that firms signal their type just by continuing in business, hence reputation can only increase over time, being undistinguishable from age. Introducing a second set of signals after continuation, our model allows reputation to be constructed, destroyed and managed.

This last characteristic of our model is related to Mailath and Samuelson [2001] who in a different setting study a problem where reputation can vary depending on results and where strategic types try to separate from ”bad” types. However, they do not consider firms can die as a result of their actions, not capturing continuation incentives on decisions.

Our model differ from these two paper in three important dimensions. First, it incorporates elements of continuation (as in Diamond [1989]) and elements of reputation based on results (as in Mailath and Samuelson [2001]). On the one hand, the

[16] Nevertheless, when relevant, we will show along the exposition how strengthening reputation also as an intrinsically valuable element reinforce the results

[17] Another relevant paper for us, even when not closely related, is Holmstrom [1999]. He suggests that managers' incentives for risk-taking depend on their career concerns. When proposing projects, managers send to owners imperfect signals about their talent to determine which ones are good projects. The better the perceived talent the higher future wages. When wages are linearly related to talents and managers are risk neutral, they are indifferent about risk-taking decisions. However, if managers are risk averse, they prefer to propose that no investment should be taken, avoiding the risk of having a bad result. Since uncertainty is shared by the manager and the owners a ”nicely behaved” pure strategy equilibrium always exists.

combination of these two types of incentives in a single framework is critical to the main fragility result, hence being more than the sum of the parts. On the other hand it allows us to separate the interactions of lenders and consumers with the firm.

Second, our model let incentives for opportunistic behavior to vary with aggregate fundamentals. This is in stark contrast with both Diamond [1989] and Mailath and Samuelson [2001] who analyze reputation in an invariant situation. This improvement sheds light on reputation effects over cycles.

Finally, as in these two papers, ours suffer a multiple equilibria problem. Diamond [1989] deals with multiplicity by focusing on the evolution of extreme equilibria (i.e., cases in which all strategic firms play risky or all of them play safe). Mailath and Samuelson [2001] only focuses on discussing the properties and conditions of the best equilibrium, the one that eliminates inefficiency completely. In our case, the introduction of fundamental-driven incentives naturally lead us to the use of a dynamic global games approach to select a unique equilibrium, which is robust to small perturbations in information about the state of the economy. This uniqueness is important to characterize risk-taking behavior by firms over economic cycles and to analyze the efficiency effects of reputation.

1.3.2 Differential gains from playing safe

Given cutoff strategies, we can redefine beliefs of risk taking at each fundamental θ as a function of cutoff beliefs. Following equation (1.6), $\widehat{x}(\phi,\theta)$ is a function of $\widehat{k}(\phi)$. Total discounted profits for a given reputation ϕ and fundamental θ, conditional on the probability of risk-taking x and on cutoff beliefs $\widehat{k}$, are:

$$\begin{aligned}\widehat{V}(\phi,\theta|x,\widehat{k}) = \quad & x \quad [p_r[\Pi_r(\theta) - R(\phi|\widehat{k})] + \beta p_r \left[\alpha_r \mathbf{V}(\phi_{g_{(\phi,\widehat{x}|\widehat{k})}}) + (1-\alpha_r)\mathbf{V}(\phi_{b_{(\phi,\widehat{x}|\widehat{k})}})\right]] \\ & +(1-x)[p_s[\Pi_s(\theta) - R(\phi|\widehat{k})] + \beta p_s \left[\alpha_s \mathbf{V}(\phi_{g_{(\phi,\widehat{x}|\widehat{k})}}) + (1-\alpha_s)\mathbf{V}(\phi_{b_{(\phi,\widehat{x}|\widehat{k})}})\right]]\end{aligned} \quad (1.10)$$

Since we are analyzing cutoff strategies we can define differential profits from playing safe rather than risky as $\Delta(\phi,\theta|\widehat{k}) = \widehat{V}(\phi,\theta|0,\widehat{k}) - \widehat{V}(\phi,\theta|1,\widehat{k})$.

$$\begin{aligned}\Delta(\phi,\theta|\widehat{k}) = {} & p_s\Pi_s(\phi,\theta) - p_r\Pi_r(\phi,\theta) + \beta(p_s - p_r)\mathbf{V}(\phi) - (p_s - p_r)R(\phi|\widehat{k}) \\ & +\beta[p_s\alpha_s - p_r\alpha_r][\mathbf{V}(\phi_{g_{(\phi,\widehat{x}|\widehat{k})}}) - \mathbf{V}(\phi)] \\ & +\beta[p_r(1-\alpha_r) - p_s(1-\alpha_s)][\mathbf{V}(\phi) - \mathbf{V}(\phi_{b_{(\phi,\widehat{x}|\widehat{k})}})]\end{aligned} \tag{1.11}$$

These are the differential gains from playing safe for a firm ϕ that observes a fundamental θ, conditional on lenders and consumers having cutoff beliefs $\widehat{k}(\phi)$ and hence beliefs $\widehat{x}(\phi,\theta)$ of risk-taking in θ. The firm decides to play safe if $\Delta(\phi,\theta|\widehat{k}) > 0$ and risky if $\Delta(\phi,\theta|\widehat{k}) < 0$.

The next lemma shows how $\Delta(\phi,\theta|\widehat{k})$ depends on θ, $\widehat{k}$ and $\widehat{x}$

Lemma 3 $\Delta(\phi,\theta|\widehat{k})$ *is monotonically increasing in* θ*, monotonically decreasing in* $\widehat{x}$ *and monotonically non-increasing in* $\widehat{k}$

Proof We divide this proof in three steps.

• *Step 1:* $\frac{\partial\Delta(\phi,\theta|\widehat{k})}{\partial\widehat{k}} \leq 0$

Regardless of θ, it is straightforward to show, from equation (1.8), that $\frac{\partial R(\phi|\widehat{k})}{\partial\widehat{k}} \geq 0$

• *Step 2:* $\frac{\partial\Delta(\phi,\theta|\widehat{k})}{\partial\widehat{x}} < 0$

By decomposing the first component, we can write it explicitly as $(p_s\Pi_s(\phi,\theta) - p_r\Pi_r(\phi,\theta)) = (p_s - p_r)A(\theta)P(\phi,\theta|\widehat{k}) - [p_sC_s(\theta) - p_rC_r(\theta)]$. First, for a given θ, $\widehat{k}$ defines a $\widehat{x}$ and $(p_s\Pi_s - p_r\Pi_r)$ only depends on $\widehat{x}$ through prices. For a given ϕ and θ, as shown in Section 1.3.1.2, $\frac{\partial P(\phi,\theta|\widehat{k})}{\partial\widehat{x}} < 0$. Since $p_s > p_r$ and $A(\theta) > 0$, $\frac{\partial(p_s\Pi_s - p_r\Pi_r)}{\partial\widehat{x}} < 0$. Second, for a given ϕ and θ, as shown in equations (1.3) and (1.4), reputation gaps $(\phi_g - \phi)$ and $(\phi - \phi_b)$ decrease as $\widehat{x}$ increases. By assumption[18], $\mathbf{V}(\phi)$

[18]This is an assumption for the moment since we will show this is the case in Section 1.5

is monotonically increasing in ϕ, hence $\mathbf{V}(\phi_g) - \mathbf{V}(\phi)$ and $\mathbf{V}(\phi) - \mathbf{V}(\phi_b)$ decrease as $\widehat{x}$ increases. The higher the beliefs assigned to the firm playing risky, the more difficult is the updating of reputation and the smaller the reputation gains from playing safe. Hence $\frac{\partial(\mathbf{V}(\phi_g) - \mathbf{V}(\phi))}{\partial \widehat{x}} < 0$ and $\frac{\partial(\mathbf{V}(\phi) - \mathbf{V}(\phi_b))}{\partial \widehat{x}} < 0$

- *Step 3:* $\frac{\partial \Delta(\phi, \theta | \widehat{k})}{\partial \theta} > 0$

As shown in *Step 1*, $(p_s \Pi_s(\phi, \theta) - p_r \Pi_r(\phi, \theta)) = (p_s - p_r) A(\theta) P(\phi, \theta | \widehat{k}) - [p_s C_s(\theta) - p_r C_r(\theta)]$. First total demand $A(\theta)$ increases with fundamentals. Second there is a reinforcement effect that comes through prices. As shown in the previous step, $\frac{\partial P(\phi, \theta | \widehat{k})}{\partial \widehat{x}} < 0$ and by cutoff strategies $\frac{\partial \widehat{x}}{\partial \theta} \leq 0$. Since $p_s > p_r$ and $P > 0$, then $\frac{\partial (p_s - p_r) A(\theta) P(\phi, \widehat{x})}{\partial \theta} > 0$. Finally, by assumption 2, $\frac{\partial [p_s C_s(\theta) - p_r C_r(\theta)]}{\partial \theta} < 0$ since $p_s > p_r$. Hence $\frac{\partial (p_s \Pi_s - p_r \Pi_r)}{\partial \theta} > 0$. Since the rest of components do not depend on θ, $\frac{\partial \Delta(\phi, \theta | \widehat{k})}{\partial \theta} > 0$.

Q.E.D.

Lemma 3 shows that differential gains from playing safe decrease as fundamentals go down (θ decreases) and as the beliefs of the firm playing risky go up, both in expectation ($\widehat{k}$ increases) and for a given θ ($\widehat{x}$ increases), which represents the source of multiplicity in this model. Intuitively, the reasons behind these relations are the following.

Differential gains from playing safe decrease as fundamentals weaken. When the state of the economy gets worse average costs in expectation increase less by experimenting than by following safe procedures, making more attractive to play risky in recessions.

Differential gains from playing safe decrease as beliefs of the firm playing risky increase. First, the price consumers are willing to pay for the good decreases because they assign a less probability of getting a good result. Since prices are lower, the gains from increasing the probability of remain alive by playing safe decrease. Second, in expectation default is more likely, interest rates are higher and firms are more prone to take risks since they become heavily indebted. Finally, a higher belief that the firm plays risky reduces the updating of beliefs, reduces the gain in terms of reputation from getting good results and makes less attractive to play safe.

We will assume Uniform Limit Dominance, which determines extreme fundamentals $\underline{\theta}(\phi|\widehat{k})$ and $\overline{\theta}(\phi|\widehat{k})$ for each reputation value ϕ when cutoff beliefs are $\widehat{k}$ such that, for all $\theta < \underline{\theta}(\phi|\widehat{k})$ it is optimal to play risky and for all $\theta > \overline{\theta}(\phi|\widehat{k})$ it is optimal to play safe, no matter what lenders and consumers believe firms decide given that fundamental θ (i.e., no matter $\widehat{x}(\phi,\theta)$). While $\underline{\theta}(\phi|\widehat{k})$ is obtained for $\widehat{x}=0$ in which reputation is heavily updated, $\overline{\theta}(\phi|\widehat{k})$ is obtained for $\widehat{x}=1$ in which reputation does not change.

Assumption 3 *(Uniform Limit Dominance)*

- *For each ϕ and $\widehat{k}$, $\exists\underline{\theta}(\phi|\widehat{k})$ such that* $\Delta(\phi,\underline{\theta}|\widehat{k},\widehat{x}=0)=0$
- *For each ϕ and $\widehat{k}$, $\exists\overline{\theta}(\phi|\widehat{k})$ such that* $\Delta(\phi,\overline{\theta}|\widehat{k},\widehat{x}=1)=0$

Following this notation, we can define $\underline{\theta}(\phi|-\infty)$ the value of θ for which it is indifferent to play risky or safe if $\widehat{x}=0$ and the lowest possible interest rate for that ϕ is charged ($R(\phi|-\infty)=\frac{\overline{R}}{p_r(1-\phi)+p_s\phi}$). We can also define $\overline{\theta}(\phi|\infty)$ the value of θ for which it is indifferent to play risky or safe if $\widehat{x}=1$ and the highest possible interest rate for that ϕ is charged ($R(\phi|\infty)=\frac{\overline{R}}{p_r}$). Naturally, $\underline{\theta}(\phi|-\infty)\leq\underline{\theta}(\phi|\widehat{k})$ and $\overline{\theta}(\phi|\infty)\geq\overline{\theta}(\phi|\widehat{k})$ for all $\widehat{k}$.

Important features are Single Crossing properties that are obtained from analyzing the differential gain from playing safe. The following two lemmas describe these properties, which allows us to identify a unique cutoff in the set of fundamentals (θ) and on the set of beliefs ($\widehat{x}$) that make a particular firm indifferent between playing risky or safe, given a fixed $\widehat{k}$.

Lemma 4 *(State monotonicity) For every reputation level ϕ and cutoff belief $\widehat{k}$, fix a $\widehat{x}(\phi,\theta)$ for all θ and there exists a unique $\theta^*\in[\underline{\theta}(\phi|\widehat{k}),\overline{\theta}(\phi|\widehat{k})]$ such that* $\Delta(\phi,\theta|\widehat{k},\widehat{x})<$

0 *for* $\theta < \theta^*$, $\Delta(\phi, \theta|\widehat{k}, \widehat{x}) = 0$ *for* $\theta = \theta^*$ *and* $\Delta(\phi, \theta|\widehat{k}, \widehat{x}) > 0$ *for* $\theta > \theta^*$*. Furthermore,* θ^* *is increasing in* $\widehat{k}$ *and* $\widehat{x}$*.*

Proof By Lemma 3 $\Delta(\phi, \theta|\widehat{k})$ is increasing in θ and by Assumption 2 there is a unique crossing on the space of fundamentals since, as they rise, the value of playing risky increases monotonically at a lower rate than the value of playing safe. Hence there is a unique θ^* such that $\Delta(\phi, \theta^*|\widehat{k}, \widehat{x}) = 0$. Since $\widehat{x} \in [0,1]$ then, by definition, $\theta^* \in [\underline{\theta}(\phi|\widehat{k}), \overline{\theta}(\phi|\widehat{k})]$. By Lemma 3, $\Delta(\phi, \theta|\widehat{k})$ is decreasing in $\widehat{k}$ and $\widehat{x}$, then θ^* is increasing in $\widehat{k}$ and $\widehat{x}$. If the beliefs of the firm playing risky or the interest rate increase, the firm will strictly prefer to play risky at the previous θ^*, requiring an increase to recover the indifference. Q.E.D.

Lemma 5 *(Belief single crossing) For every reputation level* ϕ *and cutoff belief* $\widehat{k}$*, fix a* $\theta \in [\underline{\theta}(\phi|\widehat{k}), \overline{\theta}(\phi|\widehat{k})]$ *and there exists a unique* $\widehat{x}^*$ *such that* $\Delta(\phi, \theta|\widehat{k}, \widehat{x}) > 0$ *for* $\widehat{x} < \widehat{x}^*$, $\Delta(\phi, \theta|\widehat{k}, \widehat{x}) = 0$ *for* $\widehat{x} = \widehat{x}^*$ *and* $\Delta(\phi, \theta|\widehat{k}, \widehat{x}) < 0$ *for* $\widehat{x} > \widehat{x}^*$*. Furthermore,* $\widehat{x}^*$ *is increasing in* θ*.*

Proof By Lemma 3 $\Delta(\phi, \theta|\widehat{k})$ is monotonically decreasing in $\widehat{x}$. This ensures there is a unique crossing in beliefs $\widehat{x}$. Hence there is a unique $\widehat{x}^*$ such that $\Delta(\phi, \theta|\widehat{k}, \widehat{x}^*) = 0$, where $\widehat{x}^* \in [0,1]$. Since, by Lemma 3, $\Delta(\phi, \theta|\widehat{k})$ is increasing in θ, so is $\widehat{x}^*$. If fundamentals improve the firm will strictly prefer to play safe at $\widehat{x}^*$, requiring an increase in the beliefs the firms plays risky $\widehat{x}^*$ to recover the indifference. Q.E.D.

1.3.3 Multiple Equilibria

The model exhibits multiple equilibria when firms perfectly observe fundamentals.

Proposition 6 *For all reputation levels* $\phi \in (0,1)$*, there is a continuum of equilibrium strategy cutoffs* $k^*(\phi) \in [\underline{\theta}(\phi|\underline{\theta}), \overline{\theta}(\phi|\overline{\theta})]$*. For reputation* $\phi = 1$ *there is finite multiple* *equilibria when $\gamma_\theta \rightarrow 0$. For reputation $\phi = 0$, there is always a unique equilibrium cutoff $k^*(0)$.*

Proof The Proposition follows directly from assumption 3 and lemmas 4 and 5. A cutoff $k^*(\phi)$ is an equilibrium strategy only if it's a best response for any realization of the fundamental θ. Take a cutoff $k^*(\phi)$ such that $k^*(\phi) \in [\underline{\theta}(\phi|k^*), \overline{\theta}(\phi|k^*)]$. The existence of such a case is guaranteed by assumption 3. From the cutoff strategy, $x(\phi,\theta) = 0$ for all $\theta > k^*(\phi)$ and $x(\phi,\theta) = 1$ for all $\theta < k^*(\phi)$. From Lemma 5, at $\theta = k^*(\phi)$, indifference occurs at some $0 < x^*(\phi,k^*) < 1$. The cutoff $k^*(\phi)$ is an equilibrium because, for all $\theta > k^*(\phi)$, $\Delta(\phi,k^*|k^*) > 0$ and hence it is optimal for the firm to play safe (i.e., $x(\phi,\theta) = 0$). Similarly, for all $\theta < k^*(\phi)$, $\Delta(\phi,k^*|k^*) < 0$ and hence it is optimal for the firm to play risky (i.e., $x(\phi,\theta) = 1$). Now take an arbitrarily close cutoff $k^{**}(\phi) = k^*(\phi) + \varepsilon$ such that $0 < R(k^{**}(\phi)) - R(k^*(\phi)) < \delta$, where an arbitrarily small $\varepsilon > 0$ allows to define an arbitrarily small δ. By the discontinuity on beliefs (sudden jump from $x = 1$ to $x = 0$ at $\theta = k^*$)and the same reasoning described above, $k^{**}(\phi)$ is also an equilibrium cutoff strategy. Inductively it is possible to define a continuum of equilibrium strategy cutoffs.

The bounds of the equilibrium cutoffs $[\underline{\theta}(\phi|\underline{\theta}), \overline{\theta}(\phi|\overline{\theta})]$ are determined in the following way. $\underline{\theta}(\phi|\underline{\theta})$ is the value of the cutoff that determines an interest rate $R(\underline{\theta})$ and considers the gains from reputation ($\widehat{x} = 0$). Similarly, $\overline{\theta}(\phi|\overline{\theta})$ is the value of the cutoff that determines a higher interest rate $R(\overline{\theta})$ and does not consider the gains from reputation ($\widehat{x} = 1$). The condition for these bounds to be unique and all $\theta \in [\underline{\theta}(\phi|\underline{\theta}), \overline{\theta}(\phi|\overline{\theta})]$ to constitute an equilibrium is that $p_s \frac{\partial \Pi_s}{\partial \theta} - p_r \frac{\partial \Pi_r}{\partial \theta} \geq \frac{\partial R(\phi|k^*)}{\partial k^*}$. This condition basically requires interest rates do not jump suddenly with changes in cutoffs, or in other words, since $\mathscr{V}(k^*)$ determines $R(\phi|k^*)$, the distribution of fundamentals has a variance big enough.

For $\phi = 1$ the only source of possible multiplicity comes from different fixed points of beliefs $\widehat{k} = b(\widehat{k})$, where $b(\widehat{k})$ is the best response to cutoff beliefs $\widehat{k}$. In this case there is not continuum of equilibria since there is no discontinuity of differential payoffs generated by reputation (i.e., $\Delta(1,\theta|\widehat{k},\widehat{x}=0) = \Delta(1,\theta|\widehat{k},\widehat{x}=1)$). A unique equilibrium exists when there is no jumps of lending rates as cutoffs change $p_s \frac{\partial \Pi_s}{\partial \theta} - p_r \frac{\partial \Pi_r}{\partial \theta} \geq \frac{\partial R(\phi|k^*)}{\partial k^*}$.

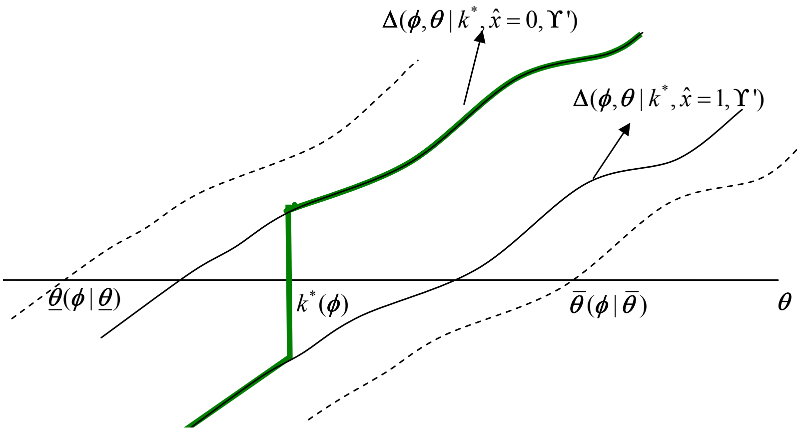
For $\phi = 0$ there is a unique equilibrium because there is a unique possible interest rate given by $R(0) = \frac{\overline{R}}{p_r}$. Furthermore, reputation updating does not happen. Hence, $\Delta(0,\theta|\widehat{k},\widehat{x}=0) = \Delta(0,\theta|\widehat{k},\widehat{x}=1)$ for $R(0)$ and $[\underline{\theta}(0|\underline{\theta}), \overline{\theta}(0|\overline{\theta})]$ collapses into a singleton given by the unique equilibrium strategy cutoff $k^*(0)$ Q.E.D.

This multiplicity characterized by a continuum of equilibrium cutoffs for each ϕ is pervasive to draw conclusions about the effectiveness of reputation to deter excessive risk-taking. Since each cutoff represents a different ex-ante probability that ϕ firms take risks ($\mathscr{V}(k^*(\phi))$), equilibria are ranked in terms of firm's instantaneous cash flows, prices paid by consumers and interest rates charged by lenders. The higher the equilibrium cutoff, the more likely it is for firms to take risks, the lower the price consumers are willing to pay in expectation and the higher the interest rates charged by lenders.

An intuitive explanation of the multiplicity is provided in Figure 1.2. Take a cutoff $k^*(\phi)$ for some reputation level $\phi \in (0,1)$ such that $k^*(\phi) \in [\underline{\theta}(\phi)|k^*), \overline{\theta}(\phi)|k^*)]$. This is an equilibrium because it is optimal to play safe for all $\theta > k^*(\phi)$ (since $\Delta(\phi,\theta|k^*,\widehat{x}=0) > 0$ for all $\theta > k^*(\phi)$) and it is optimal to play risky for all $\theta < k^*(\phi)$ (since $\Delta(\phi,\theta|k^*,\widehat{x}=1) < 0$ for all $\theta < k^*(\phi)$). The function $\Delta(\phi,\theta|k^*)$ for different fundamentals is the bald function with a discontinuity at $k^*(\phi)$ in Figure 1.2. The cutoff $k^*(\phi)$ is an equilibrium strategy because it is a best response for any realization of the fundamental θ such that beliefs are correct.

An arbitrarily small increase in the cutoff generates an arbitrarily small increase in the interest rate. If interest rates do not change suddenly, they cannot overcome the discontinuity generated by the reputation effects that sudden changes in beliefs generate. Hence, it is possible to find equilibrium cutoffs arbitrarily close and hence a continuum of equilibria. As we move the cutoff to the right of $k^*(\phi)$, interest rates increase, reducing $\Delta(\phi, \theta|\widehat{k})$ for all θ[19]. Given the discontinuity introduced by reputation at the equilibrium cutoff strategy, the new cutoffs constitute equilibria until $\overline{\theta}(\phi|\overline{\theta})$ is reached. The same is true as we decrease cutoffs from k^* towards $\underline{\theta}(\phi|\underline{\theta})$. These extremes, determined by extreme beliefs and lending rates, constitute bounds to equilibrium cutoffs.

Figure 1.2: Equilibria Multiplicity



This is the typical result of multiple equilibria in reputation settings in which ”strategic” types try to separate from ”bad” types rather than pooling with ”good” types. The multiplicity relies heavily on the impossibility of pinning down beliefs to update reputation, as discussed in Mailath and Samuelson [2006], Mailath and Samuelson [2001] and Diamond [1989].

[19] Recall that, by assuming $p_s \frac{\partial \Pi_s}{\partial \theta} - p_r \frac{\partial \Pi_r}{\partial \theta} \geq \frac{\partial R(\phi|k^*)}{\partial k^*}$, I impose changes in payoffs to be greater than changes in interest rates, for a given change in fundamentals, making this process smooth.

Up to this point we have highlighted the multiplicity that arises in a given period, for a given stream of value functions assigned to the future Υ'. Once we introduce dynamics, the multiplicity problem increases, making it very difficult to draw any conclusion about the effects of reputation on risk-taking behavior. The intuition of this result is straightforward. Since in each period multiplicity exists, multiple streams of continuation values for different ϕ, which are consistent with multiple equilibria in future periods, can be used to construct $\Delta(\phi,\theta)$. Introducing extreme continuation values determined by the highest ($\overline{\Upsilon}'$) and the lowest ($\underline{\Upsilon}'$) probability of risk-taking in all future periods for all reputation levels, it is possible to construct extreme bounds $\underline{\theta}(\phi|\underline{\theta},\overline{\Upsilon}') < \underline{\theta}(\phi|\underline{\theta},\Upsilon')$ and $\overline{\theta}(\phi|\overline{\theta},\underline{\Upsilon}') > \overline{\theta}(\phi|\overline{\theta},\Upsilon')$ such that the region of multiplicity in a given period expands when compared with the case of a unique stream of continuation values assumed so far.

It is important to distinguish the multiplicity determined just from the determination of interest rates and the multiplicity introduced by reputation. The multiplicity introduced by the determination of interest rates arises from the possibility of multiple fixed points in which the beliefs about the cutoff firms use are equal to the best response of firms to those beliefs (i.e., $\widehat{k}(\phi) = b(\widehat{k}(\phi))$). Generically this multiplicity will be finite and easy to eliminate with certain assumptions on the distribution of fundamentals[20]. The multiplicity introduced by reputation incentives arises from the discontinuity of differential payoffs at the equilibrium cutoff $k^*(\phi)$. This allows for the determination of a continuum of indeterminate equilibria, which is impossible to eliminate just with assumptions on the distribution of fundamentals.

[20] For example, if we assume fundamentals are normally distributed $\theta \sim \mathcal{N}(E(\theta),\gamma_\theta)$, the sufficient condition for uniqueness is given by $p_s \frac{\partial \Pi_s}{\partial \theta} - p_r \frac{\partial \Pi_r}{\partial \theta} \geq \frac{\phi}{\gamma_\theta} \frac{\overline{R}(p_s-p_r)^2}{\sqrt{2\pi[(1-\phi)p_r+\phi p_s-\phi(p_s-p_r)\mathcal{V}(\theta)]^2}}$. As can be seen, when $\phi=0$ the condition is always fulfilled (by assumption 6 the left-hand side term is positive), leading to the unique equilibrium discussed in Proposition 6. In general, without reputation concerns, uniqueness can be obtained when the variance γ_θ is big enough with respect to the reputation level.

In this environment comparative statics and comparative dynamics analysis are not trivial since there is no explicit theory to guide the selection of equilibrium, leaving a big role to self-fulfilling beliefs and payoff irrelevant sunspots. However, as noted by Morris and Shin [2000], what really creates the multiplicity is the assumption of complete information and common knowledge of fundamentals that at the same time implies an implausible degree of coordination and capacity to predict rivals' behavior in equilibrium. In the following section we show that the introduction of few noise in the observation of fundamentals leads to the selection of a unique equilibrium.

1.4 Uniqueness with Incomplete Information about Fundamentals

In this section we slightly modify the assumption about complete information of fundamentals and the timing in which they are realized. We assume firms observe a noisy signal of the aggregate fundamental before deciding which technology to use. After production takes place, fundamentals are realized by firms, lenders, and consumers[21]. The signal observed by the firm is not observable by lenders and consumers, who can only infer it from observing the real fundamental. This modification allows us to select a unique market's belief about the probability a ϕ strategic firm takes risks. Having a unique belief, we can select a unique equilibrium in the reputation environment. More precisely, the assumptions about the information technology are

Assumption 4 *Each firm i observes a signal $z_i = \theta + \sigma\varepsilon_i$ where $\varepsilon_i \sim F$ identically and independently distributed across i*

Given θ the distribution of signals z is then given by $F(\frac{z-\theta}{\sigma})$.

[21]The assumption about the timing fundamentals are observed is important. Otherwise, if interest rates or prices reveal, through the aggregation of information by the market, the true fundamental before production, the whole point of introducing heterogeneity through signals to pin down a unique equilibrium disappears. See Atkeson [2001]

Assumption 5 *(Monotone likelihood ratio property). For $a > b$, $\frac{f(a-\theta)}{f(b-\theta)}$ is increasing in θ*

In words, this assumption means that a firm that receives a high signal, assigns a large probability that lenders and consumers believe the firm has in fact observed a high signal.

Introducing this assumption, the firm uses a cutoff strategy in the set of signals and not on the set of fundamentals, which are no longer observable. This means that for a history of fundamentals and a current signal about fundamentals z, a strategy for a firm with reputation ϕ picks a real number $z^*(\phi)$ with the interpretation that it uses safe technologies whenever $z > z^*(\phi)$ and risky ones whenever $z < z^*(\phi)$[22]. The next proposition estates that, assuming this information structure, there exists a unique Markovian perfect Bayesian equilibrium in monotone cutoff strategies for each reputation level ϕ, when signals are precise enough.

Proposition 7 *For a given ϕ, as $\sigma \to 0$, in equilibrium there exists a unique cutoff signal $z^*(\phi)$ such that $\Delta(\phi, z|z^*(\phi)) = 0$ for $z = z^*(\phi)$, $\Delta(\phi, z|z^*(\phi)) > 0$ for $z > z^*(\phi)$ and $\Delta(\phi, z|z^*(\phi)) < 0$ for $z < z^*(\phi)$, where $\Delta(\phi, z|z^*(\phi))$ are the expected differential gains from playing safe if a ϕ firm receives a signal z and lenders and consumers believe strategic firms ϕ use a cutoff $z^*(\phi)$.*

The proof is in the Appendix. Relaxing the assumption of common knowledge about fundamentals, when signals are very precise, allows us to use the approach provided by global games to select a unique belief concerning the probability that firms take risky actions. The intuitive proof is based on the iterated deletion of dominated strategies. Assume, for example, a strategic firm with reputation ϕ observes a signal

[22] Recall each firm receives an idiosyncratic signal z_i. We get rid of the subindex for simplicity in notation. However, signals vary across firms and are not observed by lenders or consumers.

$\underline{\theta}(\phi|\underline{\theta})$. In this case the firm would like to play risky even if the market uses a belief $\widehat{x}(\phi) = 0$. By receiving a low signal the firm also believes the fundamental is close to $\underline{\theta}(\phi|\underline{\theta})$. If fundamentals in fact happen to be $\underline{\theta}(\phi|\underline{\theta})$, lenders and consumers believe with some positive probability that the firm had observed a signal below $\underline{\theta}(\phi|\underline{\theta})$ hence having a belief $\widehat{x}(\phi) > 0$ (i.e., firm takes risks with some positive probability). However, with this belief, the firm would strictly prefer to play safe, not being an equilibrium a cutoff $\underline{\theta}(\phi|\underline{\theta})$. By continuity the same reasoning can be applied to signals above $\underline{\theta}(\phi|\underline{\theta})$. The same reasoning applies also to signals close to $\overline{\theta}(\phi|\overline{\theta})$.

We require $\sigma \to 0$ so the firm put more weight to its private signal than to the public signal given by the prior distribution of θ. The previous process of iterated deletion of dominated strategies results in a unique cutoff $z^*(\phi)$ such that the firm plays risky whenever $z < z^*(\phi)$ and safe whenever $z > z^*(\phi)$.

This uniqueness result remains once we consider the full-fledged dynamic model. In the next section we consider both the finite and infinite horizon game. In the finite horizon game it is always possible to define a unique sequence of equilibrium cutoffs as signals become very precise. In the infinite horizon game we can show there is a unique limit to the sequence of perfect Markovian equilibrium for the finite game.

1.5 Dynamics

In this section we show how to solve the model dynamically such that a unique sequence of cutoffs for each reputation level ϕ is obtained. We confirm that continuation values are well defined such that we can indeed use the propositions and proofs from previous sections, where a single period was considered. First I assume all firms live for a finite period of time T such that $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ. Afterwards I extend the results to an infinite horizon game as $T \to \infty$[23].

This extension is important for two reasons. First, reputation is an intrinsically dynamic process that must be studied dynamically to fully understand it. Second, since the previous sections were based on an assumed profile of continuation values, we must confirm they are always well defined and we must understand under what conditions they are monotonically increasing in reputation and how they may change results.

The following Lemma shows how continuation values for all reputation levels $\mathbf{V}_t(\phi)$ are indeed well defined at each period t based on the boundary condition $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ[24].

Lemma 8 *For a given reputation ϕ and a period t, as $\sigma \to 0$, $x_t(\phi, z_t) = 0$ for all $z_t < z_t^*(\phi)$ and $x_t(\phi, z_t) = 1$ for all $z_t > z_t^*(\phi)$, where $z_t^*(\phi) = f(\overrightarrow{\boldsymbol{V}}_{t+1}(\phi))$ is the unique solution to the following equation (where $\overrightarrow{\boldsymbol{V}}_{t+1}(\phi) = [\boldsymbol{V}_{t+1}(\phi_b), \boldsymbol{V}_{t+1}(\phi_g)]$)*

$$\int_0^1 \Delta_t(\phi, z_t^*(\phi)|\widehat{x}_t)d\widehat{x}_t = 0 \tag{1.12}$$

$\boldsymbol{V}_t(\phi)$ is given recursively by the boundary condition $\boldsymbol{V}_{T+1}(\phi) = 0$ and by

$$\begin{aligned}\boldsymbol{V}_t(\phi) = f(\overrightarrow{\boldsymbol{V}}_{t+1}(\phi)) = &\int_{-\infty}^{f(\overrightarrow{\boldsymbol{V}}_{t+1}(\phi))} p_r[\Pi_r(\phi, \theta) - R_t(\phi) + \beta \boldsymbol{V}_{t+1}(\phi)]\nu(\theta)d\theta \\ &+ \int_{f(\overrightarrow{\boldsymbol{V}}_{t+1}(\phi))}^{\infty} p_s[\Pi_s(\phi, \theta) - R_t(\phi) + \beta E(\boldsymbol{V}_{t+1}(\phi'))]\nu(\theta)d\theta\end{aligned}$$

Proof To show the first part of the Proposition we must show we can solve the model as a series of static games that deliver a unique equilibrium (specifically, a unique cutoff for each ϕ) in each period t. At the last period T the cutoff $z_T^*(\phi)$ will be very high in general since $\mathbf{V}_{T+1}(\phi) = 0$ for all ϕ. Hence, when solving for risk-taking

[23] While previous sections results were obtained for a given period t, in what follows we use the same arguments but denote explicitly each period by subscripts t.

[24] In order to solve this finite dynamic global game we follow Morris and Shin [2003], Toxvaerd [2007], Giannitsarou and Toxvaerd [2007] and Steiner [2006]

at the last period T, there will not be any future punishment from a possible death or from a potential loss of reputation.

At the last period T, $\Delta_T(\phi,\theta)$ is well defined for all ϕ and θ. A cutoff $z_T^*(\phi)$ can be obtained as shown in Proposition 7, from $\int_0^1 \Delta_T(\phi, z_T^*(\phi)|\widehat{x})d\widehat{x}$ where,

$$\Delta_T(\phi, z_T^*(\phi)|\widehat{x}) = p_s\Pi_s(z_T^*(\phi)) - p_r\Pi_r(z_T^*(\phi)) - (p_s - p_r)\frac{\overline{R}}{(1-\phi)p_r + \phi(p_r\widehat{x} + p_s(1-\widehat{x}))}$$

Once $z_T^*(\phi)$ is determined, it is possible to define the equilibrium interest rate

$$R(\phi|z_T^*(\phi)) = \frac{\overline{R}}{(1-\phi)p_r + \phi[p_r\mathscr{V}(z_T^*(\phi)) + p_s(1-\mathscr{V}(z_T^*(\phi)))]}$$

and the expected continuation value at T for each reputation level ϕ. For signals $z_T < z_T^*(\phi)$ firms play risky and for signals $z_T > z_T^*(\phi)$ they prefer to play safe. As $\sigma \to 0$, total discounted expected profits at T are closely approximated by,

$$\mathbf{V}_T(\phi) = \int_{-\infty}^{z_T^*(\phi)} p_r[\Pi_r(\phi,\theta) - R_T(\phi)]v(\theta)d\theta + \int_{z_T^*(\phi)}^{\infty} p_s[\Pi_s(\phi,\theta) - R_T(\phi)]v(\theta)d\theta \tag{1.13}$$

Since equilibrium strategies are well defined and unique at period T (through $z_T^*(\phi)$), continuation values $\mathbf{V}_T(\phi)$ are well defined for all ϕ.

Now consider the decision of a firm ϕ at period $T-1$. The problem to be solved at $T-1$ is essentially a static one since $\mathbf{V}_T(\phi)$ are unique and well defined for all ϕ from equation (1.13). Then, $\Delta_{T-1}(\phi,\theta)$ is also well defined for all ϕ and θ. Using Proposition 7 there is a unique cutoff $z_{T-1}^*(\phi)$ and it is possible to obtain a unique and well defined $\mathbf{V}_{T-1}(\phi)$ for all ϕ.

By straightforward inductive reasoning, there will exist a unique sequence of cutoffs $\{z_t^*(\phi)\}_{t=0}^T$. Furthermore there will exist a unique sequence of expected total discounted profits for each reputation level ϕ at each period t. As $\sigma \to 0$,

$$\begin{aligned}\mathbf{V}_t(\phi) = &\int_{-\infty}^{z_t^*(\phi)} p_r[\Pi_r(\phi,\theta) - R_t(\phi) + \beta\mathbf{V}_{t+1}(\phi)]v(\theta)d\theta \\ &+ \int_{z_t^*(\phi)}^{\infty} p_s[\Pi_s(\phi,\theta) - R_t(\phi) + \beta E(\mathbf{V}_{t+1}(\phi'))]v(\theta)d\theta\end{aligned}$$

Q.E.D.

The next proposition shows that continuation values are always well defined and, given the boundary condition imposed by a final period T, the sequence of equilibrium cutoffs is unique.

Proposition 9 *In a finite game with final period T and a boundary condition* $\mathbf{V}_{T+1}(\phi) = 0$ *for all* ϕ*, as* $\sigma \to 0$*, continuation values* $\{\mathbf{V}_t(\phi)\}_0^T$ *are well defined and there is a unique equilibrium for the whole game given by a unique sequence of cutoffs* $\{z_t^*(\phi)\}_{t=0}^T$ *for each* ϕ*.*

The proof of the proposition is a direct application of Lemma 8 and is based on the idea that, if the dynamic global game has a finite final period, it can be solved as a sequence of static global games.

Before extending our conclusions to an infinite period game, we discuss how the backward determination of continuation values may lead to a convergence to a fixed point in continuation values for all reputation levels ϕ in periods t far enough from T. This is relevant because these fixed points represent bounded limits required to show that there is an infinite horizon equilibrium that is a unique limit of the finite horizon Markov perfect equilibrium[25].

First, recall that strategic complementarities that generate multiplicity in the reputational model arise endogenously from reputation formation, rather than being hard-wired into static payoffs, as is standard in the global game literature. This is important because reputation levels $\phi = 0$ and $\phi = 1$ do not show a multiplicity problem since lenders and consumers beliefs do not affect reputation updating. The fixed point of value functions for these two extreme reputation levels are given by parameters only

[25] I haven't examined yet the broader issue of what other equilibria there might be in the infinite horizon game

and do not depend on value functions for other reputational levels. Hence, $\mathbf{V}(0)$ and $\mathbf{V}(1)$ can be used as anchors to determine value functions for the rest of reputation levels. Then, we can obtain the conditions for fixed points from analyzing these two extreme cases[26].

Define the fixed point continuation value when firms $\phi = 0$ play safe for sure as

$$\overline{\mathbf{V}}(0|s) = \frac{p_s \Pi_s(0) - \overline{R}}{1 - \beta p_s} \tag{1.14}$$

where $\Pi_s(\phi) = \int_{-\infty}^{\infty} \Pi_s(\theta) v(\theta) d\theta$

Define also the fixed point continuation value when firms $\phi = 1$ play risky for sure as

$$\overline{\mathbf{V}}(1|r) = \frac{p_r \Pi_r(1) - \overline{R}}{1 - \beta p_r} \tag{1.15}$$

Let's introduce now a technical assumption to ensure a unique steady state[27]

Assumption 6 $\overline{\boldsymbol{V}}(1|r) > \overline{\boldsymbol{V}}(0|s)$

Using this assumption, the next Lemma shows the conditions for the continuation values to converge to a unique value as we iterate backwards from a large finite period T. Furthermore, we will characterize the type of behavior consistent to those conditions.

First, define the continuation value for which $z^*(0) = E(\theta)$ as

$$\widetilde{\mathbf{V}}(0) = \frac{p_r \Pi_r(0, E(\theta)) - p_s \Pi_s(0, E(\theta))}{\beta(p_s - p_r)} + \frac{\overline{R}}{\beta p_r} \tag{1.16}$$

and the continuation value for which $z^*(1) = E(\theta)$ as,

$$\widetilde{\mathbf{V}}(1) = \frac{p_r \Pi_r(1, E(\theta)) - p_s \Pi_s(1, E(\theta))}{\beta(p_s - p_r)} + \frac{\overline{R}}{\beta[p_r \mathscr{V}(E(\theta)) + p_s(1 - \mathscr{V}(E(\theta)))} \tag{1.17}$$

[26]We will assume the distribution of fundamentals has a variance γ_θ large enough such that there is a unique equilibrium for $\phi = 1$

[27]This can be justified, for example, assuming disastrous cash flows from using safe procedures in very bad times.

It is possible to show that $\widetilde{\mathbf{V}}(1) < \widetilde{\mathbf{V}}(0)$. The intuition is that, everything else constant, $z^*(1) < z^*(0)$ and then it is necessary that $\widetilde{\mathbf{V}}(1) < \widetilde{\mathbf{V}}(0)$ to compensate and to have the same cutoff $z^*(1) = z^*(0) = E(\theta)$

Lemma 10 *Convergence to a unique continuation value for each* ϕ

- *If* $\widetilde{\boldsymbol{V}}(0) < \overline{\boldsymbol{V}}(0|s)$, *(hence* $\widetilde{\boldsymbol{V}}(1) < \overline{\boldsymbol{V}}(1|r)$*), continuation values converge to a unique* $\overline{\boldsymbol{V}}(\phi)$ ***for each*** ϕ. *The probability of playing safe is close to 1 for all* ϕ.
- *If* $\widetilde{\boldsymbol{V}}(1) > \overline{\boldsymbol{V}}(1|r)$, *(hence* $\widetilde{\boldsymbol{V}}(0) > \overline{\boldsymbol{V}}(0|s)$*), continuation values converge to a unique* $\overline{\boldsymbol{V}}(\phi)$ ***for all*** ϕ. *The probability of playing safe is close to 0 for all* ϕ.
- *If* $\widetilde{\boldsymbol{V}}(0) > \overline{\boldsymbol{V}}(0|s)$ *and* $\widetilde{\boldsymbol{V}}(1) < \overline{\boldsymbol{V}}(1|r)$, *continuation values converge to a unique* $\overline{\boldsymbol{V}}(\phi)$ ***for each*** ϕ *only when* $v(z^*(0)) < 1$. *The probability of playing safe is strictly between 0 and 1 for all* ϕ.

This lemma is proved and graphical intuition is provided in the Appendix. The first two bullet points correspond to cases in which continuation values converge to a unique fixed point characterized by playing safe and risky almost surely, for all ϕ. The third case is more interesting since the fixed point is characterized by a positive probability of playing both safe and risky. However to achieve convergence to a fixed point it is also necessary that the variance of θ be big enough to avoid a cyclical pattern. For example, if θ is distributed as a normal, the condition is given by $\frac{1}{\gamma_\theta\sqrt{2\phi}} \exp -\frac{(z^*(0)-E(\theta))^2}{2\gamma_\theta^2} < 1$. If $\gamma_\theta \to 0$ this condition will not be fulfilled and a cyclical pattern will arise. If γ_θ is big enough, this condition will be fulfilled and convergence of continuation values will arise. This is in fact a similar condition to the one required to obtain a unique equilibrium in the static model without reputation.

Under the assumptions, when information becomes very precise $\sigma \to 0$ and continuation values converge to a fixed point in a finite period game, there is an infinite horizon equilibrium that is a unique limit of finite horizon Markov perfect equilibria, for all reputation values ϕ.

Proposition 11 *If $\mathbf{V}_T(\phi) \to \overline{\mathbf{V}}(\phi)$ for all ϕ as $T \to \infty$, given $\sigma \to 0$, there exists a sequence of cutoffs $\{z_t^*(\phi)\}_{t=0}^{\infty}$ for each ϕ that is a unique limit of the finite horizon Markov perfect equilibria described in Proposition 9.*

Proof Having shown uniqueness for an arbitrary finite horizon T, we must show the same reasoning is extended as $T \to \infty$. First note the value of taking safe actions and the value of taking risky actions are bounded and well behaved monotone functions of T when continuation values converges to a fixed point $\mathbf{V}_T(\phi) \to \overline{\mathbf{V}}(\phi)$ as $T \to \infty$. Second, note also $\Delta(\phi, z_t|z_t^*(\phi))$ represents the optimal trade off between the value from playing safe and the value of playing risky. Hence $\Delta(\phi, z_t|z_t^*(\phi))$ also converges to a unique limit as $T \to \infty$. Then $z_t^*(\phi)(T) \to z_t^*(\phi)(\infty)$ as $T \to \infty$, where $z_t^*(\phi)(T)$ is the equilibrium cutoff at t far enough from T and $z_t^*(\phi)(\infty)$ is the equilibrium cutoff at t in an infinite horizon game. Q.E.D.

When continuation values converge to a fixed point, which are the cases described by Lemma 10, there is an infinite horizon equilibrium that is a unique limit of the Markov perfect equilibria in the finite horizon version of the game. Rather than using the boundary condition we can use the steady state value $\overline{\mathbf{V}}(\phi)$ for each ϕ to solve backwards. When dynamics are characterized by cyclical behavior it is not possible to use a unique continuation value as a boundary condition. Hence, there would not exist a unique limit to the Markov perfect equilibrium as defined in Proposition 9.

In the full-fledged dynamic model, the ex-ante probability of risk-taking will be uniquely determined in each period by $Pr(z < z^*(\phi))$, or when $\sigma \to 0$, by $Pr(\theta < z^*(\phi))$. Since this unique belief is used in the reputation updating, a unique equilibrium exists in the reputation model. The unique equilibrium is based on payoff relevant fundamentals, rather than payoff irrelevant sunspots or self-fulfilling beliefs, which allows us to obtain conclusions about the determinants of the probability of risk-taking and about how this probability changes in response to parameters and fundamental variations. These considerations, which cannot be performed using models with multiple equilibria or models with a unique equilibrium based on sunspots or self fulfilling beliefs, are the subjects of the next section.

1.6 Reputation and Risk-Taking Behavior

In previous sections we obtained a unique equilibrium and showed how steady states continuation values are determined. In this section I use the results to analyze the determinants of risk-taking, the clustering behavior of risk-taking, and the fragility of reputation to deter inefficient risk-taking.

1.6.1 Determinants of Risk-Taking

Recall the ex-ante probability of risk-taking is given by $Pr(\theta < z^*(\phi)) = \mathscr{V}(z^*(\phi))$ when $\sigma \to 0$. Then, we have to analyze how the cutoffs $z^*(\phi)$ react to variables such as reputation levels ϕ, interest rates $R(\phi)$ and reputation concerns.

Proposition 12 *The ex-ante probability of risk-taking for a firm with reputation level ϕ decreases as reputation rewards $(\boldsymbol{V}(\phi_g) - \boldsymbol{V}(\phi))$ and reputation punishments $(\boldsymbol{V}(\phi) - \boldsymbol{V}(\phi_b))$ increase.*

Proof From equation (1.11), $\frac{\partial \Delta(\phi)}{\partial(\mathbf{V}(\phi_g) - \mathbf{V}(\phi))} = \beta(p_s\alpha_s - p_r\alpha_r) > 0$ and $\frac{\partial \Delta(\phi)}{\partial(\mathbf{V}(\phi) - \mathbf{V}(\phi_b))} = \beta(p_r(1-\alpha_r) - p_s(1-\alpha_s)) > 0$. Since the cutoff $z^*(\phi)$ is determined by equation (2.6), as the reputation rewards and punishments go up, $\Delta(\phi, \theta)$ also goes up and by Lemma 4 it is required a smaller signal as a cutoff $z^*(\phi)$ in order to maintain the indifference. Hence more reputation rewards and punishments imply reductions in the ex-ante probability of risk-taking. Q.E.D.

Reputation reduces excessive risk-taking behavior, a positive role of reputation widely discussed, informally in the press and casual discussions and formally by an extensive literature in reputation. This model also delivers this result, but explicitly solving the multiplicity that arises from different possible beliefs about the firm's actions.

Proposition 13 *The ex-ante probability of risk-taking increases with interest rates.*

Proof Since $\frac{\partial \Delta(\phi)}{\partial R(\phi)} = -(p_s - p_r) < 0$, by equation (2.6) it is straightforward to show $\frac{\partial z^*(\phi)}{\partial R(\phi)} \leq 0$ by using Lemma 4 Q.E.D.

When interest rates increase, firms are more indebted and moral hazard problems become more relevant. Incentives to follow risky procedures and hence the inefficiency of risk-taking behavior also increase. This result suggests, for example, that firms in underdeveloped countries with high $\overline{R}$, have greater incentives to take excessive risk and reputation concerns are less effective in deterring the resultant excessive risk-taking.

Proposition 14 *The ex-ante probability of risk-taking decreases with reputation in the range $\phi \in [0, \phi_M]$. Whether the probability of risk-taking increases or decreases in the range $\phi \in [\phi_M, 1]$ depends on the rate of increase $\frac{\partial[\beta \mathbf{V}(\phi) - R(\phi)]}{\partial \phi} > 0$ when compared with the rate of decrease $\frac{\partial[\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)]}{\partial \phi} < 0$. Furthermore, $\frac{\partial^2 z^*(\phi)}{\partial \phi^2} > 0$ for all $\phi \in [0, 1]$*

Proof As shown in section 1.3.1.1, $(\phi_g - \phi_b)$ achieves a maximum at ϕ_M. By assumption $\mathbf{V}(\phi)$ is monotonically increasing in ϕ (this will be discussed in detail in the next section), hence $(\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b))$ also achieves a maximum at ϕ_M, being $\mathbf{V}(\phi_g) = \mathbf{V}(\phi_b)$ at $\phi = 0$ and $\phi = 1$. Hence, in the range $\phi \in [0, \phi_M]$, as ϕ increases,

$P(\phi)$ increases, $\mathbf{V}(\phi)$ increases, $R(\phi)$ decreases and $\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)$ increases for all $\widehat{x} < 1$. From equation (1.11) it is clear that when ϕ increases in the range $[0, \phi_M]$, $\Delta(\phi, z)$ goes up and the cutoff $z^*(\phi)$ decreases, reducing the probability of risk-taking.

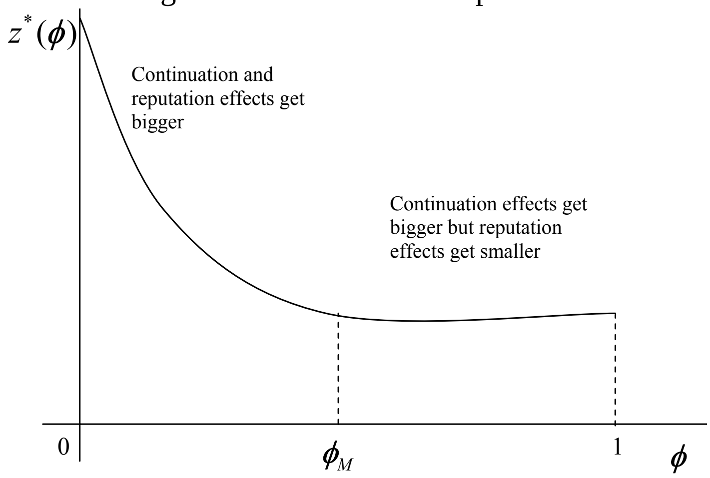
When ϕ is in the range $[\phi_M, 1]$ still $P(\phi)$ increases, $\mathbf{V}(\phi)$ increases and $R(\phi)$ decreases as ϕ increases, increasing $\Delta(\phi, z)$. We call this effect ”continuation effect”. However in this range, as ϕ goes up, $\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)$ goes down, decreasing $\Delta(\phi, z)$. We call this effect ”reputation effect”. Depending on which one of the effects is higher, $\Delta(\phi, z)$ can either increase or decrease, reducing or increasing the probability of risk-taking respectively.

Recall that, using equation (2.6) $z^*(\phi)$ is obtained by considering all possible beliefs $\widehat{x} \in [0, 1]$, so even when comparing across different levels of reputation ϕ in average the shape of reputation updating is the one shown in Figure 1.1. If the impact of a higher reputation $\phi \in [\phi_M, 1]$ on current and continuation payoffs is greater than the impact in reducing reputation effects, the probability of risk-taking continues decreasing in this range of reputation levels as well. However, even when the direction in the probability of risk-taking is not clear and depends on parameters when $\phi \in [\phi_M, 1]$, it is possible to guarantee that the rate at which the probability of risk-taking decreases in the range $\phi \in [0, \phi_M]$ is higher than in the range $\phi \in [\phi_M, 1]$.

To show $\frac{\partial^2 z^*(\phi)}{\partial \phi^2} > 0$ it is enough to show $\frac{\partial^2 \Delta(\phi)}{\partial \phi^2} < 0$ for all $\phi \in [0, 1]$. Given $\frac{\partial z^*(\phi)}{\partial \phi}$ described above, $\frac{\partial^2 P(\phi)}{\partial \phi^2} < 0$, $\frac{\partial^2 R(\phi)}{\partial \phi^2} > 0$, $\frac{\partial^2 \mathbf{V}(\phi)}{\partial \phi^2} < 0$ and $\frac{\partial^2 \mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)}{\partial \phi^2} < 0$. The intuition is that the combined decrease in $z^*(\phi)$ and ϕ creates a fast decrease in interest rates and increase in prices for low reputation levels, making $\mathbf{V}(\phi)$ concave on ϕ. The result is a convex schedule of cutoffs $z^*(\phi)$ Q.E.D.

Figure 1.3 is an example of the relation between the cutoff $z^*(\phi)$ and the reputation level ϕ. It shows the probability of risk-taking is higher for low reputation levels because continuation and reputation effects are not that important to deter these firms from taking risky actions. For low reputation levels, as ϕ increases, continuation values $\mathbf{V}(\phi)$, prices $P(\phi)$ and reputation punishments $\mathbf{V}(\phi_g) - \mathbf{V}(\phi_b)$ go up while $R(\phi)$ goes down, reducing $z^*(\phi)$. This is the case until ϕ hits ϕ_M. For $\phi > \phi_M$, as ϕ increases, continuation effects still go up but reputation pressures go down. The cutoff will increase or decrease depending on which effect dominates.

Figure 1.3: Cutoffs and Reputation



Note that in case the probability of risk-taking is minimized at a value of $\phi_m < 1$ (i.e., the cutoff schedule goes down and then increases toward $\phi = 1$), prices would be maximized and lending rates minimized at ϕ_m, hence the value function would achieve the maximum at ϕ_m. In this case continuation values would not increase monotonically with ϕ. It would increase until ϕ_m and then decrease towards one.

However, it is possible to draw some conclusions with respect to ϕ_m. First, $\phi_m \in [\phi_M, 1]$ as discussed above. Second, ϕ_m is biased towards one (i.e., the value function is almost always monotonically increasing in ϕ) when the probability of playing safe in steady state is high for all ϕ. This makes the difference between $\overline{\mathbf{V}}(1)$ and $\overline{\mathbf{V}}(0)$ big enough such that incentives from continuation are large when compared with the incentives from reputation (this is the case described in the first bullet point of Lemma 10 or in Figure 1.15 at the Appendix). At the other extreme, when steady states are characterized by risk-taking as in 1.16, $\overline{\mathbf{V}}(1) = \overline{\mathbf{V}}(0)$, both incentives from continuation and from reputation would be nonexistent.

This implies that the nonlinear schedule in cutoffs remains, even in cases where continuation values are not monotonically increasing for all ϕ but decreases for values close to one. It is difficult to draw any further analytical conclusions since value functions and cutoffs for all ϕ in equilibrium should be obtained jointly (i.e., in equilibrium, value functions of a given ϕ have an impact in determining the value functions for all other reputation levels). We will show how to solve this schedule numerically and support these considerations in Section 1.7.

1.6.2 Risk-Taking Clustering

1.6.2.1 Sensitivity of risk-taking to fundamentals

It is important at this point to analyze what happens with risk-taking in the case of a reduction in fundamentals. If $\theta < z^*(\phi)$, for a given reputation value ϕ and a signal noise $\sigma \to 0$, then most firms with that reputation level receive in expectation a signal $z < z^*(\phi)$ and decide to take risks. Since lenders and consumers observe θ and also $z^*(\phi)$ in equilibrium, they can infer a ϕ firm has a high $x(\phi)$, prices will be low and reputation would not be heavily updated. Small changes in fundamentals around $z^*(\phi)$ induce sudden changes in risk-taking for firms with reputation ϕ.

The next proposition formalizes this idea.

Proposition 15 *For highly precise signals about fundamentals (i.e., $\sigma \to 0$), small changes in fundamentals θ around the optimal cutoff $z^*(\phi)$ may induce a sudden change in risk-taking behavior for firms with reputation level ϕ.*

Proof Assume an equilibrium cutoff $z^*(\phi)$ for firms with reputation level ϕ. The market observes the fundamental value θ after firms have taken their actions from observing a signal $z = \theta + \sigma\varepsilon$. Lenders and consumers know ϕ firms will decide to play risky when $z < z^*(\phi)$, hence the probability assigned a particular firm plays risky is given by $Pr(z < z^*(\phi)|\theta)$ or which is the same, $Pr(\varepsilon < \frac{z^*(\phi)-\theta}{\sigma}|\theta)$. Since $\varepsilon \sim F$, the probability a firm ϕ plays risky $x(\phi) = F(\frac{z^*(\phi)-\theta}{\sigma})$.

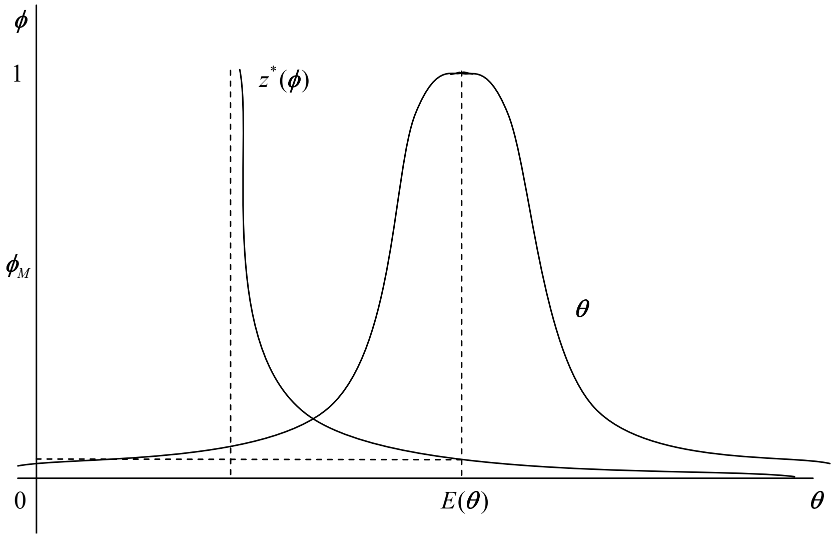
If σ is low and $\theta > z^*(\phi)$, then $x(\phi, \theta)$ is close to 1, prices are low and reputation formation is not important, reinforcing that firms want to play risky in that situation. Contrarily, if σ is low and $\theta > z^*(\phi)$, then $x(\phi, \theta)$ is close to 0, prices depend a lot on reputation and reputation is heavily updated, reinforcing that firms want to play safe in that situation. Q.E.D.

A lot of action can happen around the equilibrium cutoff to firms of the same reputation level when signals are highly precise, even when fundamentals do not show a significant change. However, the analysis so far has focused in the sudden change of behavior for firms with a particular ϕ value. What is the behavior in the aggregate assuming a particular distribution of reputation in the economy?

All firms with reputation ϕ will cluster when fundamentals go below the cutoff $z^*(\phi)$. As shown in Proposition 14 and Figure 1.3, cutoffs $z^*(\phi)$ are similar for values of reputation $\phi \in [\phi_M, 1]$. When the state of the economy is good, changes in fundamentals do not induce a change in risk-taking behavior for many reputation levels. Contrarily, in bad states of the economy, changes in fundamentals do induce a change in risk-taking behavior for many reputation levels. If the distribution of reputation levels is not heavily skewed towards low reputation firms, this will generate a big clustering in aggregate risk-taking when fundamentals weaken enough. This effect can be observed in Figure 1.4.

This property of the model implies large spikes in risk-taking behavior with a short duration, where risk-taking increases even for firms with high reputation. In fact, this

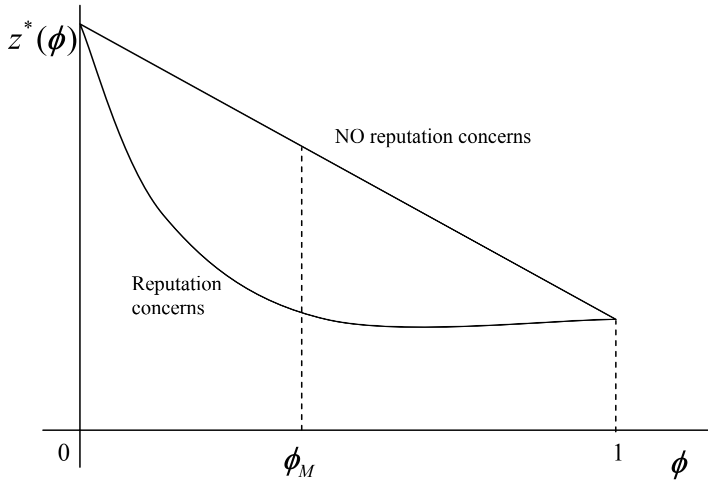
Figure 1.4: Cutoffs and fundamentals



is exactly a feature we can observe in the data (as will be discussed in Section 1.8.2.1), generating sudden and big losses for investors in those particular events.

What is even more striking is that clustering occurs exactly because of the existence of reputation concerns. Assume momentarily that firms born with a prior about the probability of being strategic ϕ that they cannot modify by gaining or losing reputation. In this counter-natural exercise, which arises for example in the case of unavailability of information about signals, differential gains from playing safe are given by equation (1.11) but without the last two components that represent the incentives from reputation. In this case, when obtaining cutoffs for each reputation level without reputation concerns, only continuation effects are present in the computation, which eliminates the nonlinearity introduce by reputation through learning, as in Figure 1.1. Figure 1.5 shows in general the relation between cutoffs with and without reputation concerns. While cutoffs are the same at $\phi = 0$ and $\phi = 1$, for all other reputation levels the benefits from playing safe are higher with reputation concerns, reducing cutoffs. In particular, the difference between the cutoffs with and without reputation concerns reaches the maximum at ϕ_M, where the reputation gains from playing safe reach the maximum. Specific examples will be shown in Section 1.7.

Figure 1.5: Cutoffs with and without reputation concerns



1.6.2.2 Considerations about the distribution of reputation

The clustering result not only depends on the non-linear schedule of cutoffs but also on the distribution of reputation levels in the economy. In reality, the distribution of reputation seems to have a high mass among intermediate reputation levels (see Tables 1.1 and 1.2 in Section 1.8.1). In the theory, since cutoffs for each ϕ are independent of the distribution, it depends on specific assumptions about the birth of new firms in the economy. Denote $\omega_t(\phi)$ the fraction of firms with reputation ϕ in the economy at period t. Its expected evolution is

$$\begin{aligned}\omega_t(\phi) = b(\phi)D + \omega_{t-1}(\phi_0)\alpha_s(1-\mathscr{V}(z^*(\phi_0))) + \omega_{t-1}(\phi_1)(1-\alpha_s)(1-\mathscr{V}(z^*(\phi_1)))\\ -\omega_{t-1}(\phi)[\mathscr{V}(z^*(\phi)) + (1-p_r)(1-\mathscr{V}(z^*(\phi)))]\end{aligned} \tag{1.18}$$

where $b(\phi)$ is the birth rate of firms with reputation ϕ as a fraction of the total population of firms, $\omega_{t-1}(\phi_0)\alpha_s(1-\mathscr{V}(z^*(\phi_0)))$ is the fraction of firms with reputation ϕ_0 at $t-1$ that in expectation will be upgraded to ϕ in period t (where $\phi_0 = \frac{p_r\alpha_r\phi}{p_r\alpha_r\phi+p_s\alpha_s(1-\phi)}$).

Similarly, ϕ_1 is the reputation level that is downgraded to ϕ. The negative expression represents the proportion of ϕ firms that die or change reputation out of ϕ.

Expressing birth rates for each ϕ as a function of a stationary expected distribution

$$b(\phi) = \omega(\phi)[2 - p_r(1 - \mathscr{V}(z^*(\phi)))] - \omega(\phi_0)\alpha_s(1 - \mathscr{V}(z^*(\phi_0))) \tag{1.19}$$
$$-\omega(\phi_1)(1 - \alpha_s)(1 - \mathscr{V}(z^*(\phi_1)))$$

Recall from the definition of $b(\phi)$ I am not restricting the economy from shrinking or growing and I am not taking any stand on the shape of the distribution. Hence, it is possible to impose any stationary distribution of reputation assigning the correct birth rate from equation (1.19). Given this degree of freedom it is relevant to discuss which are the assumptions on birth primitives required for a reputation distribution to overcome the non-linearity of cutoffs and to avoid clustering in risk-taking[28].

First, by learning properties, $\phi = 0$ and $\phi = 1$ are absorbing states. Assuming there is no renovation of firms in the economy, (i.e., $b(\phi) = 0$ for all ϕ), the economy will shrink with time and will converge to a distribution of firms with reputation 0 and 1, with a proportion given by the distribution of types in the economy. In this case the distribution of firms depends exclusively on the reputation dynamics of existing firms. Now assume all dying firms are replaced by newborns with a reputation $\phi = 0.5$[29], this renewal would fill intermediate reputation firms, making the distribution more evenly distributed or even with a greater mass at intermediate levels. Considering these effects, a distribution heavily skewed towards low reputation levels requires that most newborn firms are believed to have a very low initial reputation level, or which is the same, the proportion of strategic types is very small.

[28]Since we are dealing with simultaneous equations analytically intractable but easily solvable numerically, I will just make some general considerations based on numerical simulations available upon request

[29]This is the case, for example, if the prior is that 50% of firms are strategic and there is no further information about the firm at the time it arises

1.6.3 Efficiency considerations

In equation (1.11), the components of $\Delta(\phi,\theta)$ distinguish how effective is reputation to deal with inefficiencies that arise from adverse selection and moral hazard. Consider first the case of full information in which lenders and consumers know both firms' types and actions[30]. When strategic firms have a $\phi = 1$. The value in case they decide to play safe and risky respectively are $V(1,\theta|s)_{FI} = p_s\Pi_s(1,\theta) - p_s\frac{\overline{R}}{p_s} + \beta p_s\mathbf{V}(1)$ and $V(1,\theta|r)_{FI} = p_r\Pi_r(1,\theta) - p_r\frac{\overline{R}}{p_r} + \beta p_r\mathbf{V}(1)$.

Some specificities in the previous expressions are worth noting. First, since there are no reputation problems, the expected future value is always $\mathbf{V}(1)$. Second, interest rates reflect the real probability of default ($R_s(1) = \overline{R}/p_s$ and $R_r(1) = \overline{R}/p_r$), which means the cost of the capital for firms is always $\overline{R}$ in expectation. Finally, since consumers also observe actions, they pay a price reflecting the real probability of good results $P_s(1) = \alpha_s$ and $P_r(1) = \alpha_r$. In this sense $p_s\Pi_s(1,\theta) - p_r\Pi_r(1,\theta) = A(\theta)[(p_s\alpha_s - p_r\alpha_r) - (C_s(\theta) - C_r(\theta))]$

Hence, differential gains from playing safe under full information are,

$$\Delta(1,\theta)_{FI} = p_s\Pi_s(1,\theta) - p_r\Pi_r(1,\theta) + \beta(p_s - p_r)\mathbf{V}(1)$$

These are the first two components in equation (1.11) for the case of $\phi = 1$ when actions are observable. The value of the fundamental $\theta^*_{FI}(1)$ for which $\Delta(1,\theta^*_{FI}(1))_{FI} = 0$, determines the point below which it is efficient for strategic firms to play risky and above which it is efficient for strategic firms to play safe.

When lenders cannot observe firms' actions, they charge a unique interest rate $R(1)$, regardless of the technology used. This generates a moral hazard problem. Firms are more prone to take risks because they are not paying the premium for increasing

[30] In fact it is not interesting to have full information of actions and not types, since strategic firms can easily signal their type just by playing safe at least once

default probabilities lenders would charge for taking risks. While firms appropriate all gains from good results, they impose to lenders the losses from bad results. Formally, in the case of non-observable actions, $V(1,\theta|s) = p_s\Pi_s(1,\theta) - p_sR(1) + \beta p_s\mathbf{V}(1)$ and $V(1,\theta|r) = p_r\Pi_r(1,\theta) - p_rR(1) + \beta p_r\mathbf{V}(1)$. The main difference with full information is a common interest rate $R(1) \in [\frac{\overline{R}}{p_s}, \frac{\overline{R}}{p_r}]$. Hence,

$$\Delta(1,\theta) = p_s\Pi_s(1,\theta) - p_r\Pi_r(1,\theta) + \beta(p_s - p_r)\mathbf{V}(1) - (p_s - p_r)R(1)$$

The last component in this equation represents the moral hazard incentives to play risky that come from the lending relationship. Since $\Delta(1,\theta) < \Delta(1,\theta)_{FI}$, it is easy to show $\theta^*_{FI}(1) < z^*(1)$. $R(1)$ wil be determined in equilibrium by $z^*(1)$. For fundamentals $\theta \in (\theta^*_{FI}(1), z^*(1))$, it is efficient that firms follow safe technologies but they prefer to take risks. So, moral hazard generates excessive and inefficient risk-taking.

So far, we have been discussing the inefficiency generated by moral hazard for a firm with reputation $\phi = 1$. However, the un-observability of actions opens the room for the problem of adverse selection. This is because actions produce a non-deterministic outcome that inhibit the market to fully learn about the type of the firm. Now, considering adverse selection together with moral hazard, take a reputation value $\phi < 1$. Since $P(\phi) < P(1)$, $R(\phi) > R(1)$ and $\mathbf{V}(\phi) < \mathbf{V}(1)$, then $\Delta(\phi,\theta) < \Delta(1,\theta)$. This means that $z^*(\phi) > z^*(1)$. Hence for the fundamentals $\theta \in (z^*(1), z^*(\phi))$, it is efficient that firms take safe actions but they prefer to take risks. So, adverse selection further increases inefficient risk-taking by firms.

Equation (1.11) shows the differential profits for any ϕ (given beliefs $\widehat{k}$) for the case with adverse selection, moral hazard and the possibility of reputation formation. The last two components increase $\Delta(\phi,\theta|\widehat{k})$, reverting the inefficiencies caused by moral hazard and adverse selection.

Reputation has a bright side. It reduces inefficient risk-taking. However, reputation also has a dark side. It reduces inefficient risk-taking in a way that generates sudden and isolated events of clustering of risk-taking, loss of confidence, big increases in default probabilities and huge losses by investors. Reputation concerns are effective in reducing excessive risk-taking, but there are states of the economy bad enough that reputation incentives break down and suddenly collapse.

1.7 Simulations

In this section we develop a numerical example to show how reputation concerns generate large changes in aggregate behavior as a response to small changes in fundamentals. We also discuss about reputation efficiency effects and the sizable negative effects of net returns to investors in those extreme situations.

For simplicity, we don't introduce prices explicitly, so the market is composed only by lenders and all the results come only from interest rates. To be more specific, we assume Π_r is constant and $\Pi_s = \Pi_r + K + \psi\theta$, where $K < E(\theta)$ and $\psi > 0$, hence fulfilling assumptions 2 and 6 to ensure a unique steady state in continuation values. The computational procedure is described in the Appendix. Parameters in this particular exercise are $\beta = 0.95$, $\overline{R} = 1$, $\Pi_r = 1.6$, $K = -0.001$, $\psi = 0.4$, $p_s = 0.9$, $p_r = 0.7$, $\alpha_s = 0.8$, $\alpha_r = 0.4$ and $\theta \sim \mathcal{N}(0,1)$. These parameters have been chosen such that under full information risk-taking is efficient only for very low fundamental values, which arise with a probability 0.001%. This means risk-taking is almost never an efficient situation.

Figure 1.6 shows the ex-ante probability of risk-taking by firms with different reputation levels. The probability that intermediate firms take risks is much greater without reputation concerns than with reputation concerns. For example, the ex-ante probability a firm with a reputation level $\phi = 0.4$ takes risk is 60% without reputation concerns but only 4% with reputation concerns. Hence, the gap between the two curves shows the reduction in the ex-ante probability of inefficient risk-taking generated by reputation concerns. Even when reputation reduces inefficient risk-taking around intermediate reputation levels, it is not that successful for very low or very high reputation levels. Firms with very high reputation (ϕ around 1) and firms with very low reputation (ϕ around 0), have a probability of risk-taking around 3% and 75% respectively, whether or not they have reputation concerns.

Figure 1.6: Ex-ante probability of risk-taking - with and without reputation

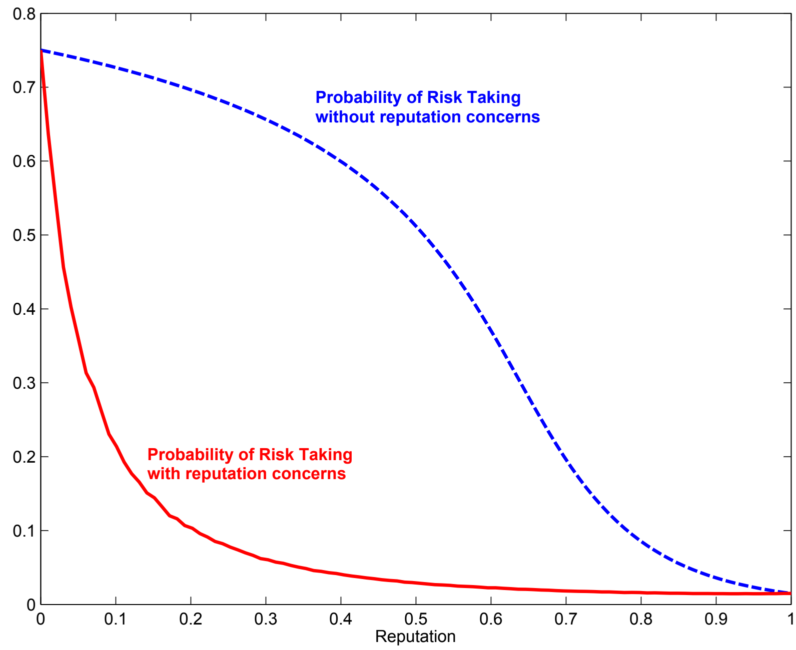
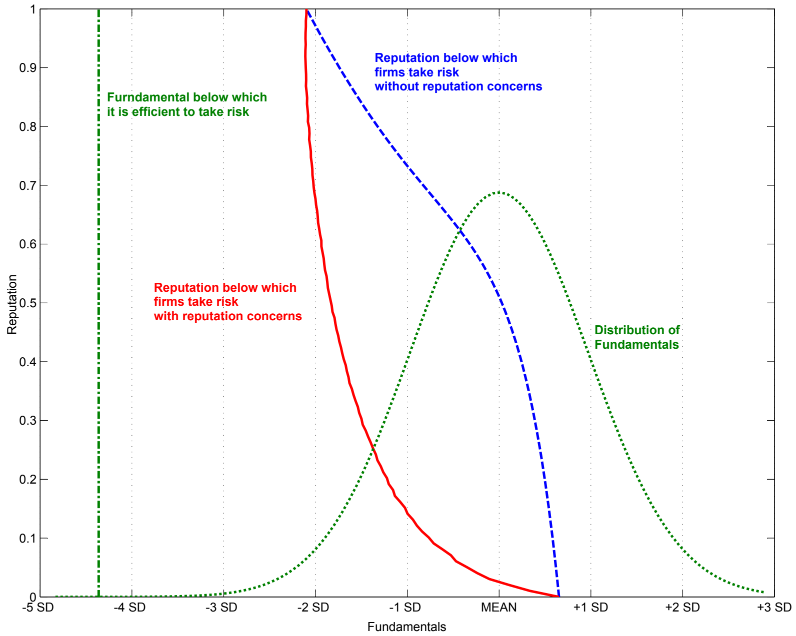


Figure 1.7 shows the same intuition of risk-taking clustering as Figure 1.5. As fundamentals decline, reputation levels that enter into a phase of risk-taking gradually grow when reputation is not a concern but suddenly grow when reputation is a concern. Figure 1.8 shows expected value functions and lending rates for firms with different reputation levels ϕ. Firms with reputation concerns pay lower interest rates and have higher expected continuation value than firms without reputation concerns.

This is because ex-ante probabilities of risk-taking are lower for all firms with reputation concerns, reducing current and future interest rates and increasing continuation values.

Figure 1.7: Cutoffs - with and without reputation

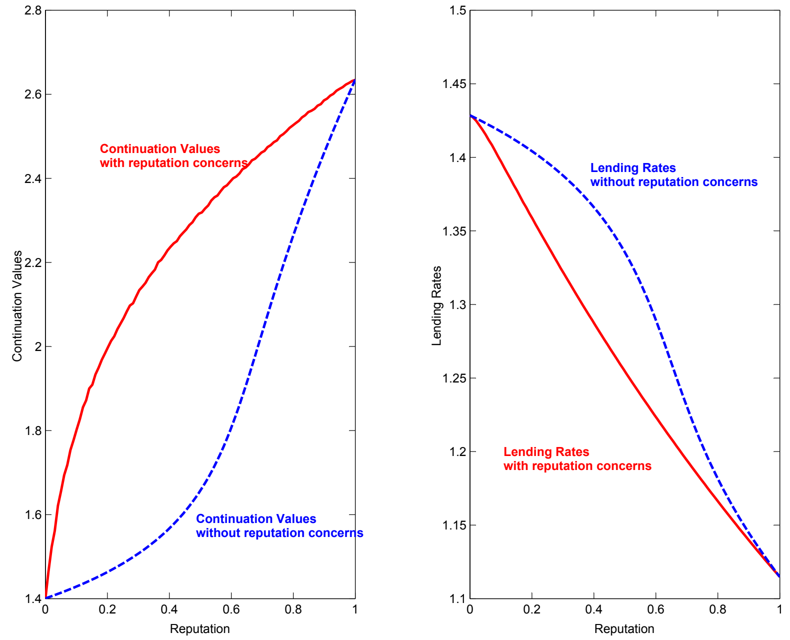


Figures 1.9 and 1.10 show the effects of sudden and isolated events of risk-taking in deep recessions, using 100 simulated periods. To do this exercise it is necessary to aggregate across firms, for which I assume a uniform reputational distribution. Since data seems to suggest this distribution has a greater mass in intermediate reputation levels, this is a conservative assumption[31]. Without reputation concerns, risk-taking is more common and arises as a result of even small declines in fundamentals. With reputation concerns, inefficient risk-taking is greatly reduced in general, except in very deep recessions when reputation does not provide enough incentives to inhibit inefficient risk-taking, even for firms with high reputation.

Figure 1.9 shows aggregate probability of default. This number goes from 10% in

[31] see Tables 1.1 and 1.2 in Section 1.8.1

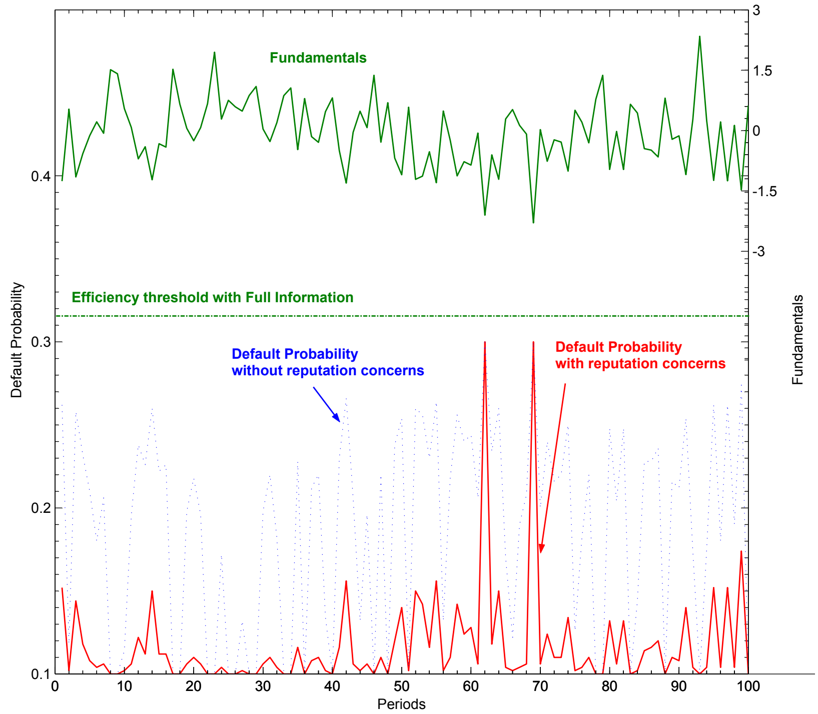
Figure 1.8: Value functions and lending rates - with and without reputation



the case no firm takes risk to 30% in the case all firms take risk. Figure 1.10 shows aggregate net returns. First we obtain individual net returns for each reputation level (computed by the lending rate charged to ϕ multiplied by the real probability of success by ϕ minus the risk free rate). Then we calculate the weighted sum of individual returns to obtain aggregate net returns, which will depend on fundamentals. When values of fundamentals decline enough, returns decline catastrophically since all firms, no matter their reputation, decide to take risks. Since lenders charge a low interest to high reputation firms, when those conditions arise, sudden losses are of high magnitude. With reputation concerns these rates are lower, then the losses are bigger.

We can also predict the returns of lending activities to firms with different reputation levels. Figure 1.11 shows simulated net returns of investors in firms with reputation levels $\phi = 0$, $\phi = 0.5$ and $\phi = 1$, for the cases with and without reputation concerns and for periods 55 to 75 in our simulation, when the two crises occur. In all cases, by the determination of lending rates in equilibrium, expected net returns

Figure 1.9: Simulated probability of default - with and without reputation

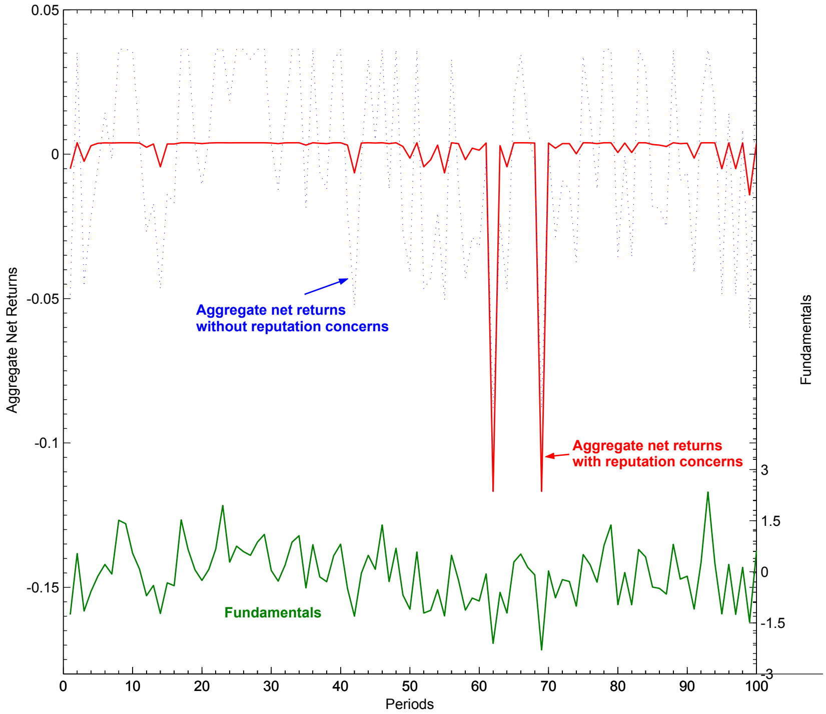


(before observing fundamentals) are zero.

Lenders to firms with very low reputation ($\phi = 0$) charge the maximum possible interest rate. Since in equilibrium they are right, all firms are of type $\mathscr{R}$ and then, no matter the fundamentals, net returns are always zero. Lenders to firms with very high reputation values ($\phi = 1$) charge interest rates that assume there will be some fundamentals under which even these firms will take risks. When fundamentals are normal, investors make a small difference because risk-taking of these firms is infrequent. When risk-taking occurs, they lose a lot.

Since reputation does not make any difference for extreme values, these two lines are the same in both panels of Figure 1.11. When reputation is $\phi = 0.5$ and reputation concerns exist, ex-ante probability of risk-taking is small and the pattern is similar to the one observed for $\phi = 1$, with less gains in normal times and less, but more frequent,

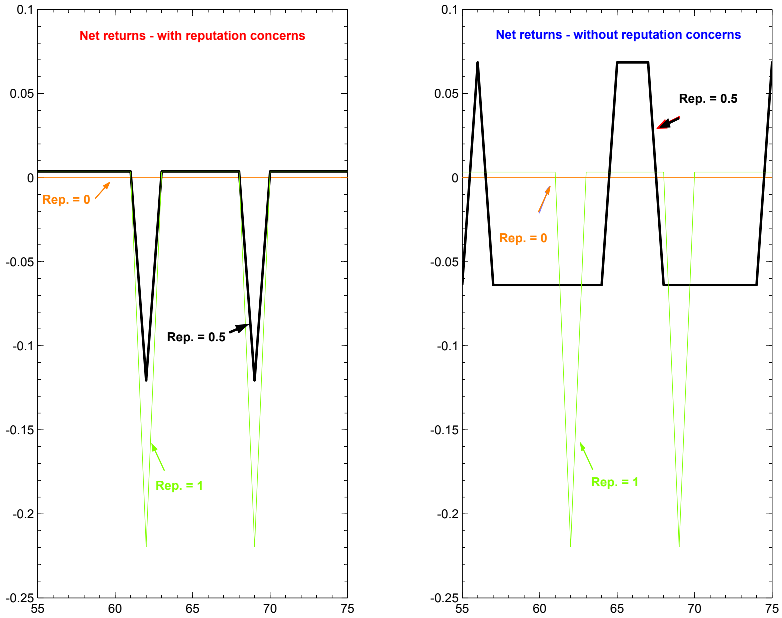
Figure 1.10: Simulated aggregate net returns - with and without reputation



looses in very bad times. Without reputation concerns, intermediate reputation firms are more volatile since they enter more frequently in the phase of risk-taking. Hence it is necessary to pay more in good times to compensate for more frequent losses in bad times.

Even when results from this simulation are based on particular and arbitrary parameters, they are highly robust to changes in the numbers used as soon as they fulfill assumptions 2 and 6. In all cases, reputation concerns introduce incentives to deter inefficient risk-taking and generate a sudden wave of risk-taking, with big losses to investors, below a certain threshold.

Figure 1.11: Simulated individual net returns - with and without reputation



1.8 Some Testable Hypothesis

1.8.1 Reputation over the Cycle

According to the model, in economic troughs we should see two patterns of reputation formation. On the one hand, we should observe higher default and downgrading, both because these are times of weakening in fundamentals (even when we neutralized this effect in this model) and because firms decide to take more risks, being more prone to exit and to have bad results. On the other hand, we should observe fewer cases of reputation revision, since it is more difficult for the market to update beliefs.

It is however difficult to directly track the evolution of reputation in the market. Here I propose a novel approach. I analyze credit ratings to capture the idea of reputation and rating transitions to capture the idea of beliefs updating and learning.

As a first step, let's divide the reputation continuum from 0 to 1 in seven bins from Aaa to C. Aaa corresponds to the highest possible reputation (ϕ close to 1) while C corresponds to the lowest possible reputation (ϕ close to 0). With this interpretation, rating transitions deliver information on how reputation varies and how beliefs are updated in different phases of economic cycles. I use a detailed view of rating migration provided by Moody's yearly rating transition matrices. These matrices summarize the size and direction of rating movements, including defaults[32], for the entire Moodys-rated universe, over specific time horizons.

In Table 1.1 I compare rating transition and default rates in 2001 (the last recession year with high recorded idiosyncratic risk as a proxy of risk-taking behavior[33]) with averages for the 20-year period 1980-2000 for broad rating categories. This is a very rough way to look for clues about general features of reputation evolution and changes in reputation over the cycle.

First, focus on the average characteristics of rating transition matrices (first panel in Table 1.1). As can be observed in the concentration of transitions around the diagonal, upgrades or downgrades in reputation are given gradually rather than suddenly. In the model this is predicted by equations 1.3 and 1.4 and graphically shown in Figure 1.1. These patterns are consistent with our model, where reputation can be constructed, destroyed, and managed. As in Mailath and Samuelson [2001], this is in stark contrast with other more standard models where reputation is intrinsically valuable (as Holmstrom [1999]), where "bad" types try to pool with "good" types and then reputation can be suddenly lost rather than managed over time (as Milgrom and Roberts [1982], Kreps and Wilson [1982] and Fudenberg and Levine [1992]) or where reputation can

[32] These rates are calculated as fractions in which the numerator represents the number of issuers that defaulted on Moodys-rated debt in a particular time period and the denominator represents the number of issuers that could have defaulted on Moodys-rated debt in that time period. Moodys defines a bond default as *"any missed or delayed disbursement of interest and/or principal, bankruptcy, receivership, or distressed exchange where (i) the issuer offered bondholders a new security or package of securities that amount to a diminished financial obligation (such as preferred or common stock, or debt with a lower coupon or par amount) or (ii) the exchange had the apparent purpose of helping the borrower avoid default"*.

[33] According to Goyal and Santa-Clara [2003] and Davis et al. [2006], 200 and 2001 were years of particularly high idiosyncratic risk

only increase over time or disappear (as Diamond [1989]).

Now we can focus on the comparison of rating transition across different states of the economy. We use the year 2001 both because it was the last recorded NBER recession date and because it was a year with a large degree of clustering in risk-taking behavior measured by firm-level volatility in returns. The shaded cells are those statistically different between the two transition matrices (at a 95% of confidence, one sided test). Comparing the two matrices, 2001 is characterized by a higher default rate (35% against 28% in average in the previous 20 years), mostly concentrated among low reputation firms.

Three facts consistent with our model predictions are worth noting. First, in recession firms take more risk, generating a higher probability of bad results (and downgrading) and a higher probability of default and exit. As can be seen, in 2001 there was more downgrading and less upgrading than on average since 1980. Second, by comparing the bolded diagonal elements of the two matrices, that indicate the frequency at which ratings have remained unchanged over respective periods, we can see that in 2001 the fraction of firms whose reputation was not updated is higher than its average since 1980. This denotes the difficulties in revising ratings in times with clustering in risk-taking. Furthermore, this difference between panels is more important for high reputation firms, which are the ones whose behavior changes the most in large recessions compared to normal times. Finally, transitions are more concentrated around the diagonal in recessions. The cells located far away from the diagonal are emptier in recessions than in normal times, showing the difficulties to update in recessions.

These patterns have also been noticed in other studies that try to document changes in ratings for other reasons. Here we highlight the finding from three sources, Bangia et al. [2000], Moody's reports and Altman and Rijken [2006].

First, in Table 1.2 we repeat results from Bangia et al. [2000], who use data from

Table 1.1: Reputation updating over the cycle

All-Corporate Average Rating Transition Matrix, 1980-2000 (percent)

Initial Rating	Terminal Rating: Aaa	Aa	A	Baa	Ba	B	Caa-C	Default	# of Firms
Aaa	**89.31**	10.15	0.50	---	0.03	---	---	---	2,585
Aa	0.96	**88.42**	10.04	0.38	0.16	0.02	---	0.04	8,085
A	0.08	2.34	**90.17**	6.37	0.81	0.22	---	0.02	15,210
Baa	0.09	0.39	6.42	**84.48**	6.92	1.39	0.12	0.20	10,066
Ba	0.03	0.09	0.50	4.41	**84.25**	8.65	0.52	1.54	8,816
B	0.01	0.04	0.17	0.58	6.37	**82.67**	2.98	7.17	7,437
Caa-C	---	---	---	1.10	3.06	5.89	**62.17**	27.77	1,025

All-Corporate Average Rating Transition Matrix, 2001 (percent)

Initial Rating	Terminal Rating: Aaa	Aa	A	Baa	Ba	B	Caa-C	Default	# of Firms
Aaa	**98.99**	1.01	---	---	---	---	---	---	121
Aa	0.31	**94.14**	5.24	0.16	---	0.16	---	---	714
A	0.26	2.38	**89.71**	6.80	0.34	0.17	0.17	0.17	1,312
Baa	0.20	0.30	3.72	**90.15**	4.32	0.80	0.20	0.30	1,081
Ba	---	---	1.23	6.94	**80.00**	9.38	1.23	1.23	540
B	---	---	0.12	1.01	5.95	**67.65**	15.50	9.78	1,055
Caa-C	---	---	---	---	---	2.67	**62.50**	34.82	266

Standard & Poor's (rather than Moody's) from 1980-2000 and show US expansion quarters against US recession quarters, defined as periods above and below the trend respectively[34]. They also noticed that defaults and downgrades are more likely in recessions and that transitions are more concentrated around the diagonal in economic troughs. The fact that the numbers in the diagonal are smaller in some cases during depressions is more than compensated for the increase in downgrades in those particular periods. Furthermore, by analyzing coefficients of variation, Bangia et al. [2000] mention that *"results suggest that migration probabilities are more stable on contractions than on average"*

Second, in several Moody's special reports, the same patterns are discussed. For example, the Historical Default Rates of Corporate Bond Issuers, 1920-1999 report states *"In spite of the higher default rates in 1999, overall rating volatility was lower than its average since 1980"*

[34]This comparison may hide our main point that under certain important weakening in fundamentals risk-taking behavior increases in a large degree as a result of a cluster behavior. Since recession dates as defined in Bangia et al. [2000] also correspond to certain reductions in fundamentals that do not justify a sudden risk-taking behavior, the comparison of the tables may hide the main source of action. This is why in our original exercise in Table 1.1 we just used an extreme year characterized by a big weakening of fundamentals as defined by NBER

Table 1.2: Reputation updating over the cycle (Bangia et al., 2000)

US Expansion Quarters (1981-1998) (percent)

	Terminal Rating								
Initial Rating	AAA	AA	A	BBB	BB	B	CCC	Default	# of Firms
AAA	**98.21**	1.66	0.11	0.02	0.02	---	---	---	6,581
AA	0.15	**98.08**	1.61	0.12	0.01	0.03	0.01	---	19,458
A	0.02	0.53	**98.06**	1.21	0.11	0.06	0.00	0.00	36,404
BBB	0.01	0.07	1.47	**96.94**	1.25	0.22	0.02	0.02	24,529
BB	0.01	0.03	0.19	1.93	**95.31**	2.25	0.16	0.12	18,161
B	---	0.02	0.07	0.10	1.70	**95.91**	1.31	0.88	20,002
CCC	0.05	---	0.19	0.23	0.47	3.57	**87.32**	8.17	2,129

US Recession Quarters (1981-1998) (percent)

	Terminal Rating								
Initial Rating	AAA	AA	A	BBB	BB	B	CCC	Default	# of Firms
AAA	**97.99**	1.76	0.25	---	---	---	---	---	795
AA	0.18	**96.89**	2.79	0.05	0.09	---	---	---	2,186
A	0.02	0.88	**96.44**	2.59	0.07	---	---	---	4,330
BBB	0.04	0.04	1.11	**96.31**	2.33	0.07	---	0.11	2,708
BB	---	0.06	0.06	1.39	**94.98**	2.72	0.42	0.36	1,655
B	---	0.06	0.06	0.11	0.72	**95.02**	2.27	1.77	1,806
CCC	---	---	---	---	---	1.20	**85.60**	13.20	250

Finally, Altman and Rijken [2006] try to rationalize the stability in rating transitions, especially on economic troughs. Their explanation is the ”through-the-cycle” methodology that rating agencies use to construct and update their estimates. According to Moody's, ratings are stable because they intend to measure default risks in the long run and because modifications are made only when rating agencies are confident that observed changes in a company's risk profile are likely to be permanent. Altman and Rijken [2006] explanation is also based on a prudent behavior of agencies. When Moody's or Standard & Poors attribute ratings to bond obligors, they are engaged in a complex judgment. In particular, rating migrations are triggered when the difference between the actual agency rating and the one predicted by the model they use exceeds a certain threshold, modifying ratings only partially. Our explanation is not based on a prudent behavior by agencies but on difficulties in evaluating the policies of firms given the point of comparison of similar firms. In fact, agencies agree that recessions inhibit rating migrations since the elements used to consider whether a permanent change in overall risk status occurred or not are noisier than in normal times

As can be seen from our own analysis and from some evidence in the literature, differences in reputation formation over the cycle constitute evidence that in recessions, when risk-taking behavior clusters, reputation is not heavily updated. Hence, the deterring effect of reputation concerns over risk-taking behavior is seriously inhibited in times of weakening fundamentals.

1.8.2 Clustering in Risk-Taking Behavior

Here we discuss some indicative evidence that risk-taking behavior measured by idiosyncratic risk tends to cluster excessively in recessions. Furthermore, corporate default rates seem to follow a similar pattern which, in our model, is a direct consequence of risk-taking clustering.

Campbell et al. [2001] analyzes the trend and cyclical behavior of idiosyncratic risk measured as a firm level profit volatility. The construction of this indicator fixes market and industry risk, reflecting variations in volatility that happen exclusively as a result of changes occurring inside a firm, such as risk-taking by managers. They show not only that idiosyncratic risk more than doubles in recessions but also that the magnitude of this clustering cannot be explained only from a weakening in fundamentals.

Similarly, Das et al. [2007] do not only show that default rates cluster in recessions but also that the correlation in default cannot be explained merely by fundamentals. They go even further and suggest there seem to be a non-observed variable that is more active in bad times than in good times and may account for the non-explained degree of clustering. Considering the evidence from Campbell et al. [2001] and the findings about reputation formation over the cycle, our model suggests risk-taking behavior may be the non-observable variable that is highly active in bad times. Even more we propose as a potential explanation that, in recessions, reputation concerns do not work as an effective mechanism to deter risk-taking.

1.8.2.1 Idiosyncratic Risk as a Proxy for Risk-Taking

Campbell et al. [2001] show that idiosyncratic risk tends to cluster excessively in recessions. In fact market volatility and recessions help to predict firm-level volatility. However, even when recessions are highly correlated to firm-level volatility, they have a smaller effect on the predictable component of volatility. Even when idiosyncratic risk doubles in recessions, the predictable component only helps to explain an increase of about 1.5. Furthermore, they show idiosyncratic risks tend to have the most negative correlation with NBER recession dates. This represents an important unexplained clustering in firm-level volatility and potentially on risk-taking behavior over the cycle.

Figure 1.12 shows the idiosyncratic risk series from Campbell et al. [2001][35]. Green bars show NBER-dated recessions. Hence, clustering occurs almost exclusively on economic downturns.

Figure 1.12: Aggregate idiosyncratic risk

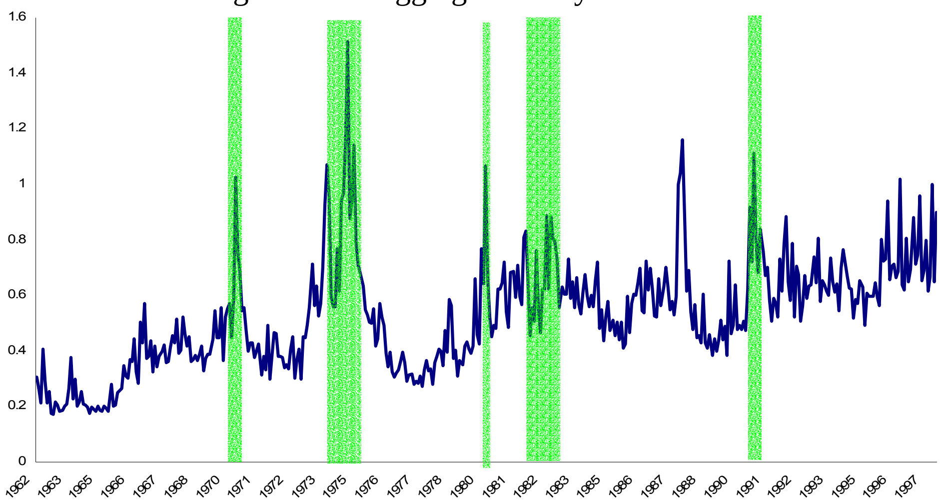
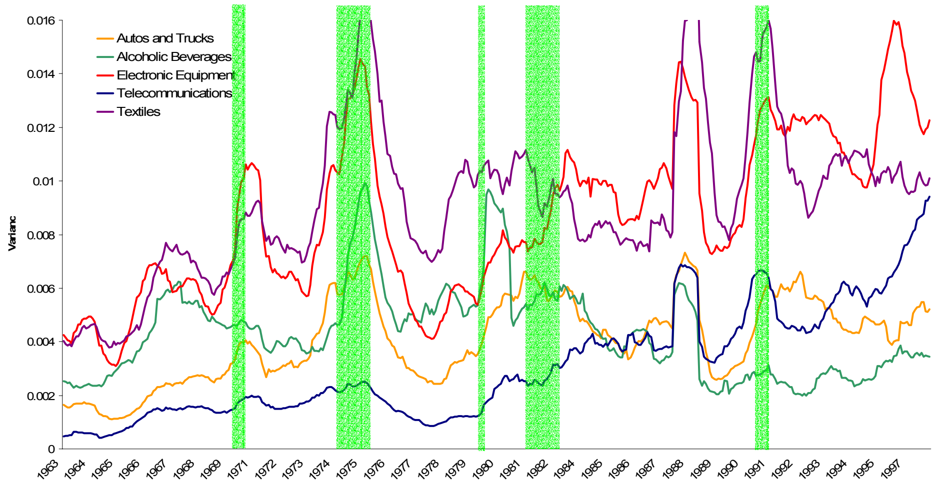


Figure 1.13 shows some representative examples of idiosyncratic risk in industries

[35] Idiosyncratic risk is measured by the monthly volatility of daily firm-level returns from the CRSP data set, including firms traded on the NYSE, the AMEX, and the Nasdaq. Monthly volatility is adjusted by subtracting market and industry volatilities. Data is recorded for 49 industries following the classification from Fama and French [1997]. Daily excess returns were calculated subtracting the 30-day T-bill return divided by the number of trading days in the month from daily returns.

with important cycles. These cycles are not perfectly correlated. Campbell et al. [2001] show these are mostly driven by shock in industry-specific fundamentals. All the plotted industries' idiosyncratic risks cluster around recessions. Outside recession dates, idiosyncratic risks seem to follow their own cycle, with industries clustering at different times when industry-specific fundamentals weaken. See, for example differences in the evolution of idiosyncratic risk between electronic equipments and telecommunications since 1992 or the differences between textiles and alcoholic beverages between the crises of 1974 and 1982.

Figure 1.13: Idiosyncratic risk across some industries



Even when industry-specific fundamentals are important, the aggregate effect of depressions for all industries is easy to observe when comparing correlation of idiosyncratic risks across industries. The weighted average correlation of idiosyncratic risk[36] of the 49 industries considering NBER recession dates is 0.48. Taking recession dates and the October 1987 market crash out of the sample, the correlation is just half of that number, 0.24. Considering NBER recession dates and the 1987 market crash represent just 50 months out of 414 months in the sample (from July 1962 to December

[36]Weights are based on market values using average market capitalization

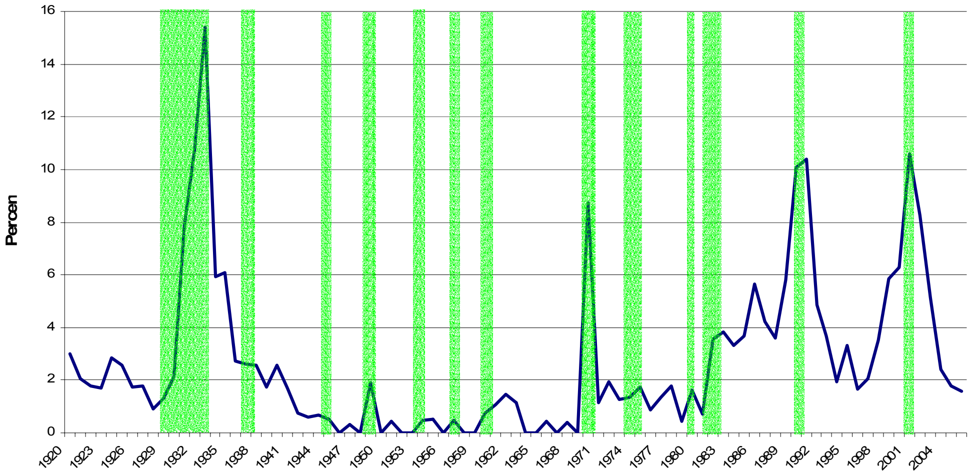
1997), it is clear the aggregate effect of depressions in all industries when compared to cycles outside economic troughs.

Our model captures exactly this feature, firm-level volatility in industries tend to cluster at the same time when a large weakening of aggregate fundamentals occurs and it tends to cluster at different times when aggregate fundamentals do not show any relevant change but industry fundamentals do.

1.8.2.2 Corporate Default Rates as a Consequence of Risk-Taking

Another possible application of our results is the explanation of a seemingly puzzling new result in the literature. Corporate default rates cluster in recessions with a magnitude that cannot be easily explained merely by a weakening of fundamentals. Figure 1.14 shows corporate default rates of speculative grade bonds collected by Moody's from 1920 to 2006.

Figure 1.14: Corporate default rates



The relation between default probabilities and macro fundamentals has been widely documented in the literature. Koopman and Lucas [2005], Bangia et al. [2000] and Nickell et al. [2000] present wide evidence of the co-cyclicality between default rates in all industries and macro fundamentals. Stock and Watson [1989] indeed highlight the important role of default rates in the construction of leading indicators.

However, the question is whether the large degree of clustering experimented in recessions can be merely explained by weakening in fundamentals. Das et al. [2007] recently make an original effort to test how well fundamentals can explain clustering in corporate default rates. They test a standard doubly stochastic model of default under which, *"conditional on the paths of risk factors that determine all firm's default intensities, firm defaults are independent Poisson arrivals with these conditionally deterministic intensity paths.*". They find in the data evidence of the existence of default clustering beyond that predicted by the doubly stochastic model. Furthermore, introducing additional variables related to fundamentals (such as GDP growth), which may be missing covariates in the model, they find a degree of clustering that is not captured by the extended model either[37]. Calibrated estimations of Gaussian copula correlation, which is a measure of the degree of correlation in default times that is not captured by co-movement in default intensities, range between 1% to 4% in Das et al. [2007] to 20% in Akhavein et al. [2005].

Das et al. [2007] propose that there seems to be a non-observed variable that is more active in bad times than in good times. In our model, this unobserved variable is risk-taking behavior, which clusters in bad times. Furthermore, we propose risk-taking is more active in bad times than in good times because the deterring effects of reputation are seriously hindered in economic depressions due to the existence of strategic complementarities in learning.

With a different methodology, Koopman et al. [2006] found that fundamentals (measured by the level of economic activity, bank-lending conditions, and financial

[37]Koopman et al. [2006] is an additional recent paper that shows cross-firm default correlation associated with observable factors cannot account for the large degree of time clustering in defaults found in the data.

markets variables) seem to be all important determinants of default rates. However, simple models seem to be significantly dynamically misspecified. Once they introduce in the model latent variables, macro fundamentals seem unable to explain the large degree of default that occurs in recessions. The question is open as to which missing latent variables capture the intensity of default. Contagion and frailty have been suggested as possible explanations[38].

Our model suggests to look at sudden changes in risk-taking behavior over the cycle as a potential factor behind a sudden jump in default rates. In the model default rates are a direct consequence of risk-taking behavior, captured by the exit state[39]. The probability of observing an exit (and hence a default) jumps from a number close to $(1-p_s)$ to a higher one close to $(1-p_r)$ in big depressions. We have shown some empirical evidence that risk-taking behavior clusters in recessions, at a higher magnitude than fundamentals can possibly explain. This suggests that risk-taking may be a good avenue to explore sudden jumps in default rates.

Finally, recall the similarities between our numerical simulations in Section 1.7 and the data, especially the similarities between Figure 1.9 and Figure 1.14. Reputation reduces risk-taking (and hence default rates) most of the time, generating sudden and isolated events of clustering in risk-taking, financial crises, big losses to investors and, eventually, credit crunches. If reputation formation is somehow inhibited, countries would experience more volatility in default rates, higher interest rates and more frequent but also less dramatic crises.

[38]Even when these answers cover part of the story they have some problems. Schonbucher [2003] shows that if ”contagion” fully explained the large degree of clustering it should not be the case that in default times both partner and competitor firms have a higher default probability. Yu [2005] has found some inconclusive evidence of ”frailty” and learning after default as a correlation device.

[39]Recall in the model we assumed there is no default from a reduction of cash flows

1.9 Conclusions

Reputation concerns deter excessive risk-taking behavior. This is a widely accepted property of reputation, both on formal and informal grounds. This paper studies these deterring effects when incentives for inefficient risk-taking vary with the state of the economy. The main finding is that reputation effects may suddenly collapse, leading to large changes in aggregate risk-taking as a response to small changes in fundamentals.

In the model reputation is the probability of being a firm with access to a safe technology. Since all firms have access to risky technologies (experimentation, for example), firms' unobservable types are defined by the unobservable actions they can take. Firms that can choose between safe and risky actions want to distinguish themselves from firms that do not have a choice. A higher reputation allows firms to pay lower interest rates and to charge higher prices. However, since types are defined by action availability, reputation does not have an intrinsic value. The reputation of being able to choose between safe and risky actions does not really matter if lenders and consumers also believe the choice will be to take risks. In this sense reputation is fragile because its value is a combination of having a certain type and behaving in a certain way. None of these two conditions is important without the other.

In the model reputation can be constructed, destroyed and managed. However, this desirable property comes with a cost in terms of equilibria multiplicity. To overcome this problem I interpret the reputation model extended with fundamentals as a non-standard dynamic global game in which strategic complementarities arise endogenously from reputation formation, and hence depend on the dynamic structure of the game, rather than being hard-wired into static payoffs as is common in the global games literature. This allows us to select a unique equilibrium robust to information perturbations.

I provide empirical support for my theory using data on corporate credit-rating transitions over the business cycle. I show that credit ratings evolve more slowly in bad times than in good times, which supports my prediction that reputation formation is more gradual and more difficult in bad times. I also discuss recent literature that shows that risk taking and defaults cluster in time, especially around recessions, at a large degree that cannot be explained just from a weakening in fundamentals. These empirical results are also consistent with the large degree of clustering predicted by the model.

Finally, the model suggests several new policy implications. First, under periods of ”clustering in risk taking” credit ratings are likely to be uninformative about default probabilities. Even firms with AAA bond ratings, for example, may have incentives in bad times to undertake risky projects which greatly increase the probability of default. This introduces a warning sign against relying on ratings as a basis to determine the right capital that banks should hold, as is the case with the Basel II regulations. My paper suggests that banking regulations which rely on official credit ratings may spread the effects of losses of confidence in borrowers more widely through the financial system, opening the doors to broader financial crises. Second, policies that promote credit bureaus facilitate learning and increase reputation incentives in domestic financial systems. My model implies that these policies have the potential to deter excessive risk taking but at the same time may exacerbate credit crises.

1.10 Appendix

1.10.1 Proof of Proposition 7

Proof To prove this proposition I proceed in four steps. First I derive the posterior density and distribution of θ given a signal z. Second, I prove there is a unique signal $z^*(\phi)$ that makes a strategic firm ϕ indifferent between taking risk or not, such that $z^*(\phi)$ is determined using Laplacian beliefs. Third I show that using $z^*(\phi)$ is a best response when the prior about θ follows a uniform distribution on the real line and both lenders and consumers believe $z^*(\phi)$ is the equilibrium cutoff. Finally we show that, as $\sigma \to 0$, the game with any prior distribution of θ uniformly converges to the unique solution proved in the previous step.

- **Step 1: Distributions of fundamentals conditional on signals**

Lemma 16 *The posterior density $f_{\theta|z}$ and distribution $F_{\theta|z}$ of θ given a signal z are given by,*

$$f_{\theta|z}(\eta|z) = \frac{v(\eta) f(\frac{z-\eta}{\sigma})}{\int_{-\infty}^{\infty} v(\theta) f(\frac{z-\theta}{\sigma}) d\theta} \tag{1.20}$$

$$F_{\theta|z}(\eta|z) = \frac{\int_{-\infty}^{\eta} v(\theta) f(\frac{z-\theta}{\sigma}) d\theta}{\int_{-\infty}^{\infty} v(\theta) f(\frac{z-\theta}{\sigma}) d\theta} = \frac{\int_{\frac{z-\eta}{\sigma}}^{\infty} v(z-\sigma u) f(u) du}{\int_{-\infty}^{\infty} v(z-\sigma u) f(u) du} \tag{1.21}$$

Proof By Bayes' rule,

$$f_{\theta|z}(\theta|z) = \frac{v(\theta) f_{z|\theta}(z|\theta)}{f_z(z)} \tag{1.22}$$

where f_z and $f_{z|\theta}$ are the densities of z and $z|\theta$ respectively. Since z is the sum of θ and $\sigma\varepsilon$, its density is given by the convolution of their densities, i.e., v and $f_{\sigma\varepsilon}$. Considering that $F_{\sigma\varepsilon}(\eta) = F(\eta/\sigma)$, $f_{\sigma\varepsilon}(\eta) = \frac{f(\eta/\sigma)}{\sigma}$, then f_z can be defined as,

$$f_z(z) = \sigma^{-1} \int_{-\infty}^{\infty} v(\theta) f\left(\frac{z-\theta}{\sigma}\right) d\theta \tag{1.23}$$

We can obtain the distribution of the observed signal z after observing a fundamental θ.

$$F_{z|\theta}(\eta|\theta) = Pr(z \leq \eta|\theta) = F\left(\frac{\eta - \theta}{\sigma}\right)$$

$$f_{z|\theta}(\eta|\theta) = \frac{dF_{z|\theta}(\eta|\theta)}{dz} = \sigma^{-1} f\left(\frac{\eta - \theta}{\sigma}\right) \tag{1.24}$$

Plugging equations 1.24 and 1.23 in 1.22, we obtain equation 1.20. The posterior distribution is obtained integrating over the density,

$$F_{\theta|z}(\eta|z) = \int_{-\infty}^{\eta} f_{\theta|z}(\theta|z)d\theta = \frac{\int_{-\infty}^{\eta} v(\theta) f(\frac{z-\theta}{\sigma})d\theta}{\int_{-\infty}^{\infty} v(\theta) f(\frac{z-\theta}{\sigma})d\theta}$$

and the expression in equation (1.21) follows from variable transformation $u = \frac{z-\theta}{\sigma}$

Q.E.D.

- **Step 2: Unique equilibrium cutoff** $z^*(\phi)$ **(using Laplacian beliefs)**.

Lemma 17 *There is a unique cutoff signal for each reputation* ϕ *such that* $\Delta(\phi, z^*|z^*) = 0$*,* $\Delta(\phi, z|z^*) > 0$ *for* $z > z^*$ *and* $\Delta(\phi, z|z^*) < 0$ *for* $z < z^*$*, where* $\Delta(\phi, z|z^*)$ *are the expected differential gains from playing safe for a firm with reputation* ϕ *that observes a signal* z *when lenders believe the cutoff the firm follows is* $z^*(\phi)$*.*

The cutoff $z^*(\phi)$ *is obtained using Laplacian beliefs, where* $\widehat{x} = F\left(\frac{z^*-\theta}{\sigma}\right)$ *is the probability the firm plays risky when the fundamental is* θ

$$\int_0^1 \Delta(\phi, z^*|\widehat{x})d\widehat{x} = 0 \tag{1.25}$$

Proof When fundamentals θ are not observed directly, the firm observes a signal z and lenders believe firms use a cutoff $z^*(\phi)$, the expected gains from playing safe are

$$\Delta(\phi, z, z^*) = E[\Delta(\phi, \theta|\widehat{x})|z] \tag{1.26}$$

where $\widehat{x}$ is just a function of the cutoff z^*.

$$\widehat{x} = F\left(\frac{z^* - \theta}{\sigma}\right) \tag{1.27}$$

Developing the expectation

$$\Delta(\phi,z,z^*) = \int_{-\infty}^{\infty} \Delta(\phi,\theta|\widehat{x}) dF_{\theta|z}(\theta|z)$$

Note that $\theta = z^* - \sigma F^{-1}(\widehat{x})$. From equation (1.21), define

$$\Psi(\widehat{x};z,z^*) = F_{\theta|z}(z^* - \sigma F^{-1}(\widehat{x})|z) = \frac{\int_{\frac{z-z^*}{\sigma}+F^{-1}(\widehat{x})}^{\infty} v(z-\sigma u) f(u) du}{\int_{-\infty}^{\infty} v(z-\sigma u) f(u) du}$$

Changing variables, from θ to $\widehat{x}$

$$\Delta(\phi,z,z^*) = \int_{-\infty}^{\infty} \Delta(\phi,z^* - \sigma F^{-1}(\widehat{x})|\widehat{x}) d\Psi(\widehat{x};z,z^*)$$

Laplacian beliefs arise from

$$\Psi(\widehat{x};z,z^*) = Pr(\theta < z^* - \sigma F^{-1}(\widehat{x})|z) = F[\frac{z-z^*}{\sigma} + F^{-1}(\widehat{x})]$$

For $z = z^*$, $\Psi(\widehat{x};z^*,z^*) = \widehat{x}$. Then, as $\sigma \to 0$

$$\int_0^1 \Delta(\phi,z^*|\widehat{x}) d\widehat{x} = 0$$

By Lemmas 4 and 5, we know there is a unique solution $z^*(\phi)$ to this equation. Q.E.D.

The intuition behind the use of a uniform distribution of beliefs $\widehat{x}$ to obtain the solution is straightforward. Adapting the discussion in Morris and Shin [2003], the key to understanding this feature is to consider the answer to the following question asked by a firm. "My signal has a realization z. What are the chances that lenders assign a probability smaller than η to me having a signal smaller than z?". If the true state is θ, the probability the firm observe a signal below z is given by $F\left(\frac{z-\theta}{\sigma}\right)$. This probability is smaller than η if $\frac{z-\theta}{\sigma} < F^{-1}(\eta)$, or when

$$\theta > z - \sigma F^{-1}(\eta)$$

The probability of this event, conditional on z

$$Pr(\theta > z - \sigma F^{-1}(\eta)|z) = Pr(z - \sigma\varepsilon > z - \sigma F^{-1}(\eta)) = F(F^{-1}(\eta)) = \eta$$

Considering in particular the cutoff $z^*(\phi)$, we can define $\widehat{x} = F\left(\frac{z^*(\phi)-\theta}{\sigma}\right)$, $Pr(\widehat{x} < \eta) = \eta$, hence the cumulative distribution of $\widehat{x}$ is the identity function, implying the density of $\widehat{x}$ is uniform over the unit interval. In words, if $z^*(\phi)$ happens to be the switching point of an equilibrium strategy, then playing safe or risky should be indifferent for the firm given lenders and consumers beliefs $\widehat{x}$ are uniformly distributed in [0,1].

- **Step 3: Best response with uniform priors over fundamentals.**

Now we need to verify that there exists indeed an equilibrium in which a firm with reputation ϕ plays risky whenever $z < z^*(\phi)$ and plays safe whenever $z > z^*(\phi)$. Signals z allow firms to have an idea not only about the fundamental but also about the signal lenders and consumers believe the firm has observed. Following Toxvaerd [2007], I first assume θ is drawn from a uniform distribution on the real line, hence an improper distribution with infinite probability mass. It is possible to normalize the prior distribution assuming $v(\theta) = 1$, simplifying the density to $f_{\theta|z}(\theta|z) = \sigma^{-1} f(\frac{z-\theta}{\sigma})$ and the distribution to $F_{\theta|z}(\theta|z) = F(\frac{z-\theta}{\sigma})$

First, we will denote $\widetilde{\Delta}(\phi, z|z^*)$ the case with a uniform prior distribution of fundamentals. We can redefine $\widetilde{\Delta}(\phi, z|\widehat{x})$ as $\widetilde{\Delta}(\phi, z|\widehat{z}(\phi))$ by writing $\widehat{x} = F\left(\frac{\widehat{z}(\phi)-\theta}{\sigma}\right)$, where $\widehat{z}(\phi)$ is the cutoff that the market believes strategic players ϕ use and z is the signal received by the firm. In words, since the market believes ϕ strategic firms follow the cutoff $\widehat{z}(\phi)$, when updating reputation and knowing the real fundamental, the market assigns a probability $\widehat{x} = F\left(\frac{\widehat{z}(\phi)-\theta}{\sigma}\right)$ the firm observed a signal z below the cutoff and played risk.

Expected payoff gains from playing safe rather than risky, given signal z when the market believes strategic firms ϕ use cutoffs $\widehat{z}(\phi)$ are given by

$$\widetilde{\Delta}(\phi, z|\widehat{z}(\phi)) = \int_{-\infty}^{\infty} \widetilde{\Delta}\left(\phi, z|F\left(\frac{\widehat{z}(\phi) - \theta}{\sigma}\right)\right) \sigma^{-1} f\left(\frac{z-\theta}{\sigma}\right) d\theta$$

Changing variables introducing $m = \frac{\theta - \widehat{z}(\phi)}{\sigma}$

$$\widetilde{\Delta}(\phi,z|\widehat{z}(\phi)) = \int_{-\infty}^{\infty} \widetilde{\Delta}\left(\phi,z|F(-m)\right)\sigma^{-1} f\left(\frac{z-\widehat{z}(\phi)}{\sigma} - m\right) d\theta$$

We can rewrite it in a simpler way as

$$\widetilde{\Delta}(\phi,z|\widehat{z}(\phi)) = \widehat{\Delta}(\phi,z|\widehat{z}(\phi),z') = \int_{-\infty}^{\infty} B(z',m)D(z,m)dm$$

where $B(z',m) = \widetilde{\Delta}\left(\phi,z'|F(-m)\right)$ and $D(z,m) = \sigma^{-1} f\left(\frac{z-\widehat{z}(\phi)}{\sigma} - m\right)$. As shown in Athey [2002], because of the monotone likelihood property, $\widehat{\Delta}(\phi,z|\widehat{z}(\phi),z')$ inherits the single crossing property of $\widetilde{\Delta}(\phi,\theta|\widehat{x})$. This means it exists a $z^*(\phi,\widehat{z}(\phi),z')$ such that $\widehat{\Delta}(\phi,z|\widehat{z}(\phi),z') > 0$ if $z > z^*(\phi,\widehat{z}(\phi),z')$ and $\widehat{\Delta}(\phi,z|\widehat{z}(\phi),z') < 0$ if $z < z^*(\phi,\widehat{z}(\phi),z')$.

Assuming $z > z'$ and $\widehat{\Delta}(\phi,z|\widehat{z}(\phi),z) = 0$, we can show

$$\widehat{\Delta}(\phi,z'|\widehat{z}(\phi),z') \geq \widehat{\Delta}(\phi,z|\widehat{z}(\phi),z') \geq \widehat{\Delta}(\phi,z|\widehat{z}(\phi),z) = 0$$

The first inequality coming from the state monotonicity and the second from the action single crossing property. A symmetric argument holds for $z > z^*(\phi)$. Hence, there exists a best response $\chi : \mathbb{R} \to \mathbb{R}$ such that

$$\begin{aligned} \widetilde{\Delta}(\phi,z|\widehat{z}(\phi)) > 0 \quad if \quad z > \chi(\widehat{z}(\phi)) \\ \widetilde{\Delta}(\phi,z|\widehat{z}(\phi)) = 0 \quad if \quad z = \chi(\widehat{z}(\phi)) \\ \widetilde{\Delta}(\phi,z|\widehat{z}(\phi)) < 0 \quad if \quad z < \chi(\widehat{z}(\phi)) \end{aligned}$$

Since there exists a unique $z^*(\phi)$ that solves

$$\widetilde{\Delta}(\phi,z^*(\phi)|z^*(\phi)) = \int_0^1 \widetilde{\Delta}(\phi,z^*(\phi)|\widehat{x})d\widehat{x} = 0 \tag{1.28}$$

Hence, $\chi(z^*(\phi)) = z^*(\phi)$, showing that there is a unique equilibrium in cutoff strategies for each ϕ such that

$$x^*(\phi,z) = \begin{cases} 0 \quad if \quad z > z^*(\phi) \\ 1 \quad if \quad z < z^*(\phi) \end{cases} \tag{1.29}$$

Q.E.D.

• **Step 4: Best response with general priors over fundamentals.**

Lemma 18 *As* $\sigma \to 0$, $\Delta(z, z-\sigma\xi) \to \widetilde{\Delta}(z, z-\sigma\xi)$ *uniformly.*

Proof First, $\Delta(z, z-\sigma\xi) \to \widetilde{\Delta}(z, z-\sigma\xi)$ continuously as $\sigma \to 0$

$$\Psi(\widehat{x}; z, z-\sigma\xi) = \frac{\int_{\xi+F^{-1}(\widehat{x})}^{\infty} v(z-\sigma u) f(u) du}{\int_{-\infty}^{\infty} v(z-\sigma u) f(u) du} \to 1 - F\left(\xi + F^{-1}(\widehat{x})\right) = \widetilde{\Psi}(\widehat{x}; z, z-\sigma\xi)$$

As in Toxvaerd [2007], we show convergence with respect to the uniform convergence norm, which implies uniform convergence. Uniformity ensures that the equivalence between the games with the two different assumptions about the prior distributions, is not a result of a discontinuity on $\sigma = 0$.

From the existence of dominance regions for each reputation level ϕ, $(-\infty, \underline{\theta}(\phi|\underline{\theta}))$ and $(\overline{\theta}(\phi|\overline{\theta}), \infty)$. Pick $\underline{z}(\phi) < \underline{\theta}(\phi|\underline{\theta})$ and $\bar{z}(\phi) > \overline{\theta}(\phi|\overline{\theta})$ and restrict attention to the compact sets $Z \equiv [\underline{z}(\phi), \bar{z}(\phi)]$ and $Z_\sigma \equiv [\underline{z}(\phi) - \sigma\xi, \bar{z}(\phi) - \sigma\xi]$. Hence $\Delta(\phi, z, z^*)$ maps into a compact set.

Define the sup-norm

$$\| \Delta(\phi) \| \equiv sup_{z,z^*}\{|\Delta(\phi, z, z^*)|\}$$

With respect to the Euclidean metric,

$$\forall \varepsilon_1 > 0, \exists \delta_1 \, |z - z'| < \delta_1 \Rightarrow |\Delta(\phi, z, z^*) - \widetilde{\Delta}(\phi, z', z^*)| < \varepsilon_1, \forall z^*$$
$$\forall \varepsilon_2 > 0, \exists \delta_2 \, |z^* - z^{*'}| < \delta_2 \Rightarrow |\Delta(\phi, z, z^*) - \widetilde{\Delta}(\phi, z, z^{*'})| < \varepsilon_2, \forall z$$

This implies

$$\sqrt{(z-z')^2 + (z^* - z^{*'})^2} < \sqrt{\delta_1^2 + \delta_2^2}$$

By triangle inequality

$$
\begin{aligned}
|\Delta(\phi,z,z^*)-\Delta(\phi,z',z^{*'})| &= |\Delta(\phi,z,z^*)-\Delta(\phi,z',z^*)+\Delta(\phi,z',z^*)-\Delta(\phi,z',z^{*'})| \\
&\leq |\Delta(\phi,z,z^*)-\Delta(\phi,z',z^{*'})|+|\Delta(\phi,z',z^*)-\Delta(\phi,z',z^{*'})| \\
&\leq \varepsilon_1+\varepsilon_2
\end{aligned}
$$

As $\sigma \to 0$

$$\parallel \Delta(\phi)-\widetilde{\Delta}(\phi) \parallel = sup_{z,z^*}\{\Delta(\phi,z,z^*)-\widetilde{\Delta}(\phi,z,z^*)\} \to 0$$

with respect to the sup-norm Q.E.D.

1.10.2 Proof and Intuition Lemma 10

Let's start with the case in which playing safe with a very high probability is a fixed point among continuation values (first bullet point in Lemma 10). If an important weakening of fundamentals occurs, firms will cluster in risk-taking behavior for a short period of time. This case is the simplest one and helps to develop a clear understanding of the intuition behind the next graphs.

Figure 1.15 shows $\mathbf{V}_t(\phi)$ as a function of the expected continuation value in $t+1$ ($\mathbf{V}_{t+1}(\phi)$). The lines labeled *"Always Risky"* and *"Always Safe"* represent cases in which strategic firms with a reputation level ϕ are believed to play always risky or always safe (as defined in equation (1.10) when $x=1$ and $x=0$) respectively. By Assumption 6, the intercept in the case in which firms are believed to play risky is higher than the intercept of the value when strategic firms are believed to play safe. Furthermore, the slope of the value when firms are believed to play risky (βp_r) is smaller than the slope of the value when strategic firms are believed to play safe (βp_s).

In Figure 1.15 we show the two extreme cases where $\phi=0$ and $\phi=1$. Take for example $\phi=0$. In this case, regardless of the beliefs of lenders and consumers,

$E(\mathbf{V}_{t+1}(\phi')) = \mathbf{V}_{t+1}(\phi)$ because reputation never improves or decays. A similar argument is true for $\phi = 1$.

However, these are the expected values in case lenders and consumers never change beliefs about risk-taking behavior, regardless of fundamentals or future payoffs. As shown in Lemma 8, the expected continuation value of a particular firm depends on the probabilities the fundamental θ lies below the optimal cutoff $z^*(\phi)$. Hence, expected values $\mathbf{V}_t(0)$ and $\mathbf{V}_t(1)$ are represented by the solid red lines.

For $\phi = 0$, $\Delta_t(0, z_t) = p_s\Pi_s(0) - p_r\Pi_r(0) + (p_s - p_r)[\beta\mathbf{V}_{t+1}(0) - R_t(0)]$. This differential determines $z_t^*(0)$, which is negatively related to $\mathbf{V}_{t+1}(0)$. It is possible to find a $\widetilde{\mathbf{V}}_{t+1}(0)$ for which $z_t^*(0) = E(\theta)$. For a small γ_θ, if $\widetilde{\mathbf{V}}_{t+1}(0)$ is to the left of the point $SS0$ in Figure 1.15, then the continuation value converges backward to $\overline{\mathbf{V}}(0)$ for a reputation level $\phi = 0$. Given a continuation value $\overline{\mathbf{V}}(0)$, strategic firms ϕ play safe with a very high probability.

A similar analysis is true at the other extreme, for $\phi = 1$. In this case $\Delta_t(1, z_t) = p_s\Pi_s(1) - p_r\Pi_r(1) + (p_s - p_r)[\beta\mathbf{V}_{t+1}(1) - R_t(1)]$. Note $\Delta_t(1, z_t) > \Delta_t(0, z_t)$ (for a given continuation value at $t+1$) from two effects. First, prices for $\phi = 1$ are greater than prices for $\phi = 0$ (in the case the market assigns some positive belief for the firm to play safe). Hence, $p_s\Pi_s(1) - p_r\Pi_r(1) > p_s\Pi_s(0) - p_r\Pi_r(0)$. Second, interest rates charged to firms with high reputation $\phi = 1$ are lower than those charged to $\phi = 0$, hence $R_t(1) < R_t(0)$.

It is important to note that even if lenders and consumers strangely believe that $\phi = 0$ is unlikely to play risky and $\phi = 1$ is unlikely to play safe, $R_t(1) < R_t(0)$ because they also believe the $\phi = 0$ firm cannot make a choice while the $\phi = 1$ firm is at least strategic for sure and sometimes will play risky.

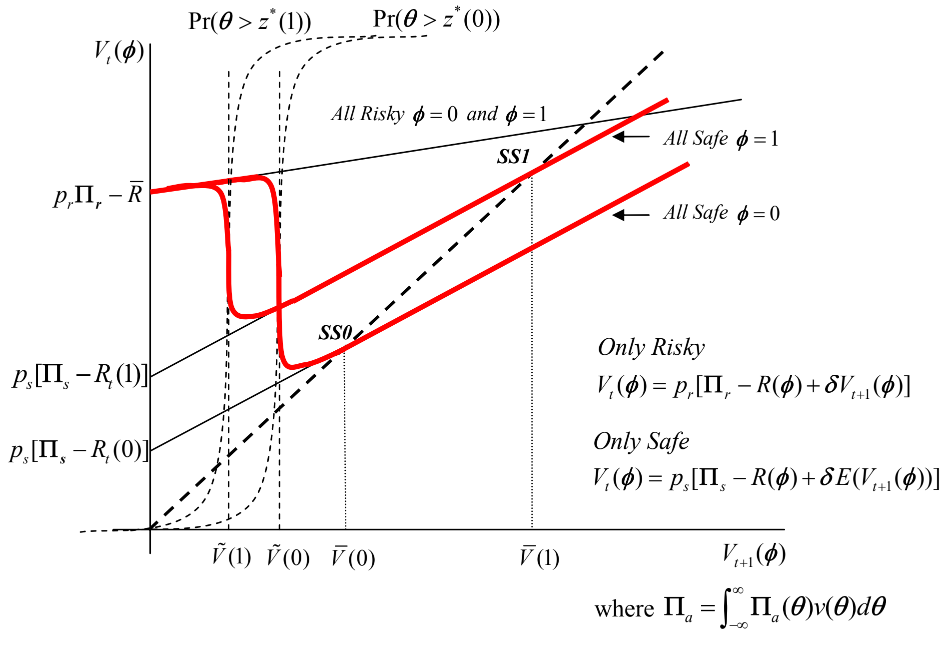
Since $\Delta_t(1, z_t) > \Delta_t(0, z_t)$, $\widetilde{\mathbf{V}}_{t+1}(1) < \widetilde{\mathbf{V}}_{t+1}(0)$ and hence $\widetilde{\mathbf{V}}_{t+1}(1)$ is to the left of $SS1$. The intercept is higher for $\phi = 1$ than for $\phi = 0$ while slopes are the same. Hence, continuation values converge to a higher level for $\phi = 1$ than for $\phi = 0$, $\overline{\mathbf{V}}(1) > \overline{\mathbf{V}}(0)$.

The lowest possible price and highest possible lending rate correspond to $\phi = 0$ because regardless of what a strategic firms would decide, lenders and consumers just believe that a firm is not strategic. For all other reputation levels, the potential behavior of strategic firms matters because lenders and consumers assign a probability that the firm is, in fact, strategic.

Hence, whenever $\widetilde{\mathbf{V}}_{t+1}(0)$ is to the left of the point *SS*0 in Figure 1.15, the continuation values for all reputation levels ϕ will converge to a unique value $\overline{\mathbf{V}}(\phi)$

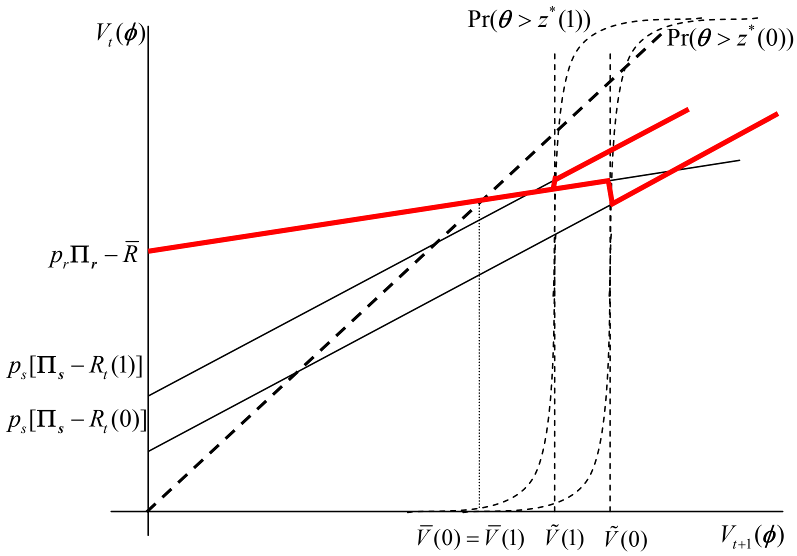
The case depicted in Figure 1.15 not only shows that continuation values are well defined but also that they converge to a fixed point such that all firms will be playing safe with a very high probability (close to 1).

Figure 1.15: Unique "Safe" Continuation Value



A similar analysis leads to convergence to continuation values dominated by risk-taking. This is the case depicted in Figure 1.16. The condition for this to be the case is that $\widetilde{\mathbf{V}}_{t+1}(1)$ exceeds the value at which $\mathbf{V}_t(1)$ crosses the 45-degree line. In this case all firms, regardless of their reputation, decide to take risk with a very high probability (close to 1). In this extreme case continuation values are well defined and converge to the same value for all reputation levels ϕ. Reputation does not play any role in introducing incentives in this particular situation where all firms almost always decide to play risky.

Figure 1.16: Unique "Risky" Continuation Value

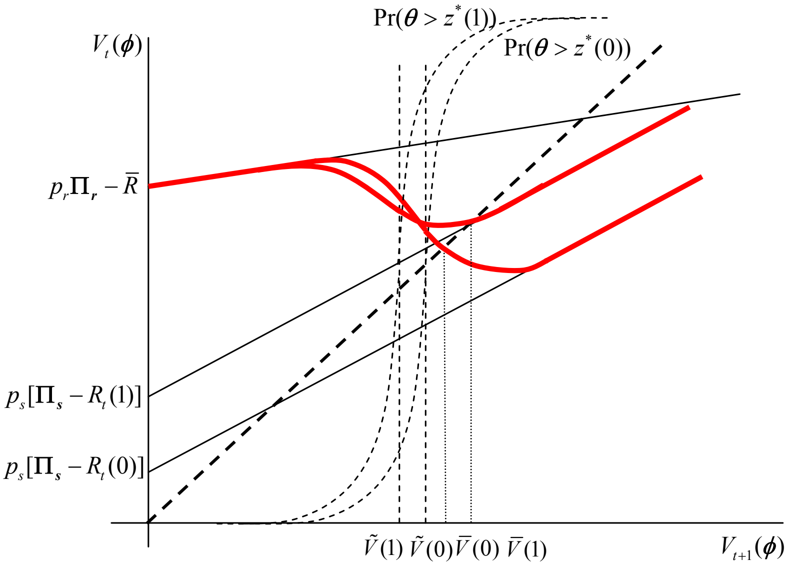


Steady states in which the market believes that firms will play safe or risky with a very high probability (close to 1) are not particularly interesting (even when the first case is able to explain situations of sudden and scarce risk-taking clustering, as shown in the data).

In Figure 1.17 we show the third possible situation in which, for $\phi = 0$ and $\phi = 1$, $\widetilde{\mathbf{V}}(0)$ and $\widetilde{\mathbf{V}}(1)$ lie between the points $SS0$ and $SS1$. To obtain convergence to a single continuation value (rather than cyclical movements) we must make the extra assumption that γ_θ is high enough.

The probability of risk-taking is higher for low reputation firms. This implies

Figure 1.17: Unique Continuation Value in Probabilities



monotonicity of continuation values on reputation levels ϕ

In the case of cyclic or even chaotic patterns, a full characterization of these cycles is always possible because continuation values are well defined, as shown in Proposition 9. However, it is possible for continuation values to behave counter-intuitively. Recall $\mathbf{V}_t(\phi)$ depends on $\mathbf{V}_{t+1}(\phi_g)$ and $\mathbf{V}_{t+1}(\phi_b)$, but these will depend on the specific part of the cycle that ϕ_g and ϕ_b will be playing at $t+1$. For example, it is possible that $\mathbf{V}_{t+1}(\phi_b) > \mathbf{V}_{t+1}(\phi_g)$ if ϕ_b will be playing risky next period while ϕ_g will be playing safe. In this case, reputation is reversed and firms may try to have a bad reputation to take advantage of the future risk-taking behavior of low reputation level firms.

In our model, reputation is not treated as an intrinsic asset that makes the market blindly assign more value to firms with higher reputation. Because of this, it may be that firms do not try to achieve a higher reputation per se, but the reputation of firms more likely to play risky in order to pool with them and get more expected profits.

These pervasive effects of cycles come from the fact that reputation is not an asset per se but a signal of how well firms commit. Introducing reputation as an asset would eliminate this particular feature of the model, preserving, nevertheless, the existence of cyclical behavior.

In any case, as all models with univariate dynamics, this is not a compelling setting to analyze cycles since it heavily depends on the definition of the period length. Making the period shorter enough, for example, generates cycles at a higher frequency. Since the period length does not have any economic meaning but it changes the results of the model, this model is not effective to discuss cyclical properties.

1.10.3 Computational Procedure

We solve the model following the next procedure.

- Set a large grid of $\phi \in [0,1]$
- Solve for the full information case (efficiency)
  - Guess a $\mathbf{V}_{FI}(1)_0 = 0$.
  - Obtain θ_0^* as the solution of

$$\Delta(1,\theta)_{FI} = p_s\Pi_s(1,\theta) - p_r\Pi_r(1,\theta) + \beta(p_s - p_r)\mathbf{V}_{FI}(1)_0 = 0$$

  - Obtain

$$\mathbf{V}_{FI}(1)_1 = \frac{\left[\mathscr{V}(\theta_0^*)[p_r\Pi_r - \overline{R}] + \int_{\theta_0^*}^{\infty}[p_s\Pi_s(\theta) - \overline{R}]v(\theta)d\theta\right]}{\left[1 - \beta(p_r + \mathscr{V}(\theta_0^*)(p_s - p_r))\right]}$$

  - Use $\mathbf{V}_{FI}(1)_1$ as the new guess and iterate until $\mathbf{V}_{FI}(1)_I - \mathbf{V}_{FI}(1)_{I-1} < \varepsilon$.
- Solve for the reputation case.
  - Guess a $\mathbf{V}(\phi)_0 = 0$ for all ϕ.
  - Using $\mathbf{V}(\phi)_0$ obtain $\Delta(\phi, z|\widehat{x})_0$ for large N_x beliefs $\widehat{x} \in [0,1]$.

- Solve for $z^*(\phi)_0$ that makes $\frac{\sum\Delta(\phi,z|\widehat{x})_0}{N_x}=0$
- For all $\theta<(>)z^*(\phi)_0$, $x(\phi,\theta)_0=1(=0)$.
  * $R(\phi|z_0^*)_0$ follows from $z^*(\phi)_0$
  * $\phi_g(\phi,\theta)_0$ and $\phi_b(\phi,\theta)_0$ follow from $x(\phi,\theta)_0$.
- Obtain $\mathbf{V}(\phi)_1$ as

$$\mathbf{V}(\phi)_1=\int_{-\infty}^{z^*(\phi)_0}p_r[\Pi_r-R(\phi|z_0^*)_0+\beta\mathbf{V}(\phi)_0]v(\theta)d\theta$$
$$+\int_{z^*(\phi)_0}^{\infty}p_s[\Pi_s(\theta)-R(\phi|z_0^*)_0+\beta E(\mathbf{V}_{t+1}(\phi|0)_0)]v(\theta)d\theta$$

- Use $\mathbf{V}(\phi)_1$ as the new guess.
- Solve for $z^*(\phi)_1$ that makes $\frac{\sum\Delta(\phi,z|\widehat{x})_1}{N_x}=0$.
- Iterate until $\mathbf{V}(\phi)_I-\mathbf{V}(\phi)_{I-1}<\varepsilon$ for all ϕ.

- Solve for the non reputation concerns case
  - Guess a $\mathbf{V}(\phi)_0=0$ for all ϕ.
  - Obtain $\theta^*_{MH}(\phi)_0$ by making $\Delta(\phi,\theta)_{MH}=p_s\Pi_s(\phi,\theta)-p_r\Pi_r(\phi,\theta)+\beta(p_s-p_r)[\mathbf{V}_{FI}(\phi)_0-R(\phi|\theta^*_{FI})]=0$
  - Obtain $\mathbf{V}_{MH}(\phi)_1=\frac{\mathscr{V}(\theta^*_{MH}(\phi)_0)[p_r[\Pi_r-R(\phi)]+\int_{\theta^*_{MH}(\phi)_0}^{\infty}[p_s[\Pi_s(\theta)-R(\phi)]]v(\theta)d\theta}{\left[1-\beta(p_r+\mathscr{V}(\theta^*_{MH}(\phi)_0)(p_s-p_r))\right]}$ for each ϕ.
  - Use $\mathbf{V}_{MH}(\phi)_1$ as a new guess and iterate until $\mathbf{V}_{MH}(\phi)_I-\mathbf{V}_{MH}(\phi)_{I-1}<\varepsilon$.

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CHAPTER 2

Slow(er) Boom, Sudden Crash. Financial Frictions and Recoveries

Asset markets are characterized by slow booms and sudden crashes. Lending rates, for example, are more likely to experience big jumps rather than big drops. We focus on the comparison of this asymmetry across countries.

First, we document that lending rates are more asymmetric in economies with poor financial systems. Second, we explain this finding by introducing financial frictions into a model with endogenous flow of information. High agency costs restrict the generation of information that fuels booms. Contrarily, they are not so important in good times, being irrelevant in determining the magnitude or speed of crashes. Finally, by calibrating the model, we show that cross-country differences in the asymmetry of lending rate fluctuations are well explained by differences in financial frictions.

2.1 Introduction

Asymmetry is a well known feature in asset markets. Lending rates, for example, exhibit sudden increases but slow and gradual reductions. The "tequila" 1994 peso crisis was a typical case of this pattern. It took just 4 months for Mexican lending rates to rose around 70 percentage points, but more than 30 months to return to pre-crisis levels [1].

Even when the explanation of this asymmetry has attracted a lot of attention in economics[2], differences across countries have been surprisingly absent from this literature. However, the study of these differences is of the utmost importance. First, high asymmetry on lending rate fluctuations may cause financial distresses, banking crises and eventually growth reductions (Bergoeing et al. [2004]). Second, since there are gains from reducing business cycles fluctuations and improve the forecasting of macroeconomic aggregates, the benefits from understanding the source of asymmetry on asset prices are non-trivial (Van Nieuwerburg and Veldkamp [2006]; Chen and Chan [1989]). Finally, a preference for (positive) skewness in rates of return is a general characteristic of investors having utility functions displaying the desirable behavioristic attributes (Kraus and Litzenberger [1976]).

In this paper we make three contributions. The first one is empirical. By focusing

[1] Many similar examples can be found in the most important recent crises. In Brazil, in October 1997, loan rates rose from 71% to 98% and it took 10 months to return to pre-crisis levels. In Indonesia, the 8 months following the Asian crisis experienced a rise in lending rates from 18% to 35% while it tooks 24 months to return to pre-crisis levels. In Russia, during the first half of 1998 lending rates rose from 24% to 48% in and it tooks 25 months for loan rates to return to pre-crisis levels.

[2] Banerjee [1992], Banerjee and Newman [1993] and Welch [1992] explained crashes from herd behavior and information cascades. Jacklin et al. [1992], based on Glosten and Milgrom [1985], used a portfolio insurance model of stock market crashes. Allen et al. [2006] used an information based model of bubbles. Zeira [1994] and Zeira [1999] proposed models of informational overshooting to explain booms and crashes in stock prices. Veldkamp [2005] used a model with endogenous flow of information to explain unconditional asymmetry.

For a review of asymmetries in real markets and aggregate economies see Van Nieuwerburg and Veldkamp [2006] and Jovanovic [2006].

on lending rates, we document a negative relationship between financial development and asymmetry. Lending rates on countries with high levels of monitoring and bankruptcy costs tend to be more asymmetric. Literature on asymmetric lending rates have been based either on evidence from a single country or on evidence from a small sample of similar countries.

The second contribution is theoretical. We explain these empirical facts by introducing financial frictions and agency costs into Veldkamp [2005] model of endogenous flow of information. In her complete information model, agents choose to invest or not in a risky asset based on an inference about the unobserved state of the economy, which is constructed from signals sent by current ventures. When agents think the state is good, many investments generate a large sample of observations. When the state changes to bad, there are a lot of signals in the economy, investors deduce easily conditions have changed and interest rates increase a lot. Contrarily, when the state is bad and changes to good, the limited number of existing ventures offer few signals about the switch, agents slowly learn about it and lending rates drop gradually.

We introduce asymmetric information between borrowers and lenders and costly state verification into this setup. High agency costs (such as monitoring and bankruptcy costs) increase lending rates in equilibrium, producing under-investment and a reduction on the number of signals available in the economy. However, the reduction of economic activity is not symmetric across states. In bad times, since the likelihood a venture fails is big, high agency costs impose big restrictions on loans, slowing down the creation of new economic activity. Contrarily, in good times agency costs are not that important in determining the number of ventures. Hence, high agency costs slow down the learning that fuels booms but not the information that sustain big crashes. Naturally this is translated into greater asymmetry on lending rates.

The third contribution is quantitative. Calibrations of the model closely match the data on cross-country differences of asymmetry in lending rates fluctuations. Using these results we estimate agency costs per country, which are consistent with the very limited (mainly anecdotical and survey based) existing estimations in the literature. Roughly speaking, data on asymmetry of lending rates is consistent with monitoring costs of around 5% over total assets for developed countries and 30% for underdeveloped ones. The model is also able to explain cross-country differences on lending spreads levels.

In Section 2.2 we report stylized facts about the negative relation between development of financial systems and asymmetry on lending rates and, particularly the positive relation between agency costs and asymmetry. In Section 2.3 we explain these findings by introducing financial frictions into a model with endogenous flow of information. In Section 2.4 we calibrate the model and obtain estimations of agency costs in different countries by matching the model with the data. In Section 2.5 we make some final remarks.

2.2 Stylized Facts

In this section we report an interesting but unexploited source of asymmetry on lending rates across countries, namely the development of financial systems in general and the magnitude of agency and monitoring costs in particular.

In the first part different exercises are made to show that the less financially developed is a country, the higher the likelihood of having changes of lending rates highly asymmetric (i.e. the more likely to have crashes when compared with booms of the same magnitude).

Since we specifically propose monitoring costs, contract enforcement and easiness in the flow of information as determinants of that relation in the financial system, the second part of this section goes deeper and uses different alternative methods to show how the skewness in lending rates is particularly tied to monitoring and bankruptcy costs.

Finally, the last part discusses the possible relation between skewness on lending rates and skewness on real variables of the economy. Results show that it is not possible to explain asymmetry in interest rates by asymmetry of real variables, which is a justification to use a model explaining the relation between monitoring costs and lending rates asymmetries independently of real variables.

Asymmetry on lending rates will be measured by the skewness of the log changes distribution. In symbols,

$$Skewness = \frac{\sqrt{n}\left[\sum_{t=1}^{n}(x_t-\overline{x})^3\right]}{\left[\sum_{t=1}^{n}(x_t-\overline{x})^2\right]^{\frac{3}{2}}} \tag{2.1}$$

where n is the number of observations (periods per country), $x_t = \ln(\rho_t) - \ln(\rho_{t-1})$, ρ_t is the lending rate at period t and $\overline{x}$ is the sample mean of the series.

Skewness is obtained using monthly data data on real lending rates from 1960 to 2004. Real lending rates are calculated based on information from the International Financial Statistics (IFS), by correcting nominal lending rates (from figure 60*P..ZF...*) by a consumer price index (from figure 64*P..ZF...*) [3]. Even when skewness was obtained for 70 countries that fulfill certain minimum requirements [4], the following analysis will be based only on those countries that exhibit positive values, which correspond to approximately 80% of them. The reason for doing this is that many studies based on individual countries, and hence more reliable information, typically have found posi-

[3]To obtain real lending rates we subtract the HP trend of inflation from the nominal figures. To use other measures of expected inflation do not change the main results.

[4]More than 100 observations, not many changes in the collection methodologies and a defined cyclical pattern.

tive skewness (see Veldkamp [2005] for a discussion). In our case we need to compare a lot of countries and we have to rely on a comparable common source of information like the IMF. A list of all countries used in the sample, their individual asymmetry levels and classifications are detailed in Appendix 2.6.1.

2.2.1 Negative relation between asymmetry on lending rates and financial development in general

2.2.1.1 Regressions

To analyze the relation between asymmetry on lending rates and financial development, the former is measured by the skewness of log changes in real lending rates (as described before), while the later is measured for each country by the credit to private sector as a percentage of GDP obtained from the World Development Indicators (WDI)

As shown in Table 2.1, just regressing these two variables for different period samples (1960-90 and 1990-2004) and different country samples (all countries and non-african countries) it is possible to find a mild but statistically significant negative relation.

However, errors from previous regressions show a structure. Figures 2.1 and 2.2 not only show the mentioned negative relation but also how many observations lie in the lower triangle part of the figure. This means countries with less developed financial systems are more prone to present high levels of skewness than countries with more developed financial sectors.

The existence of a well developed financial system does not seem to be a necessary condition to have low skewness levels, but definitely seems to restrict the possibility of having high skewness levels. Since skewness is a tail property that keeps track of

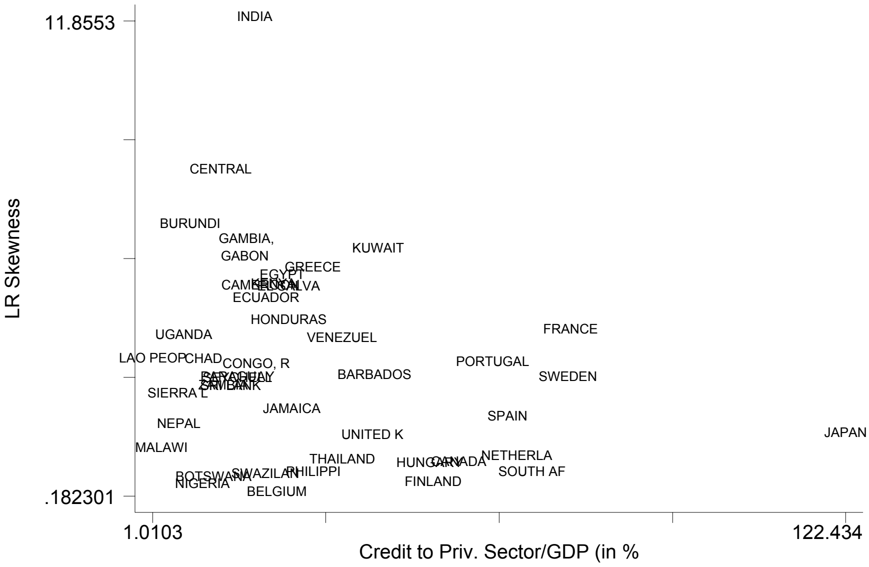
Table 2.1: Asymmetry on lending rates and financial development

Dependent Variable	All countries		Non-african countries	
Lending rates skewness	1960-1990	1990-2004	1960-1990	1990-2004
Credit to Private Sector / GDP	-0.025	-0.023	0.025	-0.008
	(0.010)**	(0.009)**	(0.014)*	(0.012)
Constant	4.06	2.77	4.15	1.62
	(0.57)***	(0.53)***	(0.90)***	(0.66)*
Observations	44	55	27	39

* Significant at 10%, ** Significant at 5% and *** Significant at 1%. Robust standard errors are reported in parentheses. The dependent variable is the skewness measured over the distribution of log changes in monthly lending rates, obtained from the IMF's IFS database. Yearly data on Credit to Private Sector as a percentage of GDP from the World Bank's WDI database. The simple average per country over the period sample is considered.

booms and crashes, a higher positive skewness means a higher probability of showing a huge crash when compared with the probability of having a boom of the same magnitude.

Figure 2.1: Skewness on lending rates and credit to private sector / GDP (1960-1990)

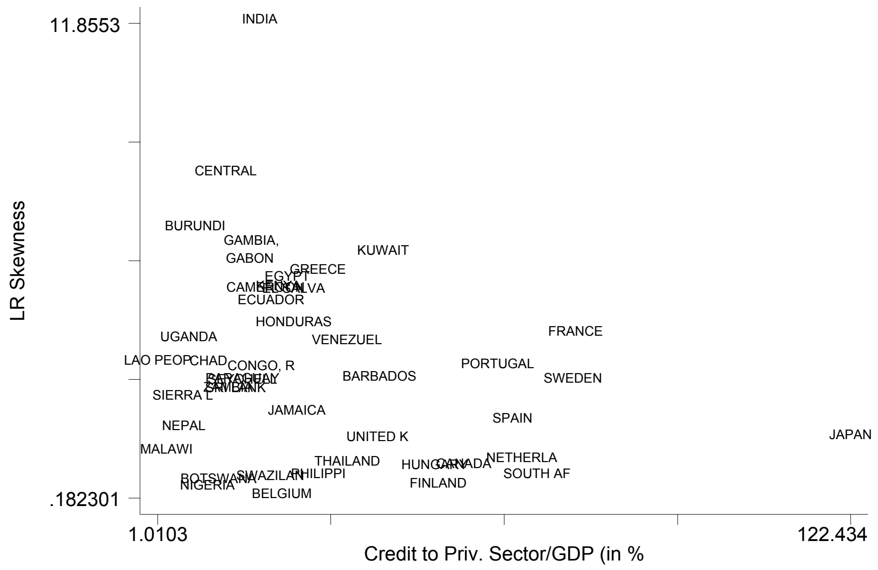


A possible reason less developed financial systems have both high and low asymmetry levels is the heterogeneity of volatility in those economies. In fact this relation between volatility and skewness will be captured in our model and shown in the simulations from Section 2.4.

To solve this problem skewness is controlled by the volatility of GDP per capita, the volatility of lending rates (both measured by the coefficient of variation), the average inflation in the period and the log of GDP per capita. As shown in Table 2.2, once controlled, a stronger negative relation between asymmetry of lending rates and financial development is obtained.

After controlling for other variables, the errors do not seem to have a structure. This can be seen in Figures 2.3 and 2.4, which reply Figures 2.3 and 2.4 but do not show the lower triangular pattern observed without controls.

Figure 2.2: Skewness on lending rates and credit to private sector / GDP (1990-2004)



2.2.1.2 Classifications

A simpler way to observe the negative relation between asymmetry on lending rates and financial development is just by dividing countries in a diversity of classifications correlated with financial development groups and checking if there is, in average, a difference between them in terms of asymmetry on lending rates. This classification will be also relevant to performs simulation exercises in Section 2.4

The following classifications are used:

a) Income groups as defined by the World Bank.

b) OECD and non-OECD countries.

c) Countries with high and low contract enforcement. A ”contract enforcement” indicator from Levine et al. [1997] is used. This measure is an average between two indicators from La Porta et al. [1998], *Rule of Law,* which is an assessment of the law and order tradition of the country and *Government Risk*, which is an assessment of the risk the government modify a contract after it has been signed. In both cases

Table 2.2: Fitted asymmetry on lending rates and financial development

Dependent Variable	All countries		Non-african countries	
Fitted LR Skewness	1960-1990	1990-2004	1960-1990	1990-2004
Credit to Private Sector / GDP	-0.019	-0.013	-0.018	-0.010
	(0.003)***	(0.003)***	(0.004)***	(0.002)***
Constant	3.89	2.43	3.85	1.69
	(0.17)***	(0.14)***	(0.28)***	(0.10)***
Observations	44	55	27	39

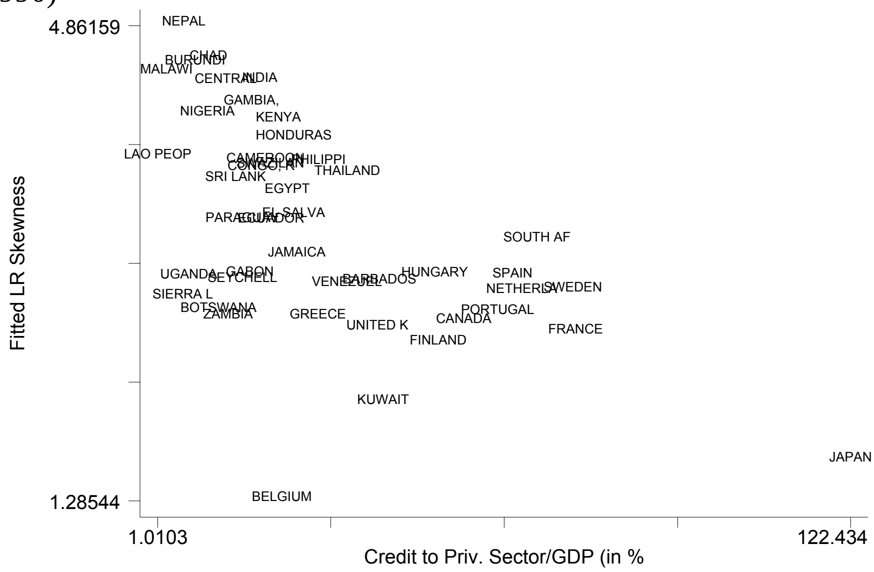
* Significant at 10%, ** Significant at 5% and *** Significant at 1%. Robust standard errors are reported in parentheses. The dependent variable is the fitted skewness controlled by log of GDPpc, volatility of GDPpc, volatility of lending rates and inflation. The simple average per country over the period sample is considered.

the indices go from 1 (the worst possible situation) to 10 (the best possible situation). The cutoff between low and high contract enforcement was set on 7 in order to have a similar number of countries in both classifications.

d) Countries with and without Private Bureau. A "Private Bureau" from Djankov et al. [2007] is defined as a private commercial firm or non profit organization that maintains a database on the standing of borrowers in the financial system and its primary role is to facilitate exchange of information amongst banks and financial institutions.

While the use of the first two classifications is justified by the well known positive relation between economic and financial development (Levine [1997]), the last two classifications reflect the situation in terms of contract enforcement and access and availability of information to lenders, more in line with the specific channels this paper focuses on to explain why financial development affects asymmetry levels.

Figure 2.3: Fitted skewness on lending rates and credit to private sector / GDP (1960-1990)



In Table 2.3 we show the simple average of skewness for each classification group and for two different periods of time. In Appendix 2.6.2, we repeat this exercise but using two different approaches. At the one hand we obtain skewness in log deviations from trend rather than in log changes. At the other hand we obtain skewness in log changes of lending rates spreads rather than levels. Results for both cases are consistent with conclusions from Table 2.3.

Richer countries, OECD countries and countries with high contract enforcement and private bureaus always show less asymmetry than poorer countries, non-OECD countries or countries with low contract enforcement and no bureaus that improve the flow of information to lenders.

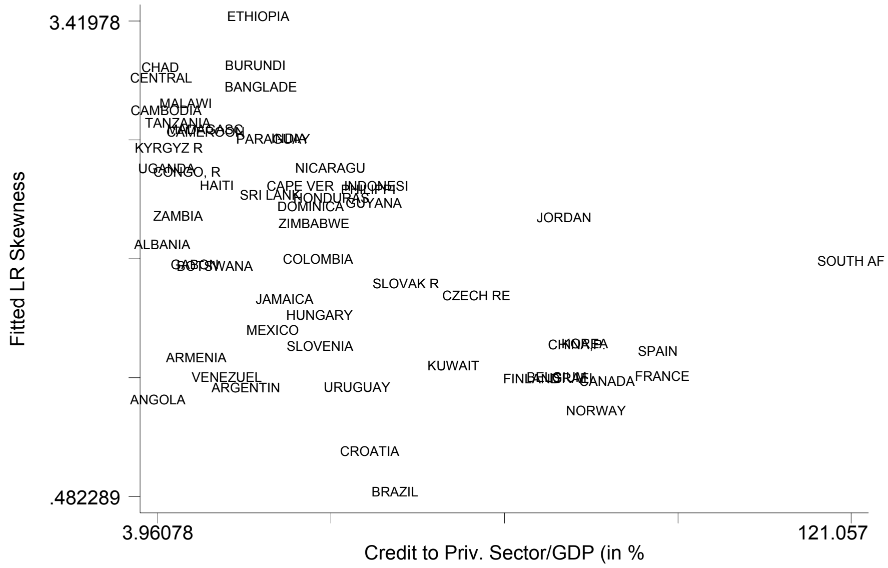
This evidence reinforces the findings from the regressions about the negative relation between asymmetry on lending rates and financial development.

Table 2.3: Asymmetry on lending rates by country classification

Countries classification	1960-1990	1990-2004	1960-2004
Income Group 1 (Richest)	2.52	0.79	1.66
Income Group 2	2.92	1.81	2.37
Income Group 3	2.65	2.44	2.55
Income Group 4 (Poorest)	4.45	2.52	3.49
OECD	2.08	1.13	1.61
non-OECD	3.77	2.13	2.95
High contract enforcement	1.84	0.23	1.00
Low contract enforcement	4.10	1.24	2.67
Private Bureau	2.40	1.49	1.95
non-Private Bureau	4.03	2.07	3.05

Income classification from the World Bank (WDI). Contract Enforcement Indicator from Levine et al.(2000). Existence of a private bureau from Djankov et al. (2004). Skewness by group is the simple average of the skewness of ”member” countries for the referred period.

Figure 2.4: Fitted skewness on lending rates and credit to private sector / GDP (1990-2004)



2.2.2 Negative relation between asymmetry on lending rates and agency costs in particular

The previous subsection shows in general the negative relation between asymmetries on lending rates and general financial and economic indicators. This paper proposes that the specific determinants are the differences in monitoring and bankruptcy costs and the degree of information asymmetry. If this is the case we should find a negative relation between asymmetry on lending rates and the level of agency costs.

The problem to perform this analysis is the unavailability of monitoring costs estimations for many countries. In fact even estimations of bankruptcy costs for the US are subjects of a great controversy (Carlstrom and Fuerst [1997]).

Given the huge scarcity of information about these costs in the literature and, even more, the big dissent about the existing estimations for the US, we use alternative indicators to understand the specific impact of monitoring costs over asymmetries on lending rates.

Three exercises are developed to cope with the unavailability of direct information on monitoring costs for many countries.

2.2.2.1 Evolution of technology and monitoring costs

Monitoring and bankruptcy costs are closely related to technology improvements since they are based on the efficiency to audit accounts and on the easiness to share and transmit information. Naturally, the better the available technology (such as computers and telecommunications), the less the monitoring costs existing on the financial sectors[5].

Table 2.3 shows that for each classification group, the asymmetry in lending rates decreases along time. At the same time, information technologies improve importantly and continuously from 1960. This positive relation between asymmetry and monitoring costs, both decreasing in the last decades, is consistent with the theoretical explanation we propose in this paper.

Monitoring costs (inextricably related to technology) increase the asymmetry on lending rates. We propose this is why we observe a decrease in skewness for all countries along time.

2.2.2.2 Proxies for monitoring costs

Another alternative method to understand the relation between asymmetry and monitoring costs, given the lack of direct information about the later, is by the use of proxy variables available for many countries. We will use two sets of proxies. The first one is based on Djankov et al. [2005], who specifically analyzes the time and cost of closing businesses. The second set of variables refers to the performance of financial and

[5] Merton [1987] constitutes an early and powerful work on the impact of the informational technologies evolution over finance and monitoring.

banking systems in general to ease the access and availability of information.

1) Bankruptcy costs and duration. (Djankov et al. [2005])

a) Cost of Bankruptcy: Costs of bankruptcy proceedings (as % of the estate value) that include court costs, as well as fees of insolvency practitioners, independent assessors, lawyers, accountants, etc. It is calculated based on answers by practicing insolvency lawyers to a multiple choice survey.

b) Time for Bankruptcy: Years to complete a procedure as estimated by insolvency lawyers.

c) Recovery Rate: Measures the efficiency of foreclosure. It shows how many cents on the dollar claimants (creditors, tax authorities, and employees) recover from an insolvent firm. The calculation takes into account whether the business is kept as a going concern during the proceedings, the discounted value due to the time spent closing down and court, attorney, etc.

Even when it seems these variables are exactly the measures of monitoring and agency costs we require, they have some drawbacks we should mention. First, the estimation of bankruptcy costs is based on a multiple-choice question, where the respondents choose among options biased towards zero[6]. Second, the variable presents a very low variance, with 30% of the countries reporting 8% of the estate value corresponds to bankruptcy costs and 30% reporting 18%. In this sense, recovery rate seems a better variable to capture our ideal measure of monitoring costs, given it is constructed considering more bankruptcy elements.

Table 2.4 shows simple OLS regressions between skewness on lending rates and these proxies. The general conclusion is that the more the monitoring costs, the more the asymmetry. This can be observed in the statistically significant positive coefficients

[6]The options in the survey are 0-2 percent, 3-5 percent, 6-10 percent, 11-15 percent, 16-20 percent, 21-25 percent, 26-50 percent, and more than 50 percent of the estate value of the bankrupt business.

Table 2.4: Asymmetry on lending rates and proxies for monitoring and bankruptcy costs

Dependent Variable Lending rates skewness	1990-2004		
Cost of Bankruptcy	0.050 (0.013)***		
Time for Bankruptcy		0.231 (0.101)**	
Recovery Rate			-0.020 (0.009)**
Constant	0.649 (0.229)***	0.734 (0.374)**	2.177 (0.425)***
Observations	48	48	48

* Significant at 10%, ** Significant at 5% and *** Significant at 1%. Robust standard errors are reported in parentheses. All independent variables were obtained from Djankov et al. [2005].

for cost and time of bankruptcy and the statistically significant negative coefficient for the recovery rate of claimants. The regressions are made only for the period 1990-2004 because proxies are measured for 2004, not being relevant to explain processes occurred 40 years before.

2) Contract enforcement and financial sector health.

a) Contract Enforcement Days: The number of days to resolve a payment dispute through courts. Variable constructed as at January 2003 by Djankov et al. [2005].

b) Legal protection to financial assets

c) Sophistication of financial markets

d) Health of banking systems

Variables based on surveys conducted by the Global Competitiveness Report, 1999 (published by the World Economic Forum and directed by Sachs, Porter and McArthur). The variables are measured by an index that goes from 1 to 7 (from the worst possible situation to the best possible situation).

Table 2.5 shows simple OLS regressions between skewness on lending rates and these proxies. The general conclusion is again that the higher the monitoring costs and contract enforcement delays and the smaller the capabilities of financial and banking sectors to ease the flow of information, the higher is the asymmetry on lending rates. This is delivered by the statistically significant positive coefficient for contract enforcement and the statistically significant negative coefficients for the other variables. The regressions are computed for the period 1990-2004 following the same logic explained for the first set of proxies.

2.2.2.3 Financial Liberalization

An additional way to see the relation between monitoring costs and asymmetry in lending rates is to follow the behavior of skewness before and after a shock in the financial system in which an abrupt change in the quality of monitoring costs occurred. Such a shock can be, for example, a financial liberalization process

Financial liberalization processes make financial systems more prone to be influenced by modern foreign auditing and bankruptcy methods. Financial liberalization processes also open financial systems to competition that propitiates the environment to adopt more efficient monitoring practices, a better enforcement of contracts and an easier flow of information.

Table 2.5: Asymmetry on lending rates and proxies for monitoring and enforcement costs

Dependent Variable Lending rates skewness	1990-2004			
Contract Enforcement Days	0.0034 (0.0016)**			
Legal protection to financial assets		-0.358 (0.185)*		
Sophistication of financial markets			-0.321 (0.156)**	
Health of banking systems				-0.469 (0.169)***
Constant	0.38 (0.62)	2.95 (1.03)***	2.53 (0.75)***	3.49 (0.94)***
Observations	45	30	30	30

* Significant at 10%, ** Significant at 5% and *** Significant at 1%. Robust standard errors are reported in parentheses. Contract Enforcement Days is the number of days to resolve a payment dispute through courts. Variable constructed as at January 2003 by Djankov et al. [2005]. Legal protection to financial assets, sophistication of financial markets and health of banking systems are based on surveys conducted by the Global Competitiveness Report, 1999 (Sachs, Porter and McArthur).

Hence financial liberalization process are events in which, suddenly, monitoring costs decrease and in general the quality of information gets better. In a similar vein, an anti liberalization process that restricts competition would lead to a worsening in the monitoring and auditing costs.

Table 2.6 shows a comparison of skewness in lending rates before and after the main financial liberalization event for 16 countries in which enough data exists to reliably obtain skewness at both sides of the liberalization event (more than 100 observations at each side).

Data on financial liberalization is obtained from Kaminsky and Schmukler [2001] for the period 1973-1998. This database includes information on liberalization of capital accounts, domestic financial sectors and stock market capitalization. For capital accounts authors consider whether corporations are allowed to borrow abroad and whether multiple exchange rate mechanisms or other sorts of capital controls are in place. Regarding domestic financial liberalization authors explored interest rate controls (lending and deposits) and other restrictions such as directed credit policies or limitations on foreign currency deposits. Their analysis of stock market liberalization encompasses the degree to which foreigners are allowed to own domestic equity and restrictions on repatriation of capital, dividends and interests.

As can be seen, 10 out of 13 countries on the table show a reduction on the lending rates asymmetry right after the main liberalization event. In the table, three countries (Chile, Indonesia and Thailand) were not reported because they had experienced both financial liberalization and financial restriction processes over the relevant period, not being relevant just to pick one event.

Another interesting exercise to check for the robustness in the results is to consider a comparison of asymmetry before and after the whole liberalization process and not just one event. To cover this possibility and to cope with the experiences of Chile,

Table 2.6: Asymmetry on lending rates before and after a main financial liberalization event

Country	Main financial liberalization event		Type of liberalization	Skewness	
	Month	Year		Pre-Event	Post-Event
Canada	March	1975	KA	0.87	0.46
Finland	January	1990	SM and DFS	0.42	0.15
France	January	1985	DFS and KA	3.91	0.04
Italy	January	1992	KA	-3.76	0.82
Japan	January	1985	SM	1.95	-0.28
Korea	January	1991	SM	-5.18	3.86
Philippines	January	1994	KA and SM	0.37	0.18
Portugal	January	1986	SM	4.00	-0.33
Spain	December	1992	KA	2.07	0.45
Sweden	January	1984	KA	3.45	0.11
UK	October	1973	KA	3.85	1.46
US	July	1973	KA	-0.17	-0.08
Venezuela	June	1995	SM	3.70	0.34

KA=Capital Account, SM=Stock Market, DFS=Domestic Financial System. Data on liberalization dates from Kaminsky and Schmukler [2001].

Indonesia and Thailand, Table 2.7 presents a summary of asymmetry before and after the whole financial liberalization process for each country, which naturally includes the main event specified in Table 2.6.

As can be seen, 13 out of 16 countries considered experiment a reduction on the lending rates asymmetry after the whole financial liberalization process. From the countries that do not follow the pattern, Italy and Korea show very strange skewness levels (negative and of big magnitude) which is due to a huge lending rate decrease experimented only once while US basically does not present any skewness (not statistically different from zero) in either case.

It is also interesting to note that the behavior of the asymmetry reverts when considering financial restrictions and not financial liberalization processes. In this sense, only Chile, Indonesia and Thailand had in the period considered financial restriction processes. When comparing before and after those processes, skewness on lending rates in fact increases, as shown by Table 2.8.

These results reinforce the idea that reductions in monitoring costs, in this case generated by sudden changes from liberalization practices, generate reductions in the level of skewness.

As a conclusion of this subsection, whether considering the historical evolution of technology for all countries, bankruptcy costs and duration, enforcement of contracts, health or sophistication of financial markets and the banking system or financial liberalization processes as proxies of monitoring costs and financial frictions in countries, it seems pretty robust the conclusion that the more the monitoring costs, the more the asymmetry on lending rates.

Exercises comparing groups of countries along time, cross sections across countries and the behavior of lending rates per country lead to the same conclusion. It definitely seems to exist a positive relation between asymmetry of changes in lending

Table 2.7: Asymmetry on lending rates before and after a financial liberalization process

Country	START of financial liberalization process		END of financial liberalization process		Skewness	
	Month	Year	Month	Year	Pre-Process	Post-Process
Canada	March	1975	March	1975	0.87	0.46
Chile	January	1984	September	1998	1.13	0.55
Finland	January	1986	January	1990	1.80	0.15
France	January	1985	January	1990	3.93	0.08
Indonesia	January	1983	August	1989	1.35	-0.09
Italy	May	1987	January	1992	-3.64	0.82
Japan	January	1979	January	1985	1.65	-0.28
Korea	January	1988	January	1996	-6.87	3.81
Philippines	January	1976	January	1994	7.85	0.18
Portugal	January	1976	August	1992	4.55	-0.07
Spain	January	1981	December	1992	2.17	0.45
Sweden	January	1978	January	1989	3.70	0.11
Thailand	January	1979	June	1992	1.56	0.14
UK	October	1973	January	1981	3.85	1.93
US	July	1973	January	1982	-0.17	-0.87
Venezuela	March	1989	April	1996	3.47	0.37

KA=Capital Account, SM=Stock Market, DFS=Domestic Financial System. Data on liberalization dates from Kaminsky and Schmukler [2001].

Table 2.8: Asymmetry on lending rates before and after a financial restriction process

Country	START of financial restriction process		END of financial restriction process		Skewness	
	Month	Year	Month	Year	Pre-Process	Post-Process
Chile	June	1979	January	1983	0.13	1.13
Indonesia	March	1991	March	1991	-0.09	5.10
Thailand	August	1995	May	1997	0.14	0.81

KA=Capital Account, SM=Stock Market, DFS=Domestic Financial System. Data on liberalization dates from Kaminsky and Schmukler [2001].

rates and monitoring costs, enforcement possibilities and the degree of information flow and availability in the system.

2.2.3 Is the asymmetry on lending rates just a reflection of the asymmetry on real variables?

An obvious question at this point is whether the results found so far is just a reflex of what happens on the real side of the economy. If this is the case, the question should change from trying to explain why lending rates are more asymmetric in less developed countries to trying to explain why booms and crashes in the real side of the economy relate with the development of financial systems.

Not only this is a completely different question but also it means that the real side of the story cannot be considered separately. Table 2.9 however shows that skewness on lending rates is not correlated with skewness on real variables such as real household consumption or real GDP[7]. This means that a country with a high asymmetry on real

[7] Real GDP was obtained by deflating nominal GDP figures by CPI and by taking directly GDP in volumes figures from the IMF database. Data were taken yearly from the IMF's IFS.

Table 2.9: Correlation coefficients between skewness on lending rates and skewness on real variables

Real variables	Correlation		
	1960-2004	1975-2004	1990-2004
Real GDP (deflated by CPI)	0.12	0.13	0.17
Real GDP (on Volume)	0.06	0.16	0.09
Real HH Consumption	0.12	0.09	0.12

Real variables are obtained yearly from the IMF's IFS. Skewness of log changes in lending rates have been obtained annually considering the information from December of each year.

GDP, for example, not necessarily presents also a high asymmetry on interest rates.

This section about stylized facts shows how the more developed a financial system in a country (and particularly the smaller the levels of monitoring and bankruptcy costs, the higher the level of contract enforcement and the better the flow of information), the less the asymmetry of lending rates. This is important because a small asymmetry on lending rates means crashes and booms of the same magnitude are similarly likely. On the other side a big asymmetry means that booms are not as likely as crashes of the same magnitude and then recoveries and reallocation of resources are more costly.

The next section proposes an endogenous information model to explain this relation.

2.3 The Model

2.3.1 Description

This model captures the previously described negative relation between asymmetry and financial development. It modifies Veldkamp's model by introducing financial frictions, agency costs and costly state verification. In this way we can analyze the impact of financial development over skewness differences across countries.

Assume a credit market with a finite number N of entrepreneurs, who are potential risk neutral borrowers and a number M of perfectly competitive and risk neutral lenders. It will be assumed that $N < M$, giving to borrowers all the negotiation power.

In each period t, each entrepreneur observes a business opportunity. All opportunities have a common probability of success across entrepreneurs but different expected profits $v_{it} \in (\underline{v};\overline{v})$ in case of success [8]. We assume entrepreneurs do not have access to initial assets, hence they need to ask for a loan of 1 unit (i.e., the normalized cost of the venture) in order to run the project.

If the entrepreneur decides to borrow, he will do that at an endogenous lending interest rate $(1+\rho)$, which depends on the expected rate of default and on the country's financial development and monitoring costs, as will be shown shortly. If the entrepreneur decides not to borrow, he can always work for a exogenously fixed wage w. If the borrower is not lucky in the venture, he will receive a zero profit.

The lender also has two possibilities. After deciding the lending rate, either to lend in case some entrepreneur is willing to borrow at that rate or just to invest the indivisible unit of capital in a risk free bond that pays an exogenous and constant rate of return $(1+r)$

The probability of a venture success depends on an unobserved state variable that

[8] Given this support, trivial agents who always invest or who never invest are not included.

can take two possibilities, a good state G or a bad state B. If there are good times the probability of a loan being repaid is θ_g while in bad times that probability is given by θ_b, such that $\theta_g > \theta_b$. Agents are not able to identify the state of the economy when trading for a loan.

Until this point, the model is very similar to the one developed by Veldkamp. The problem with the original set up such as described above is its impossibility to explain differences in skewness across countries without changing fundamentals. For this reason the model is extended to assume information asymmetry and costly state verification.

Lenders cannot see ex-post if in fact the borrower was successful or not. As in Townsend [1979] and Gale and Hellwig [1985], while cash flows are costlessly observable to entrepreneurs or borrowers, they are observable to external creditors only at some positive cost c. In return for receiving the loan in the first period, borrowers have to make a report. Depending on the report lenders may decide to monitor with some probability γ_t.

Hence lenders have to rely on standard debt contracts as the ones described by Gale and Hellwig [1985] to solve the information asymmetry. In return for receiving the loan in the first period, borrowers have to make a report. If entrepreneurs report a success they are required to repay in the next period a state-invariant amount $(1+\rho)$. If borrowers report a failure, creditors pay the monitoring costs, observe the truth and keep total profits v_i if the entrepreneur lied in the report and 0 otherwise.

The timing of the model can then be summarized as follows:

1) Agents enter each period with beliefs about the probability of being in a good state (μ_t)

2) A debt contract is determined by lenders considering the costly state verification. After this decision, entrepreneurs decide whether or not to take a loan and invest in a venture.

3) All lenders and entrepreneurs not participating in a loan contract, invest on their outside options. Lenders not making a loan invest in the risk free venture obtaining $(1+r)$ while entrepreneurs not taking a loan work in a job that pays w.

4) Borrowers report the result of their ventures, the contract is fulfilled and all payoffs are paid.

5) All reports and monitoring results are publicly observed.

6) State changes with a probability λ

7) Beliefs about the probability of being in a good state in the next period (μ_{t+1}) are updated.

2.3.2 Equilibrium

2.3.2.1 Definition

A subgame perfect equilibrium (SPNE), for an initial belief μ_0, is given by time sequences of borrowing decisions by each entrepreneur i $\{b_{it}\}$, reporting decisions by each borrower i if the venture is successful $\{z_{it}\}$, lending rates set by each lender j $\{\rho_{jt}\}$, monitoring probabilities when receiving unsuccessful reports by each lender j $\{\gamma_{jt}\}$ and beliefs (updated by Bayes formula) about the probability of being in a good state $\{\mu_t\}$, such that the following problems are solved in each period t:

a) Entrepreneurs: Given a set of available debt contracts, each entrepreneur i chooses to take or not a loan and from which lender to take it (b_{it}) and the probability of reporting the truth in case of having a successful venture (z_{it}), such that the following expected utility is maximized.

$$\max_{b_{it}\in\{0,1\};z_{it}\in[0,1];j\in\{1,\dots,M\}} b_{it}\theta_t\{z_{it}(v_i-(1+\rho_{jt}))+(1-z_{it})(1-\gamma_{jt})v_i\}+(1-b_{it})w$$

being $\theta_t = \mu_t\theta_g + (1-\mu_t)\theta_b$ the expected probability of a successful venture, which depends on the expected state of the economy.

b) Lenders: Given strategies of other agents, each lender j chooses an interest rate $(1+\rho_{jt})$ and a monitoring probability when the borrower reports the venture was unsuccessful (γ_{jt}), such that the following expected utility is maximized.

$$\max_{\rho_{jt},\gamma_{jt}} l_{jt}\theta_t\{z_{it}(1+\rho_{jt})+(1-z_{it})\gamma_{jt}(v_i-c)\}-l_{jt}\gamma_{jt}(1-\theta_t)c+(1-l_{jt})(1+r)$$

being $l_{jt}=1$ if some borrower decides to take a loan from that lender j in period t.

We define by (n_t) the total number of ventures funded in each period t, which is the same as the total number of borrowers who decide to take a loan in period t, $(n_t = \sum_{i=1}^{N} b_{it})$. This number will represent the number of signals used by the market to update beliefs about the state of the economy.

c) Beliefs: From the n_t funded ventures in period t, agents observe a number of successes and failures[9] and form posterior beliefs μ_t^P, using Bayes' rule. [10]

$$\mu_t^P = \Pr(G|s) = \frac{\theta_g^s(1-\theta_g)^{n-s}\mu_t}{\theta_g^s(1-\theta_g)^{n-s}\mu_t+\theta_b^s(1-\theta_b)^{n-s}(1-\mu_t)} \tag{2.2}$$

Adjusting these posteriors by the probability of a change in state, the probability of being in a good state in the next period is obtained by the following equation:

$$\mu_{t+1} = \Pr(G)_{t+1} = (1-\lambda)\mu_t^P + \lambda(1-\mu_t^P) \tag{2.3}$$

[9]If the real number of successes is called s^R, the number of successes observed and used in the updating will be $s = s^R z + s^R(1-z)\gamma$. In case $z=1$ then $s=s^R$ and the update proposed below is the right one. We will show later $z=1$ is in fact the case we should consider.

[10]Recall $C_s^n = C_{n-s}^n = n!/((n-s)!s!)$ and then drop from the equation.

And finally, the probability of success of a given venture in the next period is given by[11]:

$$\theta_{t+1} = \Pr(s)_{t+1} = \mu_{t+1}\theta_g + (1-\mu_{t+1})\theta_b \tag{2.4}$$

2.3.2.2 Characterization

Proposition 19 *At each period t, all lenders j set a lending rate $1+\rho_t = \frac{1+r}{\theta_t} + \frac{(1-\theta_t)}{\theta_t}c$ and always monitor a failure report (i.e., $\gamma_{jt} = 1$). All entrepreneurs i borrow (i.e., $b_{it} = 1$) indifferently from any lender $j \in \{1,2,..,M\}$ whenever $v_i \geq \widetilde{v} = \frac{1}{\theta_t}[1+r+w+(1-\theta_t)c]$. All borrowers always report the truth (i.e., $z_{it} = 1$).*

Proof To obtain the SPNE, we work by backward induction in two steps. First we obtain the optimum for lenders by characterizing the optimal debt contract to use. Second we obtain the optimum for the entrepreneurs by taking as giving the contract offered by lenders.

Step 1: Optimal decisions by lenders

Given lenders act in a competitive market, they make zero profits in equilibrium. Hence the debt contract is set such that expected profits from lending are equal to the potential profits from investing in the free risk venture $(1+r)$.

Lenders have to solve for lending rates considering the costly state verification that arises because they do not have information about the successfulness of the venture they funded. Townsend, Gale and Hellwig showed that the optimal contract is given by the standard debt contract.

When $c > 0$, the standard debt contract is characterized by a repayment on the second period of a state invariant amount $(1+\rho)$ in return for receiving one unit of

[11]Sometimes, to save notation and when no confussion may arise, I will set aside the subscript $t+1$

capital in the first period. If the entrepreneur fails to pay that amount reporting an unsuccessful activity, lenders monitor the venture (paying the monitoring costs $c > 0$) and observe and keep for themselves the true company profits. Obviously those profits are either zero, in the case the entrepreneur tells the truth and the venture was in fact unsuccessful, or $v_i > (1+\rho)$, if the borrower lied and the venture was in fact successful.

States in which monitoring occurs can be interpreted as bankruptcy and hence monitoring costs can be interpreted as bankruptcy costs for the economy as a whole.

In general, when potential profits π are a continuum variable, lenders determine $(1+\rho)$ such that expected profits are equal to the outside option $(1+r)$ after considering the payment of monitoring costs when borrowers report a failure, $\int\limits_{-\infty}^{1+\rho}(\pi - c)g(\pi)d\pi + \int\limits_{1+\rho}^{\infty}(1+\rho)g(\pi)d\pi = 1+r$

In our particular model this condition can be simply written as:

$$(1-\theta_t)(-c) + \theta_t(1+\rho_{jt}) = 1+r$$

Since the expected probability of success is the same for all ventures, lending rates apply to all loans are the same (i.e., $\rho_{jt} = \rho_t$).

$$(1+\rho_t) = \frac{1+r}{\theta_t} + \frac{(1-\theta_t)}{\theta_t}c \tag{2.5}$$

Having determined the lending rates charged by all lenders, Gale and Hellwig [1985] show it is optimal for lenders to always monitor when a failure is reported (i.e., $\gamma_{jt} = 1$), such that in expectation they get the outside option.

Summarizing this part of the proof, at each period t, all lenders j will set a standard debt contract such that if the borrower reports a success, he pays a state independent rate given by equation (2.5) and if the borrower reports a failure, the lender monitors for sure, (i.e., $\gamma_{jt} = 1$).

Step 2: Optimal decisions by entrepreneurs

Given this behavior by lenders, it is optimal to successful borrowers to always report the truth (since they gain $(v_i - (1+\rho)) > 0$ rather than 0 for sure in case of lying).

Since lending rates charged by all lenders is the same it is irrelevant for borrowers which lender j to take the loan from.

The only choice left to determine in equilibrium is whether entrepreneurs should borrow or not (i.e., $b_{it} \in \{0,1\}$). Since potential profits are different across entrepreneurs, this choice is given by a cutoff value over v_{it} such that an entrepreneur i borrows ($b_{it} = 1$) at period t whenever $\theta_t(v_{it} - (1+\rho_t)) \geq w$.

Given $(1+\rho_t)$ from equation (2.5), the rule for borrowing is then,

$$v_{it} \geq \widetilde{v}_t = \frac{1}{\theta_t}[1 + r + w + (1-\theta_t)c] \tag{2.6}$$

Q.E.D.

As can be seen, when the state verification is costless to the lenders ($c = 0$), this solution coincides with Veldkamp's original model solution.

One of the most important results to trace from here is the number of ventures funded in the economy because this is the number of signals used by agents to update beliefs and to modify interest rates. The number of funded ventures is given by the sum of the entrepreneurs who decide to take the loans. Hence, in equilibrium.

$$n_t = \sum_{i \in \{1,\dots,N\}} 1_{\{v_{it} \geq \widetilde{v}_t = \frac{1}{\theta_t}[1+r+w+(1-\theta_t)c]\}} \tag{2.7}$$

The number of ventures depends positively on the probability of a venture success θ_t in three ways. A higher θ_t increases the expected payoff of borrowing, decreases the market interest rate ρ and reduces the necessity of monitoring the venture because it reduces the probability of a false unsuccessful report.

Formally, the derivative of $\widetilde{v}_t$ with respect to θ_t is negative ($\frac{\partial \widetilde{v}_t}{\partial \theta_t} = -\frac{1+r+w+c}{\theta_t^2} < 0$). Of course a smaller $\widetilde{v}_t$ implies a higher number of signals n_t as long as the cumulative distribution of v_{it} is monotonically increasing or, which is the same, whenever the density function has mass in all points $v_i \in (\underline{v};\overline{v})$.

Because θ_t depends also positively on the probability of being in a good state μ_t ($\frac{\partial \theta_t}{\partial \mu_t} = \theta_g - \theta_b > 0$ since $\theta_g > \theta_b$ by assumption), the number of funded ventures will depend also positively on the probability of being in a good state μ_t. Formally this can be seen in the derivative of the cutoff $\widetilde{v}_t$ with respect to μ_t ($\frac{\partial \widetilde{v}_t}{\partial \mu_t} = -(\theta_g - \theta_b)\frac{[1+r+w+c]}{(\mu_t \theta_g + (1-\mu_t)\theta_b)^2} < 0$) since $\theta_g - \theta_b > 0$ by assumption.

This is important for the determination of signals in the economy. The greater the value for μ_t, the greater is θ_t, the smaller the cutoff value $\widetilde{v}_t$ and the more the number of funded ventures.

At this point it is important to see which are the main differences of introducing agency problems in this model. Two important properties are added by agency costs.

First, the greater the monitoring costs c the greater the lender interest rates because

$$\frac{\partial(1+\rho_t)}{\partial c} = \frac{1-\theta_t}{\theta_t} > 0 \tag{2.8}$$

Second, the greater c, the greater the cutoff value $\widetilde{v}_t$ entrepreneurs profits v_{it} have to exceed in order to borrow. In this sense, the greater the monitoring costs, the smaller the number of funded ventures in the economy and hence the number of signals where to learn from. Formally,

$$\frac{\partial \widetilde{v}_t}{\partial c} = \frac{1-\theta_t}{\theta_t} > 0 \tag{2.9}$$

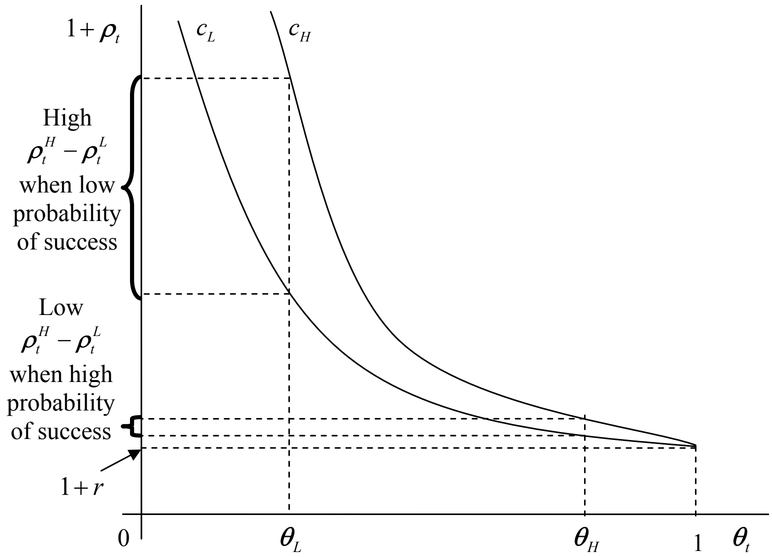
Two important conclusions arise. First, monitoring costs generate underinvestment in all states. Second, the reduction in signals is not constant across states because, the worst the belief, the more the restriction imposed by agency costs on investment. This is because, when θ_t varies, c is scaled by a double effect in the numerator $(1-\theta_t)$ and in the denominator (θ_t).

Figure 2.5 shows the relation between lending rates $(1+\rho_t)$ and the expected probability of success (θ_t) for different levels of monitoring costs c. When the market believe the probability of success is very high it is not very likely to spend on bankruptcy later, hence monitoring costs do not impose serious restrictions on the levels of lending rates and then on the determination of signals. Contrarily, when the market believe the probability of success is very low it is very likely to spend on bankruptcy later, hence monitoring costs matter a lot for the determination of lending rates and signals.

This non-linear relation is important to understand the different relative impact of monitoring costs on the formation of signals, rates and new ventures in different phases of the cycle.

Since the number of signals is changing continuously in this model, to write an explicit result as an analytical solution is intractable. This is why the results and conclusions from the model will be discussed with the help of Monte Carlo simulations in Section 2.4.

Figure 2.5: Lending Rates and Expected Probability of Success



2.3.3 Asymmetry implications

This model generates time-irreversible lending rate changes on lending rates, which basically means changes on lending rates have an asymmetric unconditional distribution where the probability of a large interest rate increase is higher than the probability of a decrease of the same magnitude.

This feature of the framework contrasts with a constant information economy, where the number of signals is given exogenously and changes on interest rates are time-reversible and symmetric.

Our intention is not to provide a full proof of how the endogenous formation of signals leads to time irreversibility and to asymmetric distribution of changes on lending rates (Veldkamp [2005] provides a ”four propositions” formal proof of this) but to show how agency costs shape this property of endogenous information.

The following proposition analyzes how agency costs have an impact on asymmetry and how differences in financial development may imply differences in skewness on the distribution of lending rates changes. However, we also sketches the main points about why endogenous information generates asymmetry in the first place.

Proposition 20 *In an endogenous information economy, assuming $\theta_g > \theta_b$, agency costs increase asymmetry on lending rates.*

Proof This proof proceeds in three steps. First, the concept of time reversibility is introduced showing why a constant information economy does not present asymmetry. The second step shows why an endogenous information economy is time irreversible and then, asymmetric. Finally, the third step shows agency costs increase asymmetry in such a context.[12]

Step 1: Time reversibility in a constant information economy

Time reversibility is defined as the property of a stochastic process in which beliefs in a good state are the time-reverse of beliefs in a bad state. In symbols, $\Pr[\mu_{G,t+1} = x|\mu_{G,t} = y] = \Pr[\mu_{B,t+1} = x|\mu_{B,t} = y]$. In plain words, the increase in beliefs of being in good times if, for example, all signals are successful should have the same magnitude than the decrease of beliefs if all signals were unsuccessful.

Going to this extreme case, which represents the situation where the maximum possible booms and crashes are obtained, consider the prior for the probability of being in a good state is $\mu_t = x$. If suddenly, all n_t signals fail ($s_t = 0$), $\mu_{t+1} = y < x$. If in the following period all n_{t+1} signals are successful ($s_{t+1} = n_{t+1}$) and the process is time reversible, we should obtain that $\mu_{t+2} = z = x$.

[12]This proof is based on the case in which there is no state change ($\lambda = 0$) just to sketches the main points about why the endogenous information model delivers asymmetry on interest rates. This is not a critical assumption to show the impact of agency costs. A more general proof (with $\lambda > 0$) can be found in Veldkamp [2005].

If the economy has constant information, then the number of signals are the same (say n) no matter the prior belief μ. Considering, without loss of generality, the case of equally informative signals $\theta = \theta_g = 1 - \theta_b > \frac{1}{2}$ and assuming no state change ($\lambda = 0$), it is easy to show time reversibility.

Assume in period t initial beliefs are $\mu_t = x$ and all n signals fail ($s = 0$). Using equations (2.2) and (2.3).

$$\mu_{t+1} = y = \frac{(1-\theta)^n x}{(1-\theta)^n x + \theta^n (1-x)} \tag{2.10}$$

If in the following period $t+1$ all n signals are successful ($s = n$), then

$$\mu_{t+2} = z = \frac{\theta^n y}{\theta^n y + (1-\theta)^n (1-y)} \tag{2.11}$$

and replacing (2.10) into (2.11), $\mu_{t+2} = z = x$. Hence, in a constant information environment, beliefs respond to a time reversible stochastic process.

Step 2: Time irreversibility in an endogenous information economy

In an endogenous information economy, the number of signals is not independent on the beliefs of being in a good state. In fact, the greater the probability assigned to be in good times μ_t, the less the cutoff $\widetilde{v}_t$ given in equation (2.6) and the more the ventures funded (the signals n_t). Considering the same arguments and assumptions used in step 1, it's possible to show the stochastic process is not time reversible anymore.

Assume as before that in period t, $\mu_t = x$ and all n_t^x signals fail ($s_t = 0$) (the subscript t is now necessary because n varies on time and depends on beliefs. The superscript x denotes n_t depends on beliefs $\mu_t = x$).

$$\mu_{t+1} = y = \frac{(1-\theta)^{n_t^x} x}{(1-\theta)^{n_t^x} x + \theta^{n_t^x} (1-x)} \tag{2.12}$$

Now, given $y < x$, borrowers are less confident about being in good times, the number of ventures decline and hence the number of signals in the economy becomes $n^y_{t+1} < n^x_t$

If in the following period all n^y_{t+1} signals are successful ($s_{t+1} = n^y_{t+1}$), then

$$\mu_{t+2} = z = \frac{\theta^{n^y_{t+1}} y}{\theta^{n^y_{t+1}} y + (1-\theta)^{n^y_{t+1}} (1-y)} \tag{2.13}$$

now replacing (2.12) into (2.13),

$$\mu_{t+2} = z = \frac{\left[\theta^{n^y_{t+1}} (1-\theta)^{n^x_t}\right] x}{\left[\theta^{n^y_{t+1}} (1-\theta)^{n^x_t}\right] x + \left[(1-\theta)^{n^y_{t+1}} \theta^{n^x_t}\right] (1-x)} \tag{2.14}$$

and

$$z - x = \frac{\left[\theta^{n^y_{t+1}} (1-\theta)^{n^x_t} - (1-\theta)^{n^y_{t+1}} \theta^{n^x_t}\right] x(1-x)}{\left[\theta^{n^y_{t+1}} (1-\theta)^{n^x_t}\right] x + \left[(1-\theta)^{n^y_{t+1}} \theta^{n^x_t}\right] (1-x)} \tag{2.15}$$

It is straightforward to check that $z < x$ as long as $\theta > \frac{1}{2}$ and $n^y_{t+1} < n^x_t$. This basically means that highest possible decreases in beliefs (from x to y) are more likely than increases in beliefs (from y to z) of the same magnitude. This is the same to say, considering equation (2.5), that highest possible increases in lending rates are more likely than decreases in lending rates of the same magnitude, which is a necessary and sufficient condition for the existence of positive asymmetry on lending rates.

Exactly the same conclusion (that $z < x$) can be obtained reverting the order of successes and failures.

Hence, in an endogenous information economy, beliefs respond to a time irreversible stochastic process and lending rates show positive asymmetry.

Step 3: The effect of monitoring costs on lending rates asymmetry

The magnitude and importance of the asymmetry is summarized by the difference $z-x$ (equation 2.15) since it shows the degree of irreversibility in the stochastic process and the gap in the probability of obtaining an increase in lending rates over the probability of having a decrease of the same magnitude.

The gap $z-x$, for a given starting belief x and a given θ, only depends on the difference (not on the levels) between n^y_{t+1} and n^x_t. The difference in the number of signals given different beliefs are generated by differences on cutoffs for those beliefs $\widetilde{v}^y_{t+1}-\widetilde{v}^x_t$ (assuming the cumulative distribution of v is monotonically increasing). For example, if v is distributed uniformly $n^y_{t+1}-n^x_t$ is a negative and linear function of $\widetilde{v}^y_{t+1}-\widetilde{v}^x_t$.

The difference in cutoffs between beliefs x and y is given by

$$\widetilde{v}^y_{t+1}-\widetilde{v}^x_t=\frac{(x-y)}{xy}[1+r+w+c] \tag{2.16}$$

This expression is positive when $x>y$ because confidence on good states decrease, $\widetilde{v}^y_{t+1}>\widetilde{v}^x_t$ and the number of funded ventures decreases ($n^y_{t+1}<n^x_t$). The opposite is true when $x<y$.

Hence, the impact of monitoring costs c over the gap $n^y_{t+1}-n^x_t$ can be obtained from its impact over $(\widetilde{v}^y_{t+1}-\widetilde{v}^x_t)$. Taking derivatives.

$$\frac{\partial(\widetilde{v}^y_{t+1}-\widetilde{v}^x_t)}{\partial c}=\frac{(x-y)}{xy} \tag{2.17}$$

which is positive when $x>y$ and negative when $x<y$

Two conclusions can be drawn from the last equation. First, the higher the differences in beliefs $(x-y)$, the greater the impact of c on the number of funded ventures. Second, monitoring costs do not have the same effect in the change of beliefs if μ_t is closer to 1 than to 0. For a given difference in beliefs $(x-y)$ the less confident agents are about being in good times (x close enough to 0), the more important is the impact of c on the gap between signals because agency costs become more restrictive.

To check the impact of agency costs over symmetry assume the initial belief is $\mu_t = x$ and all ventures fail such that $x > y$. By equation (2.16), $(\widetilde{v}^y_{t+1} > \widetilde{v}^x_t)$ and $n^y_{t+1} < n^x_t$. We will show the gap is greater under high agency costs than under low agency costs.

By equation (2.17), the greater the agency costs c the greater are the gaps $(\widetilde{v}^y_{t+1} - \widetilde{v}^x_t)$ and $(n^y_{t+1} - n^x_t)$. This is because, fixing x the greater is $\widetilde{v}^y_{t+1}$) and the smaller is n^y_{t+1}). Considering equation(2.15) it is clear that high monitoring costs then widen time irreversibility (given by $z - x$).

Hence, in an endogenous information economy with financial frictions, the greater the agency costs c, the more important the asymmetry on lending rates.

Q.E.D.

This Proposition shows that when comparing two countries with different levels of monitoring costs c, both countries experience similar magnitude of crashes (given by increases of lending rates) but the country with highest c show slower booms (decreases in lending rates) than a country with low agency costs. This translates into a greater asymmetry of the changes on lending rates, the greater the levels of monitoring costs.

This is a result shown empirically in subsection 2.3.4, where we found asymmetry is mostly due to slower booms rather than sharper crashes. Even more, literature on slow recoveries shows additional elements to confirm the prediction that monitoring costs increase asymmetry fundamentally by making booms slower [13].

[13] See for example Bergoeing et al. [2004].

2.3.4 Additional testable predictions of the model

This model delivers a serie of testable predictions Naturally, the most important one is that agency costs increase asymmetry on lending rates, as shown in Proposition 19. This was the fact that motivates the introduction of agency costs in an endogenous information model and was carefully tested in Section 2.2.

But there are also a couple of conclusions from the model that can be also tested in the data.

2.3.4.1 Countries with less developed financial systems show higher lending rates

A testable prediction from the model is that countries with less developed financial systems show, in average, higher levels of lending rates than countries with highly developed financial systems or, which is the same, with less financial frictions and agency costs.

Formally, from equation (2.5),

$$\frac{\partial(1+\rho_t)}{\partial c} = \frac{1-\theta_t}{\theta_t} > 0$$

Even when this relation seems very natural from a casual observation of economic data, some basic regressions were estimated to check whether lending rates in countries with less developed financial systems in general and high monitoring costs in particular are high in average.

Table 2.10 shows a couple of regressions between the average level of lending rates and financial development (again measured by credit to private sector as a percentage of GDP). The estimations are made only for the period 1990-2004 using both a sample of all countries and a restricted sample of OECD countries.[14],

[14]We only the period 1990-2004 because, unlike skewness, which is calculated using changes on

Table 2.10: Lending rates average and financial development

Dependent Variable	1990-2004	
Lending rates average	All countries	OECD countries
Credit to Private Sector / GDP	-0.195	-0.227
	(0.039)***	(0.027)***
Constant	28.69	26.24
	(2.63)***	(1.93)***
Observations	59	12

* Significant at 10%, ** Significant at 5% and *** Significant at 1%. Robust standard errors are reported in parentheses.

In general more developed financial systems imply lower levels of lending rates. In fact, an increase of 1% in credit to private sectors as a percentage of GDP implies a reduction of around 0.2% in lending rates.

Table 2.11 is a mixture between Tables 2.4 and 2.5 but using as a dependent variable average levels of lending rates. The goal is to measure more specifically the relation between levels on lending rates and proxies for the health of the financial system and monitoring, enforcement and flow of information costs. Variables not reported (cost and time of bankruptcy and contract enforcement days), even when having the correct sign, are not statistically significant.

An important drawback is that, unlike regressions to explain skewness, comparisons of lending rate levels across countries may be capturing important differences in methodologies and definitions from the dataset.

lending rates along time for each country, averages of lending rates are highly dependent on the used measurement methodology. IMF's information for the nineties is more standardized across countries, making comparisons more reliable. for the same reason we took out most of African countries, restricting the same to OECD countries rather than non-African ones.

Table 2.11: Lending rates average and proxies for monitoring and enforcement costs

Dependent Variable Lending rates average	1990-2004			
Recovery rate	-0.28			
	(0.07)***			
Legal protection to financial assets		-5.63		
		(0.98)***		
Sophistication of financial markets			-3.71	
			(1.00)***	
Health of banking systems				-2.76
				(0.90)***
Constant	32.11	44.31	32.24	30.44
	(4.31)***	(5.45)***	(4.98)***	(5.20)***
Observations	49	29	29	29

* Significant at 10%, ** Significant at 5% and *** Significant at 1%. Robust standard errors are reported in parentheses.

All in all, even when we have to be more careful with these regressions than those explaining skewness, results seem consistent with the particular prediction of the model. Agency costs seem to increase lending rates, leading to under-investment and slower generation of signals necessary to fuel booms.

2.3.4.2 A higher asymmetry on lending rates is related to slower booms rather than to sharper crashes

The model also predicts that big asymmetries generated by high monitoring costs are characterized by slower booms rather than by sharper crashes. The intuition is that, in good times investors become very confident about the probability of success. In this context monitoring costs lose importance to determine the number of signals in the economy. Hence, when a crash occurs, it is based on similar conditions, no matter the magnitude of agency costs.

Contrarily, when times are bad, monitoring costs introduce serious borrowing constraints and reduce importantly the signals in the economy. If times change, booms are slower the fewer the number of signals. In this sense financial frictions introduce a sharper effect in booms rather than in crashes.

To show this formally, consider a country A with monitoring costs (c_A), higher than those on country B (c_B). It is possible to obtain the difference in the number of signals by the difference on cutoffs from equation (2.6).

$$\widetilde{v}_{t,A} - \widetilde{v}_{t,B} = \frac{(1-\theta_t)}{\theta_t}(c_A - c_B)$$

This difference increases monotonically as θ_t decreases. Formally, $\frac{\partial(\widetilde{v}_{t,A}-\widetilde{v}_{t,B})}{\partial\theta_t} = -\frac{(c_A-c_B)}{\theta_t^2} < 0$. For example, if $\theta_t = 1$ (very good times) there is no difference in cutoffs, which means agency costs do not affect at all the construction of signals in the economy. Contrarily, if $\theta_t = 0$ (very bad times) the difference in cutoffs is infinite.

Given crashes occur after good times (where θ_t is high), the difference in their magnitude between the two countries is almost unaffected by differences in monitoring costs since the number of signals are very similar. Contrarily, booms occur after bad times when θ_t is low and monitoring costs reduce the number of signals a lot. In this context booms in country A will be slower than booms in country B. In other words, recoveries are slower in countries with high monitoring costs and financial frictions.

To test this particular prediction from the model we generate an indicator per country called *Booms duration* that measures the proportion of periods the economy is below the trend, recovering from a crash.

The trend of lending rates is obtained using a standard HP filter on the series of the changes on lending rates ($ln(\rho_{t+1}) - ln(\rho_t)$). We define recovery periods as those in which lending rates decrease in comparison to the trend. Other periods are considered crash periods. Since there is not a standard measure in the literature for this concept we propose just a ratio between numbers of recovery periods over the total periods in the sample.

Table 2.12 presents OLS regressions between skewness on lending rates and the *Booms duration* for the samples used before. A positive coefficient means countries with high asymmetry on lending rates are characterized by booms and recoveries that take in average more time to occur than crashes.

As can be seen it is possible to find a positive relation between the magnitude of the skewness and the duration of booms. Furthermore, this relation is statistically significant in the whole sample. This means high asymmetry is mostly characterized by slower booms rather than by sharper crashes.

Table 2.12: Asymmetry on lending rates and duration of booms

Dependent Variable	All countries		OECD countries	
Lending rates skewness	1960-1990	1990-2004	1960-1990	1990-2004
Booms duration	7.69	6.89	2.03	3.03
	(2.47)***	(2.22)***	(2.37)	(1.68)*
Constant	-1.87	-1.82	0.80	-0.28
	(1.20)	(1.03)*	(1.29)	(0.60)
Observations	67	57	15	12

* Significant at 10%, ** Significant at 5% and *** Significant at 1%. Robust standard errors are reported in parentheses.

Table 2.13: Parameters used in the simulation

θ_g	θ_b	λ	**r**	**w**	**N**
0.97	0.95	0.027	0.0042	1	25

2.4 Simulations

In this section an endogenous information economy with agency costs, as the one discussed above, will be calibrated to see if the model is able to replicate the magnitude of differences in skewness found in the data.

When possible, simulations are performed using the same calibration parameters than Veldkamp [2005] in order to make our results comparable with hers. Table 2.13 summarizes the list of parameters.

Veldkamp [2005] obtained θ_g and θ_b from default rates on US speculative grade bonds given the unavailability of default data for emerging markets bond. The probability of a state transition λ was obtained using World GDP from the Penn World

Table 2.14: Montecarlo results

Monitoring Costs (c)	**0**	**0.1**	**0.2**	**0.3**	**0.4**
Skewness of $(ln(\rho_t) - ln(\rho_{t-1}))$	1.60	1.79	2.08	2.56	3.49
MonteCarlo S.E.	0.05	0.06	0.07	0.09	0.12

tables. The largest potential number of independent observable signals N was sharply overcame measuring the speed of price adjustments in the US. Parameters r and w only affect the scale of the lending rate and skewness is invariant in scale[15]. The same numbers can be used as benchmark in this exercise, even when trying to match a greater number and diversity of countries, given the parameters are obtained either from US or from the whole world.

Ten thousand repeated simulations, each with 10,000 periods, produce average skewness estimates depending on monitoring costs. Monte Carlo standard errors are also reported for each case. Since we assumed the initiation cost for each venture is 1, a monitoring or bankruptcy cost given, for example, by $c = 0.3$ represents a cost of 30% of total asset values. Table 2.14 shows the asymmetry implied by the model for different monitoring costs possibilities.

As formally shown above, the greater the monitoring costs in this simulated economy the greater the skewness of changes in lending rates. Even more, Monte Carlo standard errors show that differences in asymmetry caused by different monitoring costs are statistically significant at standard confidence levels.

The result without monitoring costs (skewness=1.60) is the same as in Veldkamp [2005] when using uniformly distributed investment payoffs. One of the drawbacks in that paper is the difficulty to match successfully the data about asymmetry on lending rates for 13 emerging markets she analyzes (skewness=2.9).

[15] Skewness is independent on r and w since the support of v_i is $[\underline{v}, \overline{v}]$, where $\underline{v} = \frac{1+w+r}{\overline{\theta}}$, $\overline{v} = \frac{1+w+r}{\underline{\theta}}$, $\overline{\theta}$ is the most optimistic probability of success and $\underline{\theta}$ the most pessimistic one.

At this point Veldkamp experimented with different parameters to match the data. She was able to increase the simulated skewness, for example, by decreasing the probability of state switching (by reducing λ), generating clearer signals (by increasing $\theta_g - \theta_b$) or changing the assumed distribution of potential profits ν_i. However these changes imply countries with a very stable state or those with clearer signals are those with higher asymmetry. Since these characteristics are more common in developed countries than in developing ones, Veldkamp [2005] results seem contradictory with the data.

Introducing monitoring costs, and without modifying the parameters calibrated from real information, the skewness based on Veldkamp's 13 emerging markets (2.9) is consistent with bankruptcy costs of 35% over total asset values.

Now we can face, as shown by Figure 2.1 and 2.2, the relation between these results and the fact that less developed countries can present either high or low skewness levels. By introducing monitoring costs we are introducing a compensating force to more volatile states or noisy signals in developing countries, which tend to reduce asymmetry levels. Considering the high dispersion of parameters such as λ, θ_g and θ_b among developing countries in comparison with developed ones, developing countries high monitoring costs, but also with unstable states and noisy signals, can in fact show low relative skewness.

This is exactly why in Table 2.2 and Figures 2.3 and 2.4 we controlled skewness by the volatility of GDP per capita, lending rates and consumer prices, raw estimations for λ in each country.

An interesting question we can answer from these exercises is: What is the magnitude of monitoring costs consistent with skewness differences reported in Section 2.2?. The idea is to obtain from this very basic and rustic model an idea of differences in agency costs across countries. This is a straightforward application of the model to

Table 2.15: Implied monitoring costs to match real data on lending rates asymmetry

Countries classification	**Real Skewness**	**Cost of Bankrupt**	**Consistent *c* (in %)**	
			Point	**Range**
Income Group 1	1.66	7.1	7	2 – 11
Income Group 2	2.37	18.5	25	22 – 28
Income Group 3	2.55	18.6	31	28 – 33
Income Group 4	3.49	23.6	39	37 – 41
OECD	1.61	9.0	7	2 – 11
non-OECD	2.95	20.1	33	30 – 36
High contract enforcement	1.00	6.5	0	0
Low contract enforcement	2.67	16.8	26	24 – 30
Private Bureau	1.95	11.6	15	12 – 18
non-Private Bureau	3.05	20.3	34	32 – 36

Countries classification and Real Skewness columns are taken from columns 1 and 5 of Table 2.3. Bankruptcy costs are taken from Djankov et al. [2005]. ”Consistent c” refers to monitoring costs that, given the parameters, allows to match real skewness. The range is determined using two Montecarlo standard deviation at each side of the point estimation.

offer an, surprisingly missing, information about the magnitude of monitoring costs differentials across countries.

Table 2.15 shows the results. Monitoring costs consistent with skewness in each classification group and the range within two Montecarlo standard deviations are reported. As can be seen estimations are significant since ranges do not intersect.

Table 2.15 also shows bankruptcy costs indicators used in Section 2.2. The reason to include them in the Table is to compare the monitoring costs implied by the model with the subjective measure of foreclosure costs offered by Djankov et al. [2005].

As can be seen, even when bankruptcy costs obtained from surveys by Djankov et al. [2005] (column 3) and consistent c obtained from a simulation of our model (column 4) are two imprecise measures of monitoring costs, it is impressive the high correlation between them. In fact monitoring costs delivered by the calibration are consistently higher than bankruptcy costs reported by Djankov et al. [2005]. This sustained bias can be rationalized at least in two ways.

First, monitoring costs from the model replicate skewness measured for the period 1960-2004 while bankruptcy costs from Djankov et al. [2005] is measured for the year 2004. As discussed in Section 2.2, technologic improvements and an easier flow of information imply that modern measures of bankruptcy costs are lower than older ones. Doing the same exercise for the period 1990-2004 closes the gap between columns 3 and 4 for poorer countries but delivers zero monitoring costs for richer ones.

Second, bankruptcy costs as measured by Djankov et al. [2005] exclude bribes, which can raise monitoring costs considerably. Even more, this will be true fundamentally for poorer countries with low contract enforcement. This may be the reason why the gap between monitoring costs implied by the model are bankruptcy costs by Djankov et al. [2005] is not only positive but also increasing as countries become less financially developed.

At this point it is important to put these results into context with a brief discussion about the literature on monitoring technology and bankruptcy costs, where a great debate exists about the correct way to measure them.

One of the first attempts to estimate bankruptcy costs was done by Warner [1977] who, considering only direct costs of bankruptcies, and using data on the railroad industry, found a cost of around 4% of total firm's assets. Altman [1984] included also indirect costs, raising the estimation at about 20% of total firm's assets. Indirect costs include financial distress, such as lost sales and lost profits. Another way bankruptcy costs were estimated in the literature is due to Alderson and Betker [1995], by comparing the value of the firm as a going concern with the liquidation value of the firm. This calculation of bankruptcy costs accounts for approximately 36% of firm's assets.

As can be seen, the possible range for bankruptcy costs given by the literature is very wide and imprecise. Furthermore, the few available estimations are typically based on the US or another developed country. This controversy lies fundamentally on differences in definitions. The interpretation most closely related to the concept of bankruptcy costs used here is the one that only considers direct costs, as Warner did. This is because no indirect cost can arise in the environment described by the model and no liquidation value of the firm can be obtained.

The model seems very successful in matching the magnitude of asymmetry from the data not only with previous estimations of monitoring technology but also with new subjective indicators across countries. Even when the model is very basic and simple, it can offer common-sense consistent bankruptcy costs, with sensible differences among various countries' classifications. This is particularly important given the nonexistence of direct estimations of this type for developing countries.

All in all, this exercise can offer an idea of monitoring costs in developing countries. Even when the method to obtain them is very indirect and based on a very limited and simple model, the results in fact make a lot of sense and seem to be robust to different specifications.

2.4.1 What about levels?

A natural question at this point is whether monitoring costs are able to explain the big differences we observe in levels of lending rates across countries. This is not an easy task for a simple model such as the one proposed in this paper, mostly considering that lending rates in countries with poor financial systems almost double those existing in developed markets [16].

In our model, levels on lending rates depend exclusively on free risk interest rates, default rates and monitoring costs. In fact equation (2.5), can be re-written as,

$$\rho_t = r + \frac{(1-\theta_t)}{\theta_t}(1+r) + \frac{(1-\theta_t)}{\theta_t}c \tag{2.18}$$

This means lending rates can be expressed as the sum of three terms: A risk free interest rate, a risk premium (which depends on the risk free interest rate adjusted by default rates) and costs that arise from financial frictions and costly state verifications.

From this formula, our model can explain big differences on lending rates only by a combination of big differences on risk free interest rates, on probabilities of success and on monitoring costs. In this subsection we show that most of the differences in levels in fact comes from differences in ”risk free interest rates”, (as typically measured by the literature), which affect levels but not skewness.

Up to this point, to make the skewness simulations we just considered the US risk free interest rate, we used default rates from the riskiest speculative-grade US bonds to obtain probabilities of success and, from there, we estimated consistent monitoring costs. Now it’s important to discuss the role of each one of these components before simulating levels.

First, in the previous calibration we used as a risk free interest rate (r) the average of 3-month US Treasury Bills Yields from 1990 to 2005. Since in our model skewness is invariant in scale, the specific number used did not matter for skewness comparisons and for the determination of consistent monitoring costs. However, to simulate levels we need to obtain risk free interest rates for other economies as well. In this sense we use 3-month Treasury Bills Yields for the countries in the sample [17]. We find

[16] See for example the first column in Table 2.16

[17] This information was obtained from the Global Financial Dataset, taking averages per country between 1990 and 2005 when available.

surprising disparities among them, as shown in column 3 of Table 2.16, which suggests government bonds in developing countries are not really ”risk free” since they include default risks, country risks, exchange volatility risks, etc.

Second, default rates are obtained from Moody’s bonds information from 1970-2000. In skewness simulations, probabilities of success are US speculative-grade bonds and not ”all corporate” figures, since emerging markets bonds (whose default rates are not available) are likely to be riskier than typical US corporate bonds. Hence, to simulate skewness we used a 5% probability of default in recession years ($\theta_b = 0.95$) and 3% in non-recession years ($\theta_g = 0.97$). Even when this may be a good assumption for developing countries, this is not necessarily true for developed ones. Hence to do the simulations about levels for developed countries we obtain default rates from US ”all corporate” bonds ($\theta_b = 0.97$ and $\theta_g = 0.98$)[18].

Finally, to simulate levels we use the monitoring costs that make the model consistent with skewness data, as shown in column 4 of Table 2.15. Results from the simulations as well as the three components of lending rates from equation (2.18) are displayed in Table 2.16.

As can be seen, the importance of monitoring costs on levels of lending rates are low when compared with the importance of differences in risk free interest rates and the multiplicative effects of default rates (through θ). However, it’s important to recognize that monitoring costs accounts for more than 20% of lending rates spread in developing countries ($1.7/7.6$ for income group 4) and less than 5% in developed ones ($0.1/3.0$ for income group 1).

All in all, even when it seems monitoring costs are not very important to explain differences on lending rates levels, their importance to explain the spread decreases as

[18]These default rates reduce the estimation of monitoring costs for developed countries in the previous exercises when skewness was simulated. However this reduction was not very important since monitoring costs were already low.

Table 2.16: Real vs. Estimated lending rates

Countries classification	**Real lending rates**	**Estimated lending rates**			
		Total	*Components*		
			r	$\frac{(1-\theta)(1+r)}{\theta}$	$\frac{(1-\theta)}{\theta}c$
Income Group 1	**9.6**	**9.4**	6.6	2.7	0.1
Income Group 2	**18.5**	**19.2**	13.5	4.7	1.0
Income Group 3	**18.5**	**18.0**	12.0	4.7	1.3
Income Group 4	**24.4**	**23.4**	16.8	4.9	1.7
OECD	**10.8**	**11.2**	8.4	2.7	0.1
non-OECD	**20.9**	**20.4**	14.3	4.7	1.4
High contract enforcement	**8.4**	**8.8**	6.0	2.0	0.0
Low contract enforcement	**20.7**	**22.3**	16.3	4.9	1.1
Private Bureau	**14.3**	**16.9**	11.6	4.7	0.6
non-Private Bureau	**23.2**	**21.4**	15.2	5.8	1.4

financial systems become more developed.

In Appendix 2.6.3 we also discuss the capacity of the model to deliver the big differences in volatility of real lending rates across countries. However we show, as in this section, most of the action comes directly from risk free interest rates, about which the model does not have much to say.

Since the model assumes fixed risk free interest rates, the fact that they are much higher and much more volatile in underdeveloped countries, reduces the magnitude in which the model may contribute to match the data. Even when differences across countries in the first and second moments of lending rates seem to be led by the differences in the first and second moments of risk free interest rates, this does not seem to be the case for the third moment of changes.

Asymmetry in changes of risk free interest rates do not differ significantly across countries. Hence monitoring costs are very important in understanding the differences in asymmetry we show exists across countries.

2.5 Conclusions

A well documented characteristic of financial markets is their asymmetry in changes over cycles. While booms are slow and gradual, crashes are sudden and sharp. This feature represents a non trivial fact to countries since it may generate economic problems such as financial distress, banking crisis and costly reallocation of resources.

But, aside from the existence of asymmetry in a country along time, an interesting characteristic that surges from the data is that less financially developed systems, with high monitoring and bankruptcy costs, show in average higher levels of asymmetry.

While a diverse and rich literature tries to explain why asymmetry exists, this is the first attempt to understand why asymmetry differs across countries.

We introduce agency costs into an endogenous information model (which has the property of generating unconditional asymmetry) to replicate differences observed in the data. The idea of a model with endogenous flow of information is that, in good times there is more economic activity than in bad times, generating a greater number of signals and more information. The asymmetry in the rate of transmission of information across states is the origin of the asymmetry on lending rates. Booms and recoveries are gradual because agents learn slowly about better conditions when few signals are available. Contrarily, crashes are sharp because agents learn quickly that worse conditions arose since a lot of information is available.

When agency costs are introduced in this environment it is possible to generate even more asymmetry. The main reason is that agency costs reduce investment (the number of signals), but their impact is not the same across states. These costs are more restrictive in bad times since an agency problem is more likely to arise. After a crises high monitoring costs prevent a fast renew of economic activity, slowing the generation of signals, making harder for agents to learn about the new conditions and slowing down recoveries.

Even when strikingly simple, the simulation of this model delivers an estimation of cross-country differences in monitoring costs that match observed skewness differences.

Direct monitoring costs of around 5% match the data for developed countries while monitoring costs of around 30% match the data for developing countries. These figures are consistent with some new ”survey-based” evidence of differences in bankruptcy costs across economies. Furthermore the model is able to match differences on levels of lending spreads across countries.

2.6 Appendix

2.6.1 Sample of Countries

The sample of countries used, based on the classification by income levels, is shown in the following table. We also report the skewness of log changes of lending rates for each country considering the period 1990-2005.

Table 2.17: Countries included in classification by income

LR Skewness (1990-2005) by Country Classification			
Income Group 1 (Richest)			
UNITED STATES	-1.11	CANADA	0.27
UNITED KINGDOM	-1.01	SPAIN	0.40
JAPAN	-0.42	NORWAY	0.41
GREECE	-0.42	KUWAIT	0.54
CHINA,P.R.:HONG KONG	-0.38	SLOVENIA	0.69
PORTUGAL	-0.12	NETHERLANDS ANTILLES	0.70
BELGIUM	0.06	CHINA,P.R.:MACAO	0.85
FRANCE	0.09	ISRAEL	1.30
FINLAND	0.16	KOREA	3.96
Income Group 2			
BARBADOS	-1.49	CROATIA	1.35
VENEZUELA, REP. BOL.	0.17	ARGENTINA	1.55
SLOVAK REPUBLIC	0.39	MEXICO	1.55
URUGUAY	0.60	BRAZIL	2.54
HUNGARY	1.02	CZECH REPUBLIC	3.42
SOUTH AFRICA	1.19	GABON	6.12
Income Group 3			
THAILAND	-1.15	SRI LANKA	0.76
EL SALVADOR	-1.01	JORDAN	1.54
EGYPT	-0.62	JAMAICA	1.97
SWAZILAND	-0.02	GUYANA	2.75
PHILIPPINES	0.12	INDONESIA	4.80
DOMINICAN REPUBLIC	0.12	BOTSWANA	6.95
COLOMBIA	0.13	CAPE VERDE	6.99
PARAGUAY	0.76		
Income Group 4 Poorest)			
KENYA	-1.42	ANGOLA	1.39
NEPAL	-1.01	BANGLADESH	1.68
LAO PEOPLE S DEM.REP	-0.44	MADAGASCAR	1.75
SIERRA LEONE	-0.05	ZIMBABWE	1.91
UGANDA	0.05	HONDURAS	2.19
ARMENIA	0.17	MALAWI	3.25
BURUNDI	0.38	CAMEROON	3.82
HAITI	0.45	CHAD	4.94
INDIA	0.46	CONGO, REPUBLIC OF	5.10
CAMBODIA	0.55	CENTRAL AFRICAN REP.	5.14
ZAMBIA	0.74	ALBANIA	8.24
NICARAGUA	0.88	ETHIOPIA	8.41
TANZANIA	1.36		

The countries included in the other classifications we use are the following.

Table 2.18: Countries included in other classifications

OECD (15 countries)	Non-OECD Countries (55 countries)
Belgium, Canada, Czech Rep., Finland, France, Greece, Hungary, Japan, Korea, Mexico, Norway, Portugal, Spain, UK, US.	Albania, Angola, Argentina, Armenia, Bangladesh, Barbados, Botswana, Brazil, Burundi, Cambodia, Cameroon, Cape Verde, CAR, Chad, Colombia, Congo, Croatia, Dominican Rep., Egypt, El Salvador, Ethiopia, Gabon, Guyana, Haiti, Honduras, Hong Kong, India, Indonesia, Israel, Jamaica, Jordan, Kenya, Kuwait, Lao, Macao, Madagascar, Malawi, Nepal, Netherlands Ant., Nicaragua, Paraguay, Philippines, Sierra Leone, Slovak Rep., Slovenia, South Africa, Sri Lanka, Swaziland, Tanzania, Thailand, Uganda, Uruguay, Venezuela, Zambia, Zimbabwe.
High Contract Enforcement (10 countries)	**Low Contract Enforcement (16 countries)**
Belgium, Canada, Finland, France, Japan, Norway, Portugal, Spain, UK, US.	Argentina, Brazil, Colombia, Greece, India, Israel, Kenya, Korea, Mexico, Philippines, South Africa, Sri Lanka, Thailand, Uruguay, Venezuela, Zimbabwe.
Private Bureau (27 countries)	**Non-Private Bureau (33 countries)**
Argentina, Botswana, Brazil, Canada, Colombia, Czech Rep., El Salvador, Finland, Greece, Hong Kong, Hungary, Israel, Japan, Kenya, Korea, Kuwait, Mexico, Norway, Paraguay, Philippines, Portugal, South Africa, Spain, Thailand, UK, Uruguay, US.	Albania, Angola, Armenia, Bangladesh, Belgium, Burundi, Cambodia, Cameroon, CAR, Chad, Congo, Croatia, Egypt, Ethiopia, France, Haiti, Honduras, India, Indonesia, Jamaica, Jordan, Lao, Madagascar, Malawi, Nepal, Nicaragua, Sierra Leone, Slovak Rep., Slovenia, Uganda, Venezuela, Zambia, Zimbabwe.

2.6.2 Robustness on Skewness Classification

In the main text we analyze the differences in skewness in log changes of real lending rates across countries, using several classification related to financial development. In this Section of the Appendix we extend that exercise following two alternative approaches.

First, we use the same classification but we obtain skewness in the distribution of log deviations from trend. Trends of lending rates for each country are obtained using an HP filter. For each month we obtain the difference between the log of lending rates and the log of the trend. The skewness is calculated over such distribution per country.

Table 2.19: Asymmetry on lending rates by country classification (using differences in the log deviation from trend)

Countries classification	1960-1990	1990-2004	1960-2004
Income Group 1 (Richest)	2.49	1.00	1.74
Income Group 2	2.82	1.97	2.40
Income Group 3	2.60	2.14	2.37
Income Group 4 (Poorest)	4.32	2.46	3.39
OECD	2.05	1.25	2.15
non-OECD	3.66	2.11	2.89
High contract enforcement	1.84	0.32	1.08
Low contract enforcement	4.00	1.22	2.61
Private Bureau	2.37	1.51	1.94
non-Private Bureau	4.93	2.14	3.54

Following this approach we are able to discuss whether the results depend or not on specific properties of the trend across countries. Table 2.19 shows that the results are basically the same than the obtained in the main text using log changes of lending rates along time.

Second, the model in this paper is in fact a model of lending rates spreads rather than a model of lending rates levels since we do not discuss the determination of risk free interest rates. Hence, in the next Table we analyze cross country differences in skewness of log changes in spreads rather than levels. As above, we use the same classifications than in the main text in order to easy comparisons.

Lending rates spreads are obtained subtracting risk free interest rates (measured by the average of 3-month Treasury Bills Yields for each country from the Global

Financial Dataset in the period 1990-2005) from the real lending rate, using as a base the year 1995. The skewness information is obtained from the log changes distribution of this spread variable.

Table 2.20 shows the results are consistent with the ones obtained in the main text. Spread Asymmetry seems to be higher among poor, non-OECD countries with low enforcement of contracts. However we should be cautious with the conclusions since data about risk free interest rates is not available with a high quality for underdeveloped countries.

Since the main point of the paper is to compare developed and underdeveloped countries, the analysis of spreads is seriously hindered. However, for the 37 countries we have consistent information about risk free interest rates, the correlation between skewness based on levels of lending rates and skewness based on spreads of lending rates is positive (0.84) and statistically significant at 1% of confidence. This is the main reason we focus directly in analyzing levels of lending rates in the main text rather than spreads.

2.6.3 Volatility of Lending Rates

As discussed in the main text, the model is a model of spread of lending rates rather than a model of levels of lending rates. This is because the model just takes risk free interest rates as fixed. Hence, in the case of levels we assigned the corresponding risk free interest rates to each country and obtained spread levels in the data. As shown, the model cannot match the level of lending rates but it can match the levels of spreads in lending rates.

We follow a similar procedure for volatilities. In Table 2.21 we can see that lending rates are more volatile in poor countries, with underdeveloped financial systems. However this seems to be due to higher volatility in risk free interest rates rather than

Table 2.20: Asymmetry on lending rates spread by country classification

Countries classification	1960-1990	1990-2004	1960-2004
Income Group 1 (Richest)	NOT	0.48	0.48
Income Group 2	Enough	0.79	0.67
Income Group 3	Data	0.92	0.90
Income Group 4 (Poorest)		0.83	0.78
OECD		0.58	0.56
non-OECD		0.77	0.69
High contract enforcement		0.26	0.43
Low contract enforcement		1.15	1.16
Private Bureau		0.89	0.75
non-Private Bureau		0.72	0.75

by higher volatility in spreads. While the volatility in 3-month T-Bill yields is more volatile in poorer countries this is not the case for spreads, which do not seem to follow a clear pattern across countries. The data was obtained from the same sources as in the exercise about levels and the period 1980-2004.

Even when the model is consistent with the data in terms of not not delivering any patter of spread volatility across countries, it fails importantly in matching the level of volatility.

Hence, differences in monitoring costs and financial systems do not seem to have an effect on volatility of spreads. The higher volatility of lending rates in poorer countries seem to be exclusively the result of higher volatility in risk free interest rates in those countries.

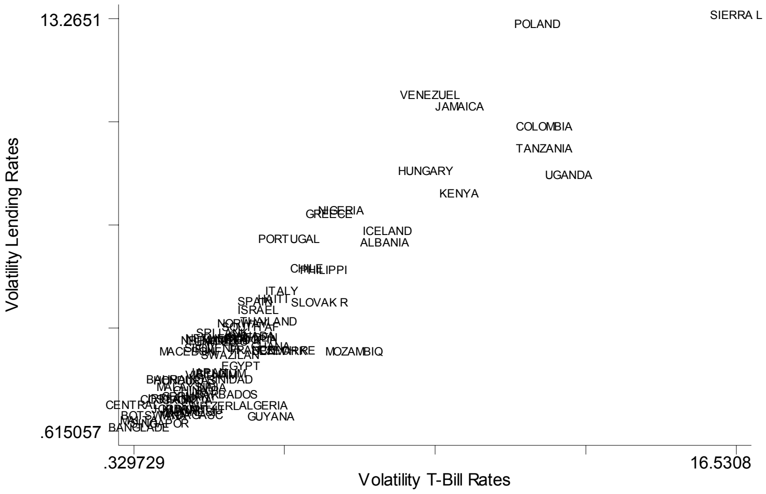
Figure 2.6 shows the highly significant (economically and statistically) positive

Table 2.21: Data vs. Estimated Volatility in changes of lending rates

Country classification	**Volatility (in % - St. Dev.)**			
	Data			Model
	Lending Rates	T-Bill Rates	Spread	Spread
Income Group 1	4.2	10.8	29.3	**1.1**
Income Group 2	4.4	6.6	28.2	**1.0**
Income Group 3	9.6	12.2	32.3	**0.9**
Income Group 4	6.3	12.4	31.9	**0.9**
OECD	4.5	7.2	32.9	**0.9**
non-OECD	6.9	12.2	28.1	**0.9**
High contract enforcement	3.5	7.2	32.4	**0.9**
Low contract enforcement	8.6	10.3	29.9	**0.8**
Private Bureau	6.1	10.4	29.5	**0.7**
non-Private Bureau	7.1	13.5	33.7	**0.7**

relation between volatilities of lending rates and risk free interest rates. Countries with high volatility of lending rates are also countries with high volatility of T-Bills rates. This relation suggests the high volatility of lending rates in poor countries are mostly due to high volatility of risk free interest rates, not high volatility of spreads. This last result is consistent with the model. However, the model is completely unable to explain the high magnitude of spread volatility.

Figure 2.6: Volatility of Lending Rates and Risk Free Interest Rates



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CHAPTER 3

Why are Wages Smoother than Productivity? An Industry-Level Analysis

(with David Lagakos)

In this paper we document a new fact about the cyclical behavior of productivity and wages. Using industry-level time series on wages and labor productivity, we show that in high-wage industries, wages respond relatively little to industry productivity shocks, whereas in low-wage industries, productivity movements result in relatively large movements in wages. In other words, wages are substantially ”smoother” than productivity over time in high-wage industries, while wages are comparatively less smooth in low-wage industries. To explain this fact we develop a variant of the Thomas and Worrall [1988] wage contracting model. The two key features of our model are match specific skills, which serve to increase wage smoothing in the contract, and exogenous match separations, which serve to reduce smoothing. We show that, empirically, a higher fraction of the skills of the high-wage workers are match-specific than the skills of the low-wage workers, and that job separation rates are lower for high-wage workers than low-wage workers. A calibrated version of the model accounts quite well for the facts at hand.

3.1 Introduction

In this paper we document a new fact about the cyclical behavior of productivity and wages. Using industry-level time series on wages and labor productivity, we show that in high-wage industries, wages respond relatively little to innovations in industry productivity, whereas in low-wage industries, productivity movements result in relatively large movements in wages. In other words, wages are substantially ”smoother” than productivity over time in high-wage industries, while wages are comparatively less smooth in low-wage industries. We show that this finding is robust within manufacturing and service industries, and in both the US and the majority of OECD countries for which we have data. To the best of our knowledge we are the first to document this fact.

We find this fact puzzling in light of standard implicit-contract theory (Baily [1974] and Azariadis [1975]). According to implicit-contract theory, workers and firms agree to a wage contract in which the worker receives wages that are smoother over time than the worker's marginal product, and in return the firm pays the worker wages that are on average lower than the worker would earn in spot markets. The key assumption in this theory is that workers are more risk averse than firm owners. This assumption is typically justified by arguing that workers have worse access to asset markets than firm owners, and hence have less means of otherwise smoothing their consumption. However, since low-wage workers generally have less access to asset markets than high-wage workers, the theory suggests that low-wage workers would have the smoother wages rather than high-wage workers. Our findings show exactly the opposite.

To help resolve this puzzle, we develop a variant of the Thomas and Worrall [1988] wage contracting model, in which a risk-neutral firm and risk-averse worker agree upon an optimal wage contract under limited commitment. The limited commitment is two-sided: both the worker and the firm can renege on the contract after any history. The state of the world in each period is characterized by the worker's productivity, which evolves exogenously. Following Thomas and Worrall [1988], we restrict our study to wage contracts that are self-enforcing, meaning that neither party has an incentive to renege on the contract in any state of the world.

The optimal contract in this environment specifies wage smoothing: wages move as little as possible after any productivity realization to keep both parties at least indifferent to remaining in the match. The amount of wage smoothing sustainable in equilibrium depends on the outside options of the worker and firm. The better the outside options, the less smoothing can be sustained. In our model, the firm and worker have the option of leaving the match and going to spot markets in any period. If the worker goes to spot markets she earns her marginal product in every subsequent period. If the firm fires its current worker, it may match up with another worker, but competition among firms leads to zero expected profits from any given match.

We depart from Thomas and Worrall [1988] by adding two new features into the environment, each of which qualitatively affects the amount of wage smoothing sustainable in the optimal wage contract. The first feature is match-specific skills, which are lost if either party leaves the match. In the model, wage smoothing is increasing in the fraction of the worker's skills that are match specific, since the worker can take fewer of her skills to a new firm, and hence is less productive in any new firm. The second feature we add is the possibility of an idiosyncratic, exogenous job separation. In the model, smoothing is decreasing in the probability of a separation to the match. Intuitively, the more likely the worker and firm are to separate, the less either party will value promises of higher future payoffs in exchange for lower payoffs in the present, as is required in order to smooth wages.

Our hypothesis about why the high-wage industries get relatively more wage smoothing is as follows. First, a higher fraction of the skills of high-wage workers are match-specific compared to low-wage workers. In other words, high-wage workers stand to loose relatively more from leaving their current job, and hence have relatively worse outside options than low-wage workers. Second, the probability of a job separation is higher for low-wage workers. In our model, both of these features lead qualitatively to smoother wages for high-wage workers.

Since our theory rests on these two differences between high and low-wage jobs, we document that both components are in line with empirical evidence. The more well-known of the two is the differences in separation rates across the two sectors. In the literature on job turnover, numerous studies document higher separation rates for low-wage industries; two recent examples include Davis et al. [2006] and Fallick and Fleischman [2004]. Regarding industry average wages and the fraction of skills that are match specific, we provide our own supporting evidence using estimates of wage losses for workers separated in mass layoffs as a proxy for match-specific skills. Carrington and Zaman [1994] estimate the average wage losses for displaced workers by detailed industry, which we match with our own measures of industry average wages. We show that workers in high-wage industries tend to lose a higher fraction of their wages than low-wage workers after a large layoff, which we interpret as evidence that a higher fraction of the skills of high-wage workers are match-specific.

As a test of our theory, we ask whether a calibrated version of our model can match the degree of wage smoothing we observe in the the cross section of industries. Specifically, we calibrate two versions of our model: one to represent a typical high-wage industry and one to represent a typical low-wage industry. We treat the empirical match-specificity of skills and separation rates as exogenous characteristics of the two sectors. We find that the two calibrated versions of our model predict degrees of wage smoothing that are quite similar to their empirical counterparts.

Our paper contributes to two distinct literatures. The first is the recent labor search and matching literature in macroeconomics, which seeks to explain equilibrium unemployment through matching frictions. A major challenge in this literature has been to explain how relatively small exogenous productivity shocks translate into relatively large movements in hiring, and hence equally large movements in the unemployment rate. [1] One potential resolution of the puzzle, explored by Hall [2005], Menzio [2005] and Rudanko [2006], among others, centers around wage contracts in which wages move little in response to a productivity shock, giving firms large incentives to hire new workers after small exogenous increases in productivity. [2]

Our paper contributes to this literature in two ways. First, we document the cross-industry variation in the response of wages to productivity, which can be used to test among existing theories of why wages respond little to productivity shocks, such as those described above. Second, we propose an alternative mechanism for the response of wages to productivity that is grounded in the empirical response observed in the cross section of industries. Our paper contrasts with those of Hall and others in that we focus specifically on explaining how and why wages respond to productivity shocks rather than the implications for unemployment volatility. In a related paper (Lagakos and Ordonez [2007]) we relate our empirical findings directly to the unemployment volatility puzzle using an industry version of Shimer [2005].

The second literature to which our paper is related is the one on risk sharing among private agents in the economy. This literature has focused in large part on impediments to risk sharing and the welfare implications of imperfect risk sharing. Recent papers by Krueger and Perri [2005], and Heathcote et al. [2004] have demonstrated that the

[1] Using a stochastic version of the Mortensen and Pissarides [1994] model calibrated to match important moments of the US labor productivity series, Shimer [2005] finds that the model predicts just 10% of the volatility of unemployment and vacancy postings seen in the data.

[2] This literature describes the unresponsiveness of wages to productivity as ”wage rigidity” or ”wage stickiness.”

wage volatility has first order effects on consumption volatility and large effects on welfare more generally. Our paper contributes to this literature by analyzing how wage volatility arises in the first place for employed workers. Since the firm is an important vehicle for sharing risk, it is important to understand the impediments to risk sharing between firms and workers. Our paper highlights two specific impediments: a lack of match-specific skills for the worker, which forces wages to respond to changes in either party's outside option, and the likelihood of an exogenous separation, which serves to discount the future of the match.

The remainder of the paper is as follows. In Section 3.2 we discuss our industry-level wage and productivity data, our measure of wage smoothing, and our empirical findings about the pattern of wage smoothing across industries. In Section 3.3 we present our wage contracting model, and in Section 3.4 we calibrate the model and describe our quantitative findings. In Section 3.5 we conclude.

3.2 Wage Smoothing: The Industry-Level Facts

3.2.1 Description of data

We use two main sources of data on industry productivity and wages in the study. The first source, available for the US, is the value-added by industry data constructed by the US Bureau of Economic Analysis (BEA). We use two different BEA data sets, each with annual industry-level measures of value added, employment, and compensation of labor. Our longest data set uses the 1972 Standard Industry Classification (SIC) codes, covers the entire private economy, and is available annually from 1947-1987. The second uses the 1987 SIC codes, and covers the shorter period from 1987-1997.[3]

[3] Unfortunately we cannot conduct a similar analysis for the North American Industrial Classification System (NAICS), which is the BEA's preferred industry definition, because the BEA has not yet released historical employment data by NAICS industries.

We could not combine these time series into one long panel of industries, since several important industries changed definitions in 1987.

Our second data source is the OECD Structural Analysis (STAN) database, which is constructed using the national accounts of major OECD nations, including the US, and supplemented with data from national surveys of firms. For each country, our data set contains annual industry-level measures of value added, employment, and compensation of employees. The data is available from 1970 or later to 2000, depending on the country. The industries comprise all sectors of the economy, and are standardized across countries according to 2-digit International Standard Industrial Classification (ISIC) codes.[4]

The two main variables of interest are labor productivity and average compensation of labor. In each of our data sets, total compensation of labor consists of all salaries, bonuses, contributions to medical and pension plans, and any other compensation that is not in-kind. Our measure of average compensation per employee is total industry labor compensation divided by full-time-equivalent (FTE) employees. Throughout, we use the term 'wages' to mean 'average compensation of employees' for expositional purposes. Our measure of labor productivity is real industry value added divided by FTE employees.[5] Employees consist of both production and non-production workers. For all countries we create real wage and value added data by deflating nominal values by a national consumer price index or its closest equivalent. To capture the relatively high-frequency component in our variables we de-trend productivity and wages in each industry, in each country, using an HP filter with smoothing parameter $\lambda = 100$.

[4]The STAN dataset is available for purchase online directly from the OECD at http://www.oecd.org/sti/stan/. Documentation can be freely downloaded from the same site. The list of industries comprising the STAN database can be found at http://www.oecd.org/dataoecd/33/19/1830838.html.

[5]For countries in which FTE employment is not available, we use the total number of employees instead. The choice of labor measure does not seem to drive any of our results: for countries with both labor input measures the results were very similar under the two measures.

Value added is difficult to measure in industries that do not have market-determined output prices. For this reason we drop any industry that consists of non-market activities, or in which market prices are not readily available.[6] In particular, we drop public administration, defense, and compulsory social security; education; health and social work; real-estate activities; other community, social, personal services; and private household employees. We also drop agriculture since problems in measuring employment (particularly for non-paid family workers) exist in several countries.

3.2.2 Our Measure of Wage Smoothing

We measure the amount of smoothing in industry j as ε_j in the linear model:

$$w_{j,t} = \varepsilon_j p_{j,t} + u_{j,t} \tag{3.1}$$

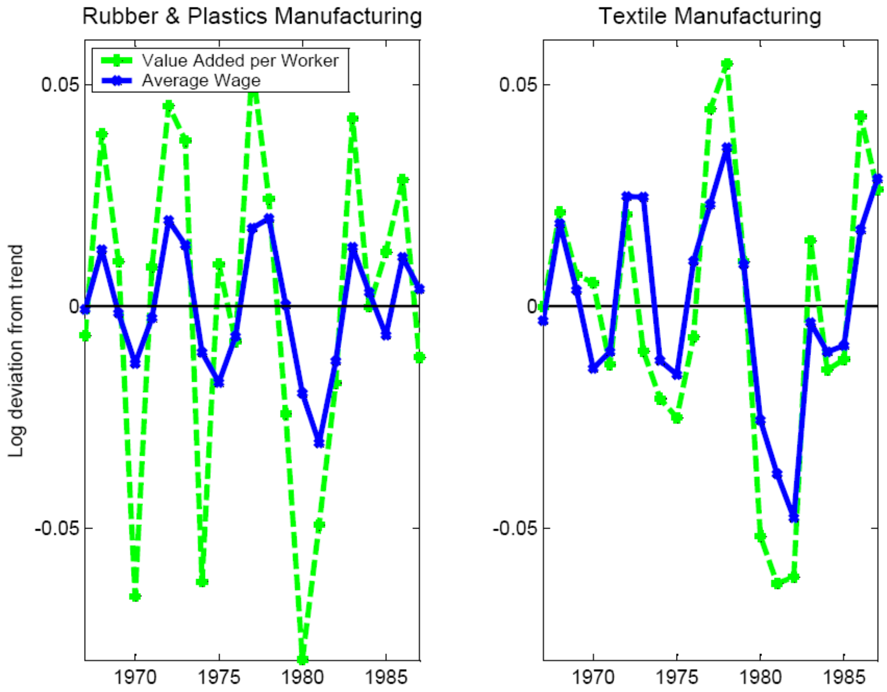
where $w_{j,t}$ and $p_{j,t}$ are average wages and labor productivity in industry j at time t, each expressed in log deviation from trend, and $u_{j,t}$ is an error term. Our wage-smoothing measure ε_j has the interpretation of the elasticity of wages in deviation from trend with respect to productivity in deviation from trend.[7] We refer to this elasticity as the *wage-productivity elasticity* throughout the paper. Industries with wage-productivity elasticities close to one are industries with little wage smoothing, and industries with elasticities close to zero have a high degree of smoothing.

To illustrate this measure of wage smoothing, we plot de-trended wages and productivity in two select US industries in Figure 3.1. Each plot in the figure shows value added per worker (green lines with +'s) and compensation per worker (blue lines with x's) expressed in log deviations from trend. For expositional purposes we selected two industries that display clear differences in their degrees of wage smoothing. The

[6] Here we follow the STAN documentation. See pages 6-9.

[7] We omit the constant term in this regression under the assumption that when productivity is at trend, then wages will be at trend as well.

Figure 3.1: Wage-Productivity Elasticities in Two Select US Industries.



$\varepsilon^{w,p} = 0.18$ $\varepsilon^{w,p} = 0.51$

wage-productivity elasticity in Rubber & Plastics manufacturing is 0.18, indicating a high degree of smoothing, while in Textile manufacturing, the wage-productivity elasticity is 0.51.[8] As is apparent in the figure, wages in textiles are considerably more responsive to a change in productivity than in Rubber & Plastics.

Figure 3.1 presents summary statistics for these elasticities for all industries, as well as for just service industries and just manufacturing industries. We define services here to be all industries in our data set that do not constitute manufacturing, mining, or mining-related industries. The statistics are all weighted by industry employment, which is crucial since industry sizes vary substantially.[9] The first thing to take away

[8] We omit the standard errors of industry elasticities here and elsewhere. We find that they are relatively small: on the order of 0.05 or smaller in almost all cases, without substantial variation across industries.

[9] To save space we omit summary statistics for US industries in the OECD STAN database and for

from this table is that mean (and median) elasticities are roughly comparable across manufacturing and services, with services above the overall average at around 0.5, and manufacturing below at around 0.3.[10] The second finding, and perhaps the more interesting one, is that both services and manufacturing exhibit large variation in elasticities, with standard deviations of around 0.15 and 90-percentile ranges from around 0 to 0.5 in manufacturing and from around 0 to 0.8 in services. These results show that there have been vast differences in wage smoothing across industries of all types over the post-war period. We now turn to the question of which types of industries tend to have the highest degrees of smoothing.

Table 3.1: Summary Statistics for US Wage-Productivity Elasticities, 1947-1987.

Industry Type	**Mean**	**Median**	**Standard Deviation**	**5th Percentile**	**95th Percentile**
All	0.41	0.39	0.18	0.06	0.73
Manufacturing	0.28	0.27	0.12	0.05	0.51
Services	0.47	0.46	0.17	0.07	0.79

3.2.3 High-Wage Industries Have the Most Smoothing

In this section we detail our main empirical finding with regard to wage-productivity elasticities, namely that elasticities tend to be lower in industries with high average

the SIC 1987 industry definition. These statistics were extremely similar.

[10]The first paper we are aware of to measure the wage-productivity elasticity in the US is by Hagedorn and Manovskii [2006], who arrive at an estimate 0.45 for aggregate BLS data. This estimate is entirely in line with our findings for the average US industry in each of our data sets.

wages. We document this pattern first for US industries, using our three different data sets, and then for industries in OECD countries, using the STAN data set. Within the US, we show that this pattern shows up within manufacturing industries as well as within service industries.

Figure 3.2 displays our main result for the US, using the BEA time series from 1947-1987. Each "bubble" on the figure represents one industry, where the size of each bubble is the industry employment weight. The *y*-axis represents the wage-productivity elasticity and the *x*-axis represents the industry average wages in 1987 in thousands of 2005 dollars.[11] The main feature of the graph is the strong negative relationship between industry average wages and elasticities, demonstrating that higher wage industries tend to get the most smoothing. The employment-weighted correlation across industries is -0.53, indicating a robust negative relationship between the industry average wage and the wage-productivity elasticity. We obtained similar results in the STAN dataset for the US, with a correlation of -0.66, and in the shorter BEA series, with a correlation of -0.38.

Figure 3.2: Wage-Productivity Elasticities and Average Wages in US Industries.

[11]We choose 1987 because it is the latest year of our series.

We explore the robustness of this result in two ways. First, we show that the negative correlation between the average wage and the wage-productivity elasticity appears *within* both manufacturing and service industries. Table 3.2 shows the correlations in each data set for just the manufacturing industries, and just the service industries (as well as for the whole economy). In all cases the correlation between the wage productivity elasticity and average wage is negative.

Table 3.2: Correlations of Wage-Productivity Elasticities and Average Wages in US Industries.

Data Source	**Industry Definition**	**Industries**	**Correlation of $\varepsilon^{w,p}$ and w**
OECD STAN, 1970-2000	ISIC, Rev 3.	All	-0.66
		Manufacturing	-0.48
		Services	-0.58
US BEA, 1947-1987	SIC 1972 definition	All	-0.53
		Manufacturing	-0.48
		Services	-0.42
US BEA, 1987-1997	SIC 1987 definition	All	-0.38
		Manufacturing	-0.27
		Services	-0.33

The second way we check robustness is to regress the wage-productivity elasticity on the industry average wage and other salient industry characteristics, using our entire sample of industries as observations. Table 3.3 shows the results of this regression, where the independent variables are a manufacturing dummy plus (the logs of) industry average wages, the volatility of industry productivity, the autocorrelation of industry productivity, and the industry's labor share in value added. As is evident from the regression results, the only industry characteristic that turns out to be statistically significant from zero is the industry wage. Furthermore, it has an economically significant coefficient. A hypothetical doubling of an industry's average wage holding all else constant yields an elasticity that is lower by 0.19, which constitutes roughly 25% of the entire range in elasticities we see in the data. We conclude from the regression results that whatever drives wage smoothing is closely related to the average industry wage.

Next we examine whether this result holds in other OECD countries. Table 3.4 shows the same correlations over other OECD countries in our sample, using the STAN data. Just as for the US, we compute the correlations weighing each industry by total employment in the last year in which data is available. Due to short samples in some countries for some industries, we drop any industry whose elasticity standard error was greater than 0.1. Next, we drop any country with less than 10 industries, so as not to generalize about too few particular industries.[12]

The results for OECD countries largely mimic the US in that there is a negative correlation between the wage level and the wage-productivity elasticity. For 8 countries out of 17, the correlation is below -0.2, indicating a reasonably strong negative relationship, for an additional 6 countries the relationship is weaker, but still negative, and for just 3 of the 17 countries the relationship is positive. The main limitation of this analysis is that the time series are relatively short for most countries, with just 20

[12]While these choices are fairly arbitrary, we find that the results do not change in any important way for other similar choices. For instance when taking 0.05 as our cutoff for the standard error, and 20 for our cutoff on the number of industries, we end up with more industries and fewer countries but the same overall result.

Table 3.3: Regression of Wage-Productivity Elasticity on Industry Characteristics, US Industries.

Independent Variable	**Coefficient (standard error)**
Log(Industry Average Wage, 1987)	-0.19**
	(0.07)
Log(Volatility of Industry Productivity)	-0.07
	(0.09)
Log(Autocorrelation of Industry Productivity)	0.08
	(0.06)
Manufacturing Dummy	-0.08
	(0.05)
Log(Industry Labor Compensation / Value Added)	-0.07
	(0.10)
Constant	0.95*
	(0.56)
R-squared	0.46
Number of Observations (Industries)	52
Dependent Variable: Wage-Productivity Elasticity	
**significant at 5% level, *significant at 10% level	

years of data or less available in most cases.[13]. Even so, at the very least our findings suggest that the negative relationship between elasticities and wage levels is not idiosyncratic to the US, but in fact exists in a number of modern economies.

[13]A more comprehensive comparison of results across OECD and developing countries, using longer time series, would be both useful and interesting, although it is outside of the scope of this paper. In particular, it would be interesting to relate this correlation and the elasticities more generally to differences in labor regulations and unionization rates across these countries, which are likely to play an important role.

Table 3.4: Cross-Industry Correlations of Wage-Productivity Elasticities and Average Wages, OECD Countries.

Country	**Time Period**	**Correlation of $\varepsilon^{w,p}$ and w**
US	1970 - 2001	-0.66
Spain	1986 - 2000	-0.61
Belgium	1980 - 1999	-0.45
Netherlands	1980 - 2000	-0.44
Sweden	1980 - 1999	-0.43
Germany	1991 - 2000	-0.39
Luxembourg	1985 - 2001	-0.36
Austria	1980 - 1999	-0.24
Portugal	1988 - 1999	-0.18
Finland	1975 - 2001	-0.18
Norway	1980 - 1997	-0.13
France	1980 - 2000	-0.08
Japan	1980 - 1998	-0.08
Korea	1980 - 1997	-0.01
Australia	1980 - 1999	0.10
Italy	1980 - 2001	0.11
Denmark	1980 - 2001	0.33

3.2.4 Characteristics of High and Low-Wage Industries

In this section we explore further the characteristics of high-wage industries and low-wage industries. In particular, we show that there are two important dimensions along which high-wage and low-wage industries differ: separation rates, and the extent to which human capital is match specific.

First, separation rates are lower in high-wage jobs. This could be because of the nature of the work, or perhaps that low-wage industries jobs tend to be disproportionately in smaller firms, which have relatively high exit rates. Cite a couple of sources.

Second, a higher fraction of skills are match-specific in high-wage jobs (Jacobson et al. [1993] and Carrington and Zaman [1994]).

Figure 3.3 shows our results. They are for a worker with the economy-wide average tenure and experience in Carrington and Zaman [1994]'s sample. The graph shows that industries with higher average wages have higher average displacement costs. It is important to note one of the major limitations of the Carrington and Zaman [1994] findings is that the elasticities tend to be very imprecisely measured for any given industry. For example the average industry in their sample had a standard error roughly the same size as the point estimate of displacement costs. We should therefore take great caution when interpreting the coefficient of any individual industry.

Finally, it is worth noting that the goal of this paper is not to analyze too deeply the origins of higher separation rates and match-specific capital for high-wage industries, but rather just to take them as given and consider their implications for wage smoothing.

3.3 Model

In this section we develop a model of wage contracting between a worker and firm, and we use the model to demonstrate how match-specific capital and separation rates influence the degree of wage smoothing present in the optimal wage contract. We show that a higher degree of match-specific skills leads to smoother wages, as does a lower probability of an exogenous separation. In the calibration section to follow, we document empirically that a higher fraction of the skills of the high-wage workers are match-specific than the skills of low-wage workers, and that job separation rates are lower for high-wage workers than low-wage workers. The model therefore predicts that high-wage workers will have smoother wages than low-wage workers.

3.3.1 Environment

A risk-averse worker and risk-neutral firm are matched. We assume that the worker prefers higher values of

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{3.2}$$

where E_0 is the expected value operator at time 0, c_t is consumption, and $\beta \in (0,1)$ is the worker's discount factor. The utility function $u(\cdot)$ is assumed to be strictly increasing and strictly concave. Workers are endowed with one unit of labor each period which they supply inelastically to the firms for wage w_t. We abstract from asset markets or storage possibilities, and so the worker's consumption each period equals her wage: $c_t = w_t$.

The firm operates a constant-returns technology that uses labor from one worker as the sole input to produce output y_t. The firm keeps the output, which it sells for a (normalized) price of 1, and pays the worker a wage w_t. The firm prefers higher values of

$$E_0 \sum_{t=0}^{\infty} \beta^t (y_t - w_t) \tag{3.3}$$

A worker matched to a firm can either be skilled in the match, or unskilled in the match. We let $\theta \in \{0,1\}$ represent the possession of skill in the match, where $\theta = 1$ represents a worker possessing match-specific skills and $\theta = 0$ represents a worker without match-specific skills. We let $m \in [0,1]$ represent the fraction of a worker's skills that are match-specific. Finally, let $p \in \mathbb{R}^+$ represent the current realization of productivity in the match; we will explain how productivity evolves in detail below. The production function is given by:

$$y = F(p,m,\theta) = p(1-m(1-\theta)) \tag{3.4}$$

In other words when the worker possesses match-specific skills, output from the match is simply p, whereas without the match specific skills output is $p(1-m)$.

$$F(p,m,\theta) = \begin{cases} p & \text{if } \theta = 1 \\ p(1-m) & \text{if } \theta = 0 \end{cases} \tag{3.5}$$

where m represents match-specific capital and $\overline{\tau}$ represents the amount of tenure required to acquire the capital. We assume that the firm keeps all the output from production and pays the worker a wage in each period. Productivity p is stochastic and takes on one value each period in the set $\mathbb{P} \equiv \{p^1, p^2, \ldots, p^S\}$ where $p^i < p^j$ for $i < j$. We assume that productivity evolves as a first-order Markov chain, where $\alpha_{p',p}$ is the probability of transitioning to state p' from state p.

At the beginning of each period t the productivity state p_t is realized. If they both decide to stay in the match then output y_t is produced, and the worker gets wage w_t. Either party may leave the match after p_t realizes, however, in which case they both get their respective outside options. Let $\overline{\Pi}(p)$ denote the value of the firm's outside option in productivity state p. We assume that if the firm breaks the match it can match up with a new worker, but the new worker does not trust the firm and will only accept a wage equal to her output in each period. In this case the firm will earn zero profits in each period. So $\overline{\Pi}(p) = 0 \; \forall p \in \mathbb{P}$.

The worker's outside option is to join another firm. Let $\overline{V}(p)$ denote the worker's outside option in state p. We assume that if the worker leaves the match she becomes unskilled in the new match for n periods, and only afterwards does she gain match-specific skills in the new match. In addition, the new firm will not trust the worker and will pay her a wage each period exactly equal to her output. We can express her outside option in state p as:

$$\overline{V}(p) = u(p(1-m)) + E\left[\sum_{t=1}^{n-1}\beta^t u(p_t(1-m)) + \sum_{t=n}^{\infty}\beta^t u(p_t)\right]. \tag{3.6}$$

The final component of the environment is the possibility of an exogenous separation, which we assume occurs with probability $s \in [0,1)$ each period. If an exogenous separation occurs in productivity state p then the worker gets $\overline{V}(p)$ and the firm gets $\overline{\Pi}(p)$.

3.3.2 Wage Contracting Problem

We now consider the optimal wage contracting problem in this environment. For now assume that at the initial period the worker is entitled to a particular utility promise v, and the worker begins as skilled in the match. Following Thomas and Worrall [1988] we consider only contracts that are self-enforcing, in the sense that in no state of the world does either party have incentive to break the contract.

Let $\Pi(v,p)$ be the firm's value function given a promised utility v for a worker in productivity state p, which represents the maximized expected discounted profits from the match. The firm's problem can be written as

$$\Pi(v,p) = \max_{w,\{v'(p')\}_{p'}}\left\{p - w + \beta\sum_{p'}\alpha_{p'|p}\Pi(v'(p'),p')\right\} \tag{3.7}$$

subject to a promise-keeping constraint

$$v = u(w) + \beta\sum_{p'}\alpha_{p'|p}v'(p') \tag{3.8}$$

to worker self-enforcement constraints for every future state:

$$v'(p') \geq \overline{V}(p') \;\; \forall p', \tag{3.9}$$

and to firm self-enforcement constraints

$$\Pi(v'(p'),p') \geq \overline{\Pi}(p') \ \ \forall p' \tag{3.10}$$

The self enforcing constraints guarantee that neither party ever wants to leave the contract. Separations only occur exogenously. As in Thomas and Worrall [1988], the optimal wages in the contract will be functions of current and one-period-prior productivities (p, p_{-1}), and the optimal wages will move as little as possible to satisfy the self-enforcing constraints.

Proposition 21 Thomas and Worrall [1988] *Let* (p_{-1}, p, p') *be any productivity history in* $\mathbb{P}\times\mathbb{P}\times\mathbb{P}$*, and let* $w \equiv w(p, p_{-1})$ *and* $w' \equiv w(p', p)$ *be the optimal wage after history* (p, p_{-1}) *and* (p', p)*. Then* w' *and* w *satisfy*

1. $w' > w \Rightarrow v(p') = \overline{V}(p')$
2. $w' = w \Rightarrow v(p') > \overline{V}(p')$ *and* $\Pi(v, p') > \overline{\Pi}(p)$
3. $w' < w \Rightarrow \Pi(v, p') = \overline{\Pi}(p)$

Proof See Appendix 3.6.1. Q.E.D.

Proposition 22 Thomas and Worrall [1988]. *Let* (p_{-1}, p, p') *be any productivity history in* $\mathbb{P}\times\mathbb{P}\times\mathbb{P}$*, and let* $w \equiv w(p, p_{-1})$ *and* $w' \equiv w(p', p)$ *be the optimal wage after history* (p, p_{-1}) *and* (p', p)*. Then*

1. *if* $w' > w$ *then* $v(p') = \overline{V}(p')$
2. *if* $w' = w$ *then* $v(p') > \overline{V}(p')$ *and* $\Pi(v, p') > \overline{\Pi}(p)$
3. *if* $w' < w$ *then* $\Pi(v, p') = \overline{\Pi}(p)$

Proof See Appendix 3.6.1. Q.E.D.

The proposition says that if wages rise from one period to the next, they do so just to the point where the worker is indifferent between staying in the match. Similarly, if wages fall they do so until the firm is indifferent. Finally, if wages stay the same then it must be the case that both parties strictly prefer the match to their respective outside options. In short, wages are smoothed as much as possible such that both parties are willing to stay in the match. This result highlights the fact that the amount of wage smoothing will depend in large part on the outside options for each party.

Also just as in Thomas and Worrall [1988] we have the following corollary about the domain on which optimal wages must lie.

Proposition 23 *For all $p \in \mathbb{P}$ there exists an interval $[\underline{w}_p, \overline{w}_p]$ such that*

1. $w(p, p_{-1}) \in [\underline{w}_p, \overline{w}_p] \; \forall p_{-1}$
2. *when* $w(p, p_{-1}) = \underline{w}_p$ *then* $v = \overline{V}(p)$*, and*
3. *when* $w(p, p_{-1}) = \overline{w}_p$ *then* $\Pi(v, p) = \overline{\Pi}(p)$

This result says that the range of optimal wage always lives in an interval where the worker is indifferent between staying in the contract or not at the lowest wage in the interval, and the firm is indifferent at the highest wage in the interval. This result will be used to greatly simplify the quantitative analysis to come later.

3.3.3 Two-state Version

For the remainder of the paper we consider a two-state version of the model. As we will show below, two features of the productivity process in the model have direct implications for the amount of wage smoothing present in the optimal contract. These are the volatility and autocorrelation of the productivity series. We note that the two-state version of the model will not restrict our quantitative analysis with regards to autocorrelation and volatility: both can be captured with a 2-state Markov chain representing the productivity process.

In this version we consider the set of productivity states to be $\mathbb{P} = \{p_L, p_H\}$ where $p_L < p_H$. Let α be the persistence parameter in the transition matrix. Denote the two intervals described in Proposition 23 by $[\underline{w}_L, \overline{w}_L]$ and $[\underline{w}_H, \overline{w}_H]$ in states p_L and p_H. We immediately get the following two corollaries, which depend on whether or not the two intervals $[\underline{w}_L, \overline{w}_L]$ and $[\underline{w}_H, \overline{w}_H]$ overlap or not.

Corollary 24 *If $\overline{w}_L \geq \underline{w}_H$ then the optimal wages are constant after the first time productivity switches states.*

To see this result, let $\overline{w}_L > \underline{w}_H$, and take an arbitrary initial state (for exposition say p_L) and an arbitrary initial wage w_0 that satisfies Proposition 23. Once the state switches to p_H, we know by Proposition 22 that if $w_0 < \underline{w}_H$ then the wage rises until the worker's self-enforcement constraint binds, i.e. until $w = \underline{w}_H$. But this wage is now incentive compatible in both states, and hence by Proposition 22 it remains constant for all future periods. If on the other hand $w_0 \geq \underline{w}_H$ then it is incentive compatible to both parties in both states to begin with, and hence it remains constant.

Corollary 25 *If $\overline{w}_L < \underline{w}_H$ then the optimal wages (w_L, w_H) after the first time productivity switches states are given by $w_L = \overline{w}_L$ and $w_H = \underline{w}_H$ for all remaining periods.*

The intuition for this corollary is seen as follows. Take an arbitrary initial state (for exposition again say p_L) and an arbitrary wage that satisfies Proposition 23. Once the state switches to p_H, we know by Proposition 22 that the wage rises until the worker's constraint binds, i.e. until $w = \underline{w}_H$. By Proposition 22 again we know that while at p_H the wage remains constant. When p_L realizes the wage must fall until the firm's constraint binds, i.e. until $w = \overline{w}_L$. Similarly, wages remain constant while in p_L. When p_H realizes again we have $w = \underline{w}_H$.

These corollaries describe the two possible types of wage dynamics in the model: perfect smoothing and imperfect smoothing. So far we have said nothing about the initial wage, or alternatively the initial utility promise to the worker. However, the initial split of the surplus is not of particular importance in this model, since the wage dynamics are set as soon as the productivity switches states. Therefore, we focus on the long-run implications of the wage contract, which we define to be after the state has switched at least once. We also focus on the case of imperfect wage smoothing (i.e. $\overline{w}_L < \underline{w}_H$), since this is the empirically relevant case. Now, by Corollary 25, we conclude the wages in the optimal contract are $w_L = \overline{w}_L$ and $w_H = \underline{w}_H$ in *all* periods.

Let $V_L(w_L, w_H) \in \mathbb{R}+$ and $V_H(w_L, w_H) \in \mathbb{R}+$ be the worker's expected discounted utilities in states p_L and p_H under wages (w_L, w_H). Similarly, let $\Pi_L(w_L, w_H) \in \mathbb{R}+$ and $\Pi_H(w_L, w_H) \in \mathbb{R}+$ be the firm's expected discounted profits in the optimal contract in states p_L and p_H. The optimal contract can then be pinned down by the following system of two equations and two unknowns, w_L and w_H:.

$$V_H(w_L, w_H) = \bar{V}(p_H) \tag{3.11}$$

$$\Pi_L(w_L, w_H) = \bar{\Pi}(p_L) = 0. \tag{3.12}$$

We now turn to how wages in the optimal contract depend on fundamentals of the environment, in particular the match-specific capital m, separation rate s, and volatility and autocorrelation of the productivity process.

Proposition 26 *Wage smoothing is increasing in m, i.e.* $\frac{\partial w_H}{\partial m} < 0 < \frac{\partial w_L}{\partial m}$.

The intuition for this result is that losing match specific skills has a first-order negative effect on the value of the worker's outside option in all states. Reducing this outside option allows the firm to obtain lower average wages that are smoother.

Proposition 27 *Wage smoothing is decreasing in s, i.e.* $\frac{\partial w_L}{\partial s} < 0 < \frac{\partial w_H}{\partial s}$.

The separation rate acts to discount future utility promises for both parties, since there is less expected future in the match. But discounting the future of the match more heavily reduces the willingness of either party to sacrifice in the present in exchange for future payoffs. Hence, the worker and firm can only agree upon wages that are relatively close to the worker's marginal product.

The final proposition gives the model's implications for the autocorrelation and volatility of productivity on wage smoothing.

Proposition 28 *Wage smoothing is decreasing in the autocorrelation of productivity* (α)*, i.e* $\frac{\partial w_L}{\partial \alpha} < 0 < \frac{\partial w_H}{\partial \alpha}$*, and increasing in the volatility of productivity* (p_H/p_L).

The intuition for the autocorrelation result is that the higher the autocorrelation, the higher is the worker's value from deviating in the high state, since she can expect to be in the high state for longer. Thus her wage in the high state must be higher to keep her in the contract, and her low-state wage lower to keep the firm in the contract. The volatility result is straightforward: the higher is the volatility of shocks, the worse the value of the worker's outside option relative to the contract, which allows for more smoothing.

Recall that our theory about why high-wage industries had smoother wages than low-wage industries is that high-wage industries have a higher degrees of match-specific skills and lower separation rates than low-wage industries. Our theory predicts that these two differences between high and low-wage industries will result in more wage smoothing in high-wage industries. In the following section we provide empirical support for these two differences between high and low wage industries, and we compare the quantitative implications of the model to the facts documented in Section 3.2.

3.4 Quantitative Analysis

In this section we parameterize the model developed above and assess its quantitative predictions. In parameterizing the model, we also document directly that high-wage industries have a higher degree of match specific capital than low-wage industries and lower separation rates than low-wage industries. While the previous section demonstrated that match-specific capital and separation rates lead *qualitatively* to more smoothing in high-wage industries, this section assess whether the model can match the *quantitative* degrees of wage smoothing found in the data. In particular, we quantify two versions of our model, one to represent the average high-wage industry and one to represent the average low-wage industry. We allow three key differences between these two sectors: to the fraction of skills that are match-specific m, to the separation rate s, and to the productivity series themselves. Quantitative success will be if the calibrated versions can match the average wage-productivity elasticity for low-wage industries, which is 0.34, and the average elasticity for high-wage industries, which is 0.59.

3.4.1 Differences Between High and Low-Wage Industries

The first part of our hypothesis is that a higher fraction of the skills of high-wage workers are match-specific than the skills of low-wage workers. However, measuring the extent to which skills are match specific is not a straightforward exercise. How does one distinguish between match-specific and general skills? A seminal paper by Jacobson et al. [1993] uses worker wage loss after a mass layoff as a proxy for the value of skills that are match specific. They treat mass layoffs as an event unrelated to worker ability, and compare the post-layoff wages of workers that were laid off (leavers) to those that stayed with their respective companies (stayers). If all skills were general, rather than specific to the match, there is little reason to expect that the leavers would have substantial wage reductions after a layoff – they could simply take their (general) skills to a new match and earn a comparable wage as in their old match.

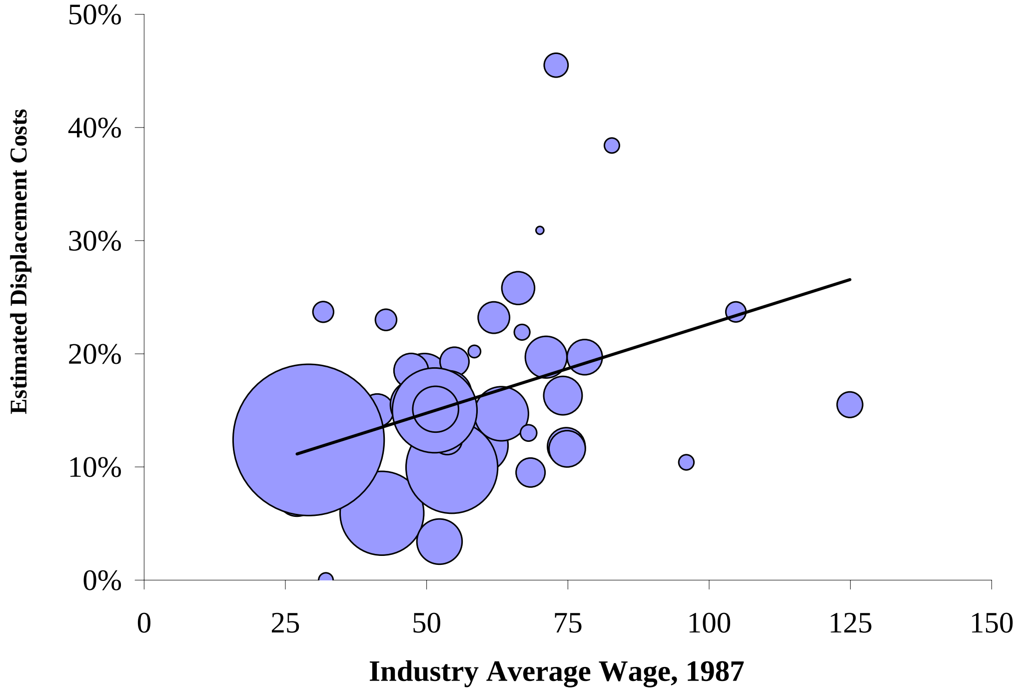
In fact Jacobson et al. [1993] find that wages of stayers and leavers, while almost indistinguishable before the layoff period, depart drastically after the layoff. The wages of stayers stay roughly constant while the wages of leavers fall by around 40% or more. Furthermore, the wages of leavers stay depressed for a long period after their layoff, returning to their pre-layoff wage only after around 4 years. The authors conclude that these drastic layoffs provide direct evidence that match-specific skills form a very large fraction of a worker's total human capital. One caveat here is that the authors consider only the losses of workers with 6 years of tenure or more, who are much more likely to have acquired match-specific skills than workers with lower tenure. In a similar study using different data and a different period, Schoeni and Dardia [1996] corroborate almost all of the findings of Jacobson et al. [1993].[14]

Following the work of Carrington and Zaman [1994] measure the costs of job displacement by industry, using the percentage wage loss after a mass layoff as a measure of match-specific portion of a worker's overall human capital. Fortunately for our study, Carrington and Zaman [1994] conduct their analysis using the same 2-digit industry classification as ours, which allows us to compare their displacement cost estimates to our industry characteristics, especially average industry wages. We present these findings in Figure 3.3. As can be seen on the graph, there is a significant positive correlation between Carrington and Zaman [1994]'s estimates of displacement costs (in percentage terms) by industry and the industry average wage. For low-wage industries, displacement costs run from around 6% to 18% of average wages. On the other hand workers in high-wage industries tend to lose between 10% to 26% of their

[14] For an engaging overview of the literature on worker displacement see Kletzer [1998].

average wage. This finding suggests that the skills of high-wage workers are generally more match specific than low-wage workers.

Figure 3.3: Displacement Costs and Industry Average Wages.



Regarding separation rates, recent estimates by Davis et al. [2006] provide evidence that separation rates are indeed higher in low-wage industries. They construct industry measures of job separation rates using micro data from the Job Opening and Labor Turnover Survey (JOLTS) for 2001-2005, which contains detailed measures of job separations by broad industry groups. They find that the highest separation rates occur in industries with the lowest average wages, such as retail trade and hospitality & leisure. Retailing for example has a monthly separation rate of 3.9%, which is equivalent to a 41% annual rate of separation. On the other hand, high wage industries have relatively lower rates of separation. In manufacturing, for example, the monthly separation rate is 2.7%, which is a yearly rate of 28%.

3.4.2 Calibration & Simulation

We begin our calibration with our measures of match specific capital and separation rates, which correspond to m and s in the model. For m, rather than picking one particular value for high-wage industries and a second value for low-wage industries, we solve each version of the model over the ranges of m observed in the data. The empirically-observed ranges of m are 6% to 18% for low-wage industries and 10% to 26% for high-wage industries. For separation rates, for the low-wage-industry version we take the observed 41% annual rate of separation in retail trade, an industry which had an average wage in 1987 of around 50% of the average industry. For our high-wage-industry version we take the 28% rate of separation observed in manufacturing, which has average wages roughly 50% higher than average in our sample of industries.

Other parameters are calibrated as follows. For the household's preferences we choose CRRA utility with risk-aversion equal to 1, and for the discount factor β we choose 0.95 as is typical in annual data. We choose the length of time the worker is unskilled in a new match following separation to be 4 years as a benchmark, which is consistent with the findings of Jacobson et al and Schoeni and Dardia mentioned above. For the productivity series we estimate, for each industry, an AR(1) process for productivity of the form

$$p_{i,t} = \phi_i p_{i,t-1} + u_{i,t} \tag{3.13}$$

where we assume that $u_{i,t} \sim N(0, \sigma_i^2)$, and where $p_{i,t}$ is the logarithm of detrended productivity in industry i. We take the average $\hat{\phi}_i$ and $\hat{\sigma}_i$ for low-wage industries and high-wage industries and approximate each sector by a 2-state Markov chain. Normalizing long-run productivity to be 1, we end up with states of $p_{H,LW} = 1.042$ and $p_{L,LW} = 0.958$ for the low-wage sector and $p_{H,HW} = 1.063$ and $p_{L,HW} = 0.937$ for the high-wage sector. The persistence parameters for the transition matrices are $\alpha_{LW} = 0.71$ for the low-wage sector and $\alpha_{HW} = 0.69$ for the high-wage industries.

For each sector, we simulate the model over the range of m values described above. For each m, we simulate 10,000 paths for productivity. Each path consists of 1,040 periods where the first 1,000 are discarded to avoid any influence of the initial state, and the next 40 (representing 1947-1987) are kept. For these 40 periods we calculate the wage-productivity elasticity in the same way as in our empirical analysis, and we take the mean elasticity over all 10,000 simulations.

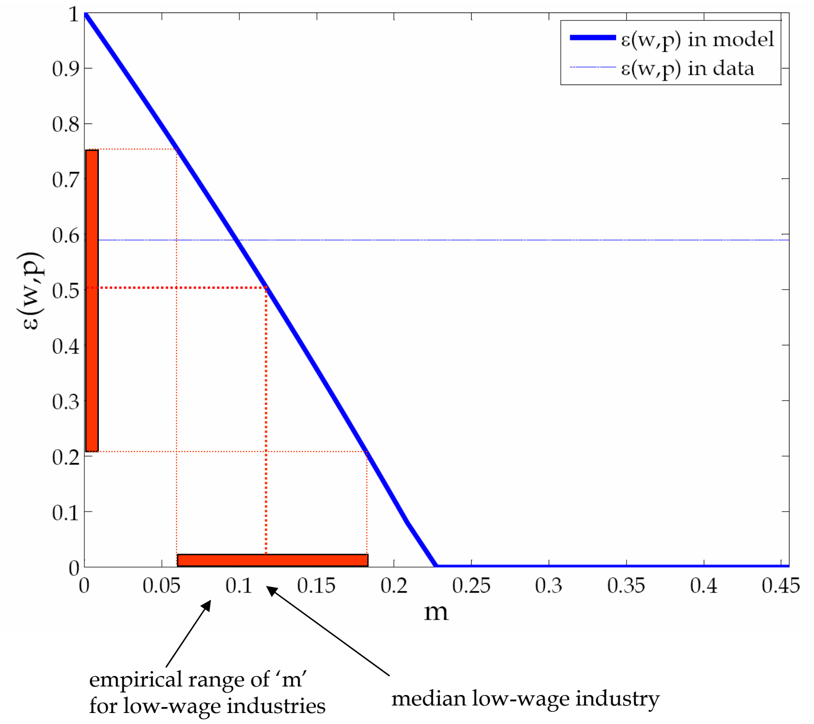
3.4.3 Simulation Results

Our simulations results are shown in figures 3.4 and 3.5. Figure 3.4 shows the results for the low-wage sector. The x-axis represents m and the y-axis represents the elasticities generated by the model and in the data. The blue horizontal line (at $\varepsilon_{w,p} = 0.59$) is the average low-wage elasticity. The thicker downward-sloping blue line is the elasticity generated by the model for each given m. At $m = 0$ the elasticity is 1, meaning that the worker earns his marginal product in every period. At $m = 0.22$ we get an elasticity of 0, or perfect wage smoothing. For intermediate m values we get imperfect smoothing. The red shaded box on the x-axis represent the empirically relevant range of match specific skills taken from Carrington and Zaman [1994]. For the empirically plausible range of m we get elasticities of between 0.2 and 0.75, which are distinguished by the red box along the y-axis. These elasticities are largely in line with the range of low-wage elasticities seen in the data.[15] The median low-wage industry (retail trade) has an estimated m of 0.12 – for this value the model yields an elasticity of 0.5 (shown as a red dotted line), which is close to the true value of 0.59 but too much smooth relative to the data. We will return to this sector shortly.

For the high-wage sector (Figure 3.5) we have an empirically plausible range of m of 10% to 26%, which the model maps into elasticities of 0 to 0.6. These are also

[15] This can be seen in Figure 3.2.

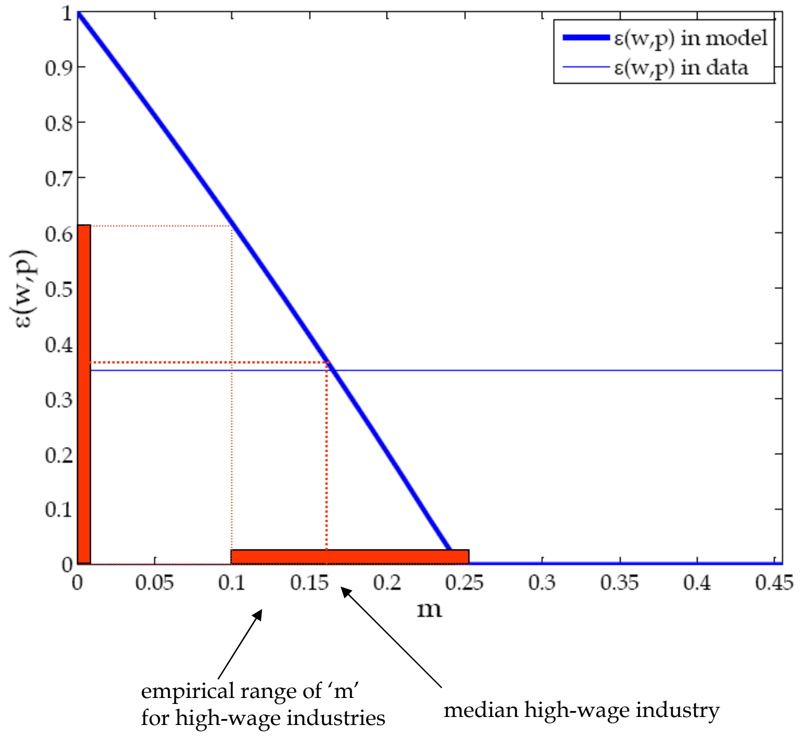
Figure 3.4: Simulated Wage-Productivity Elasticities, Low Wage Sector, n=4.



very much in line with the elasticities found in the data for high-wage industries. The median high-wage industry has *m* of around 16%, which gives an elasticity of 0.37 (shown as a red dotted line), almost exactly the 0.35 seen in the data. We conclude that this baseline version of the model does very well in matching the high-wage sector but predicts a bit too much smoothing in the low-wage sector.

How can we decrease the smoothing in the model low-wage sector? A natural choice is to reduce *n*, the number of years for which the low-wage work is not skilled in a match. This seems sensible, since low-wage jobs generally require lower (specific and general) human capital levels than high-wage jobs. When we decrease *n* to 3 years, we do much better in matching the data. In this case the median low-wage *m* of 12% implies an elasticity of 0.62, which is closer to the empirical value of 0.59. A lower *n* works just as expected.

Figure 3.5: Simulated Wage-Productivity Elasticities, High Wage Sector, n=4.



We conclude from the quantitative portion of the paper that this simple wage contracting model does surprisingly well in matching the facts documented in the empirical section of the paper. This suggests that wage smoothing in the cross-section of industries is well explained by differences in the degree of match specific skills and separation rates.

3.5 Conclusion

In this paper we document a new fact about the cyclical behavior of wages and productivity: in high-wage industries, average wages move relatively little in response to a change in labor productivity, whereas the opposite is true in low-wage industries. In other words, wages are substantially smoother than productivity in high wage industry and considerably less so in lower-wage industries. This finding appears for the US within both manufacturing and service industries and in a majority of OECD countries for which we had data. The finding also appears robust to controlling for other important industry characteristics such as the volatility of industry productivity and the industry labor share in value-added.

Our explanation of this fact is that the response of wages to productivity is determined by an optimal wage contract between a worker and firm under two-sided limited commitment. In high wage industries, high levels of match-specific capital and low separation rates lead to a greater degree of wage smoothing in the optimal contract. We provide direct empirical support for our hypothesis using industry-level data on displacement costs and separation rates, and we formalize our hypothesis in a model based on the Thomas and Worrall [1988] study of wage contracting under limited commitment. We find that the calibrated model performs quite well in explaining the facts at hand, using empirically justifiable measures of match-specific skills and separation rates by industry. Future work will explore the quantitative performance of the model in greater detail.

3.6 Appendix - Proofs of Propositions

3.6.1 Proof of Proposition 22

Fix a state (v, p) and let η be the Langrange multiplier on the promise keeping constraint (3.8). For the worker and firm self-enforcing constraints (3.9) and (3.10) let the multipliers be $\beta\alpha_{p'|p}\lambda_e(p')$ and $\beta\alpha_{p'|p}\lambda_f(p')$. The first order conditions are for choice of w and each $v'(p')$ imply

$$\frac{1}{u_w(w)} = \frac{1}{u_w(w')}(1+\lambda_f(p')) - \lambda_e(p') \qquad \forall p^{'} \tag{3.14}$$

If $w' = w$ then it must be true that $\lambda_f(p') = \lambda_e(p') = 0$, which implies that $v(p') > \overline{V}(p')$ and $\Pi(v, p') > \overline{\Pi}(p)$. If $w' > w$ then $u_w(w') < u_w(w)$ by concavity, which by (3.14) implies that $\lambda_e(p') > 0$ and hence $v(p') = \overline{V}(p')$. By a similar argument $w' < w$ implies that $\lambda_f(p') > 0$, and hence $\Pi(v, p') = \overline{\Pi}(p)$. Q.E.D.

3.6.2 Proof of Proposition 26

First we establish that $\frac{\partial\Pi}{\partial m} > 0$. Note that increasing m has a first-order negative effect on the outside options: $\frac{\partial\bar{V}(p_H)}{\partial m} < 0$ and $\frac{\partial\bar{V}(p_L)}{\partial m} < 0$. With this in mind, imagine that m increases but that the contract wages w_H and w_L stayed the same. Then $V_H(w_H, w_L)$ would remain the same while $\bar{V}(p_H)$ would fall, implying that (3.11) would no longer hold. On the other hand it would still be true that $\Pi_L(w_H, w_L) = 0$, in other words (3.12) would still hold. It follows that leaving that leaving w_L and w_H the same is clearly not optimal when m increases, and more importantly that the firm could reduce average wages in order to reduce $V_H(w_H, w_L)$ and make (3.11) hold once again. With lower average wage, it follows then that $\frac{\partial\Pi}{\partial m} > 0$.

Now there are two logical possibilities to reduce average wages: (1) the firm could reduce w_H while increasing w_L by a smaller magnitude , or (2) it could increase w_H

and reduce w_L by a larger magnitude. We show that (1) is in fact the case. From the definition of Π_L in the imperfect smoothing case we have that

$$\frac{\partial \Pi_L}{\partial m} = -\frac{\partial w_L}{\partial m} + \beta(1-\alpha)(1-s)\frac{\partial \Pi_H}{\partial m}$$

and hence

$$\frac{\partial w_L}{\partial m} = \beta(1-\alpha)(1-s)\frac{\partial \Pi_H}{\partial m} \tag{3.15}$$

using the fact that $\frac{\partial \Pi_L}{\partial m} = 0$ from (3.12). From this we conclude that $\frac{\partial w_L}{\partial m} > 0$. Using the definition of Π_H we have that

$$\frac{\partial \Pi_H}{\partial m} = -\frac{1}{1-\beta\alpha(1-s)}\frac{\partial w_H}{\partial m}$$

Combining this with (3.15) we get that

$$\frac{\partial w_L}{\partial m} = -\frac{\beta(1-\alpha)(1-s)}{1-\beta\alpha(1-s)}\frac{\partial w_H}{\partial m}$$

from which we conclude that $\frac{\partial w_H}{\partial m} < 0$ as we claimed would be the case. Q.E.D.

3.6.3 Proof of Proposition 27

First we show that $\frac{\partial w_L}{\partial s} < 0$. We start with the fact that $\Pi_L = 0$ in the optimal contract no matter what s is, so

$$\frac{\partial \Pi_L}{\partial s} = -\frac{\partial w_L}{\partial s} + \beta(1-\alpha)\left[(1-s)\frac{\partial \Pi_H}{\partial s} - \Pi_H\right] = 0 \tag{3.16}$$

which implies that

$$\frac{\partial w_L}{\partial s} = \beta(1-\alpha)\left[(1-s)\frac{\partial \Pi_H}{\partial s} - \Pi_H\right] = 0.$$

Again by the fact that $0 = \Pi_L = p_L - w_L + \beta(1-s)\Pi_H$ we have that

$$\Pi_H = (\beta(1-\alpha)(1-s))^{-1}(w_L - p_L)$$

and hence that

$$\frac{\partial \Pi_H}{\partial s} = (\beta(1-\alpha)(1-s))^{-1}\left[-(w_L - p_L)/(1-s) + \frac{\partial w_L}{\partial s}\right] \tag{3.17}$$

Combing (3.16) and (3.17) we get that

$$\frac{\partial w_L}{\partial s} = (\beta(1-\alpha))^{-1}\left[-\Pi_H - (w_L - p_L)/(1-s)\right].$$

Since $\Pi_H \geq 0$ and $w_L - p_L > 0$ the left-hand side must be negative, which implies that $\frac{\partial w_L}{\partial s} < 0$ as well.

Second, we show that $\frac{\partial w_H}{\partial s} > 0$. Using the fact that $V_H = \bar{V}_H$ in the optimal contract for any s, it follows that $\frac{\partial V_H}{\partial s}$=0, which gives

$$0 = \frac{\partial V_H}{\partial s} = \beta(1-\alpha)(\bar{V}_L - V_L) + \beta(1-s)(1-\alpha)\frac{\partial V_L}{\partial s} + u_w(w_H)\frac{\partial w_H}{\partial s}$$

and hence

$$\frac{\partial V_L}{\partial s} = \frac{\beta(1-\alpha)(V_L - \bar{V}_L) - u_w(w_H)\frac{\partial w_H}{\partial s}}{\beta(1-\alpha)(1-s)}. \tag{3.18}$$

From the definition of V_L we get

$$\frac{\partial V_L}{\partial s} = \frac{\beta\alpha(V_L - \bar{V}_L) - u_w(w_H)\frac{\partial w_H}{\partial s}}{1-\beta\alpha(1-s)}$$

which can be combined with (3.18) to give

$$(\gamma - \alpha\beta)(V_L - \bar{V}_L) = \gamma u_w(w_H)\frac{\partial w_H}{\partial s} + u_w(w_L)\frac{\partial w_L}{\partial s} \tag{3.19}$$

for $\gamma \equiv (1-(1-s)\alpha\beta)/((1-\alpha)(1-s)\beta)$. It can be shown that $\gamma > 1$, which implies that the left-hand side of (3.19) is greater than or equal to zero. Using our result that $\frac{\partial w_L}{\partial s} < 0$ it follows that in order to satisfy (3.19) that $\frac{\partial w_H}{\partial s}$ must be > 0. Q.E.D.

3.6.4 Proof of Proposition 28

We start with the proof that $\frac{\partial w_H}{\partial \alpha} > 0 > \frac{\partial w_L}{\partial \alpha}$ under imperfect smoothing. Note that increasing α increases the worker's outside option in p_H and reduces it in p_L : $\frac{\partial \bar{V}(p_H)}{\partial \alpha} >$

0 and $\frac{\partial \bar{V}(p_L)}{\partial \alpha} < 0$. As in the proof of Proposition 26, consider an increase in α while the firm hypothetically keep contract wages the same. Then $V_H(w_H, w_L)$ would remain the same while $\bar{V}(p_H)$ would increase, and on the other hand $\Pi_L(w_H, w_L) = 0$. So (3.11) would fail to hold while (3.12) would still hold. It follows that leaving that leaving w_L and w_H the same is not the optimal, and that that the firm would need to raise average wages in order to increase $V_H(w_H, w_L)$ to make (3.11) satisfied. With a higher average wage, it follows then that $\frac{\partial \Pi}{\partial \alpha} < 0$.

We can use the definition of Π_H to get

$$\frac{\partial \Pi_H}{\partial \alpha} = -\frac{1}{1-\beta\alpha(1-s)} \frac{\partial w_H}{\partial \alpha}$$

which tells us that $\frac{\partial w_H}{\partial \alpha} > 0$. From the definition of Π_L and using the fact that $\frac{\partial \Pi_L}{\partial \alpha} = 0$, we get that

$$\frac{\partial w_L}{\partial \alpha} = \beta(1-s)\left[\frac{-1(1-\alpha)}{1-\beta\alpha(1-s)} \frac{\partial w_H}{\partial \alpha} - \Pi_H\right]$$

Since $\Pi_H \geq 0$ we conclude that $\frac{\partial w_L}{\partial \alpha} < 0$, which completes the proof.

Finally, we argue that wage smoothing is increasing in the volatility of shocks, which we capture by p_H/p_L. For brevity we keep this argument short and informal as it follows almost identically the logic of the proof of Proposition 26. Increasing the volatility reduces the worker's outside options in both states, which allows the firm to reap higher profits from the match. Profits are higher the smoother the wages, which means that w_H falls and w_L rises, rather than the other way around. Q.E.D.

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CHAPTER 4

On Scapegoats, Nested Activities and Incentives

Scapegoating is often said to be a source of inefficiency in organizations. In this paper we analyze the consequences of scapegoating within a firm in a model where reputation concerns drive the actions of superiors. Consider delegation choices, for example. The hiring of efficient workers may be a good idea if successful production is the only way to build reputation. But if successful scapegoating also increases reputation, superiors will tend to hire less efficient workers and eventually blame them easily.

We discuss scapegoating as an activity ”nested” after failures. Its results do not directly affect the welfare of society but indirectly affect the decisions governing the probability of success in production. We also examine how activities ”nested” after good results may increase efficiency without relying on costly incentives and why in good times superiors tend to hire better workers than in bad times.

4.1 Introduction

A quick review of newspapers in many countries shows that people condemn scapegoating behavior. This attitude is not only due to its unfairness but also to its negative effects on efficiency and performance in organizations. In fact, this general view has been widely used to justify recent institutional reforms designed to improve efficiency by reducing superior's scapegoating[1]. However, no model has been developed so far to formalize this conventional and seemingly well-accepted wisdom.

In this paper we attempt to understand the impact of scapegoating on efficiency by focusing on delegation decisions made by reputation-concerned superiors. We offer a novel interpretation of scapegoating as an irrelevant activity that only happens after failures and that may be used by reputation-concerned superiors as an additional way to signal their competence.

Given these properties of scapegoating we introduce the idea of ”nested reputation” games as an environment where potentially irrelevant stages can be achieved only after certain situations, helping to build reputation[2]. The existence of these activities will change not only superior's incentives to make decisions but also the outcome of the game.

We introduce both delegation and scapegoating into the reputation environment developed by Mailath and Samuelson [2001]. In the model superiors can be either competents or inepts. Production outcomes are useful elements for consumers to infer

[1]The assignment of more responsibility to superiors has been a main goal of OECD institutional changes over the past decade. Examples are ”Next Steps” and ”Outcome-Output” programs of the UK and New Zealand. Art. 25 (RCSS) of the Rome Conference for an International Criminal Court also criticizes scapegoating from civilian superiors. (Martin [1997]; Polidano [1999]).

[2]To my knowledge, nested reputation models do not exist, constituting this work an initial effort to understand how the results in a standard reputation game change when introducing ”nested” activities as substitutes or complements to the original activity that generates reputation. Even when this paper shares some features with the literature on ”reputation spillover” (as in Cole and Kehoe [1996]), the logic is not the same. Spillovers deal with different and multiple types of reputation. Here reputation is based on a single aspect, but constructed through several nested stages and steps.

the superior's type. Scapegoating is introduced as an additional alternative superiors can use to signal their capability. If blaming is a clearer way to signal competence than production, superiors will prefer to make decisions that exploit blaming, not caring if only production matters to society. In this way superiors will make delegation decisions not only thinking on the way production affects reputation but also on potential scapegoating.

Conditions for an efficient equilibrium with and without scapegoating are compared, concluding that in the former case it is more difficult, and sometimes even impossible, to achieve efficiency as an equilibrium.

The main force driving this result is that hiring experts becomes less attractive when scapegoating is a possibility. First, scapegoating avoids a big decrease in reputation after a failure, reducing the expected gains from working with efficient employees. Second, hiring experts hinders the use of scapegoating to maintain reputation after a failure (since it is harder to blame experts than nonexperts), reducing the expected reputation losses from working with inefficient employees.

Even when focusing on how scapegoating, an activity after failures, negatively affects efficiency, we also extend the logic to study how irrelevant activities that only occur after successes increase the probabilities of achieving efficiency by exploiting reputation forces in the right direction, without requiring monetary resources or costly incentives. Many examples, from areas so diverse as sports and universities, are discussed in the paper.

Finally, we show that considering nested reputation games it is possible to recover a "Machiavellian Effect"[3], which says *"Superiors tend to hire nonexperts in bad times*

[3] In his famous book "The Prince", Machiavelli wrote, *"Princes should delegate to others the enactment of unpopular measures and keep in their own hands the distribution of favours"*. Machiavelli's argument was that princes should delegate when the probability of having a good outcome is low and work by themselves if it is high. In this way princes would be able to blame others if something goes wrong, maintaining their reputation. More recently, Alesina and Tabellini [2005], Alesina and Tabellini

and experts in good times".

Literature on reputation is very large[4] but literature on scapegoating is almost non-existent. Despite the recognition of strategic reasons for scapegoating in the social-psychology literature, (Bell and Tetlock [1989]; Douglas [1995]), formal economic studies of this behavior are new and sparse. Dezso [2004] analyzes the conditions under which random firing of potential innocents (scapegoats) is a reaction to failures in order to maintain reputation. He focuses in firing and not in hiring, without being able to analyze the impact on efficiency. Segendorff [2000] analyzes the possible hiring of scapegoats using a signalling game, without analyzing efficiency consequences either. Winter [2001] finds that, under some circumstances, in order to provide better incentives to top levels in an organization, it may be optimal for middle levels to bear more responsibility, an aspect he labels "scapegoating". He did not consider reputation effects nor hiring decisions though.[5]

In Section 4.2 we present the basic model of reputation with delegation and scapegoating, being an special application of the "nested reputation" environment we are proposing. In Section 4.3 we analyze the conditions for an efficient equilibrium to exist, showing how and when scapegoating leads to inefficiency and to "Machiavellian Effects". In Section 4.4 we propose a way to induce efficiency by exploiting reputation concerns, without relying on the use of costly incentives. Section 4.5 concludes.

[2007a] and Alesina and Tabellini [2007b] formally modeled this pattern among politicians.

[4] Starting with Kreps and Wilson [1982] and Milgrom and Roberts [1982], important contributions on reputation models have been Fudenberg and Levine [1989], Fudenberg and Levine [1992], Mailath and Samuelson [2001] and Cripps et al. [2004].

[5] Empirical studies about scapegoating are even less common. An exception is Huson et al. [2004] who developed a moral hazard-driven scapegoat hypothesis based on agency models to study the impact of managerial succession on firm performance.

4.2 The Model

4.2.1 Description

This model extends Mailath and Samuelson [2001] by introducing delegation and scapegoating as a nested activity.

Assume a superior (a country president, a minister, the owner of a firm or a CEO) who is responsible for providing a service, selling a good or in general achieving a target that generates utility to ”consumers” (who can also be citizens, stockholders, or even upper-level superiors in the hierarchy).

Each period the superior has to make an unobservable delegation decision to achieve the target. He can be one of two possible types, Competent (C) or Inept (I). Competents have two possible choices: To hire experts (E), paying a wage $w > 0$, or to hire nonexperts (N), paying 0. Inepts can only delegate to nonexperts (N)[6].

There are several ways we can rationalize the existence of these two types. The simplest one is to assume workers are able to observe the superior's type and experts just do not work with inepts (who may generate some disutility, such as difficulties for professional improvements). Another possibility is to assume competents have the skills to perfectly identify who is an expert and who is a nonexpert, while inepts do not have access to this screening (or interview) technology[7].

Before deciding, superiors observe the state of the nature, good (G) or bad (B), which affects the probability of success in achieving the target.

[6]It is important to note here that employees in this model will be just dummies who don't take any particular action, behaving basically as machines. Hence, to assume the rent of machines instead of delegation does not change the analysis.

[7]In the latter case, inepts may prefer to hire employees at random (by offering a wage w) if the proportion of experts in the workers' population is high enough. Even in the case they eventually hire experts by following this strategy, the model's conclusions remain unchanged provided some positive proportion of nonexperts exists (the reasons will be clarified below while discussing the model). Hence, for expositional purposes, we will just assume inepts do not have the possibility to attract experts.

After the decision is made, production happens and a non-deterministic output, which can be good (g) or bad (b), is obtained. When competents hire experts the probability of a good result in good times is $(1-\rho) > \frac{1}{2}$ and in bad times $\alpha < \frac{1}{2}$. Hiring nonexperts allows them to obtain good results with a probability $(1-\alpha)$ in good times and ρ in bad times (where $\alpha > \rho$)[8]. An important assumption is that $(\alpha - \rho) > w > 0$, which means it is efficient for society that competents always hire experts, both in good and bad times. It basically says that, if ”consumers” knew agents’ delegation decisions, they would be willing to pay a premium for competents to always hire experts.

When the outcome is finally observed and the result is a failure, the superior has to make a report about its causes, deciding the intensity and amount of evidence to be presented against workers (scapegoating). This is a nested second stage in the game that occurs only after failures in production, not after successes. Once the report is done, a non-deterministic decision about the credibility of the evidence is taken, by a ”court” for example, that concludes whether the employee (e_c) or the superior (s_c) has to be considered the culprit of the failure.[9]

Deciding the intensity of the blaming and the amount of evidence displayed, superiors choose directly the probability of the ”court” blaming the worker[10]. For example, inepts decide a probability x the ”court” pronounces against subordinates such that $x \in [0, \bar{x}]$ where $\bar{x} \leq 1$. This means there can be a maximum capacity to successfully blame workers, or which is the same, maximum blaming intensities do not necessarily guarantee the ”court” deciding against subordinates.

[8]The assumption of symmetry in probabilities does not change the main conclusions but allows the use of just two parameters (α and ρ) instead of four, eliminating awkward expressions.

[9]When referring to a ”court” we are not only thinking on a judiciary court but also in a ”Court of public opinion”, a board of directors or in general any group that decides about the assignment of responsibility by considering the existing evidence.

[10]A nil blaming intensity and no evidence, for example, makes it impossible for the ”court” to decide against the worker. Increasing blaming efforts also increases the probability the ”court” pronounces against employees.

When competents choose a probability the ”court” decides in his favor, they know if the blamed employee is an expert or a nonexpert. If competents worked with experts, they decide a probability y and if they worked with nonexperts they may choose a different one, z. These probabilities will be $y \in [0, \bar{y}]$ and $z \in [0, \bar{z}]$ where $\bar{y} \leq \bar{z} \leq 1$.

The maximum probabilities of successful blaming in all cases ($\bar{x}$, $\bar{y}$ and $\bar{z}$) are exogenous parameters known by everybody in the economy. These parameters basically describe blaming capabilities under maximum blaming intensities.

Finally, at the end of the period, the superior may be replaced by another with a fixed probability λ. The substitute is competent with a probability $\theta \in (0,1)$ [11].

”Consumers” (continuum of identical persons of unit mass such that no single individual can affect the future play of the game) repeatedly receive the output generated under superior’s commands (e.g, consumers purchase a good, citizens receive a service and stockholders obtain dividends). This generates two possible utility levels in each period, 1 if the result is a good outcome ($u(g) = 1$) and 0 if it is a bad outcome ($u(b) = 0$). Each ”consumer” receives the same public result (or signal). ”Consumers” do not get any utility from scapegoating.

Even when ”consumers” know the probability of being in a good state is $\Pr(G) = \gamma$, they are not able to see if the economy is in good or bad times nor if the superior hired experts or nonexperts. ”Consumers” can only see the results from production activities (success or failure) and from blaming activities after failures (superior or employee considered culprit).

From this information they update the probability that the superior is competent, $\Pr(C) = \phi$, (i.e. his or her reputation). This is of the utmost importance to superiors since we assume each ”consumer” has to buy the good or service before production

[11] This assumption is needed to sustain an efficient equilibrium in the long run, as discussed in Mailath and Samuelson [2001] and Cripps et al. [2007].

takes place, hence paying the expected utility and not the real utility it delivers.

The greater the reputation (probability of the superior being competent), the greater the probability assigned by ”consumers” to obtain good outcomes and the more payments they will be willing to make for the good or service. This is the reason superiors are so concerned about reputation while ”consumers” are only concerned about the utility derived from production.

4.2.2 Timing

The timing of the model is:

0) The superior receives the payment for period t, before the production takes place, which only depends on his reputation and not on his period t's true type, delegation decision or production result.

1) The superior observes ϕ, w and the environment state (G or B). Competents decide to hire experts or nonexperts. Inepts can only attract and hire nonexperts. ”Consumers” do not observe this decision, nor whether there are good or bad times.

2) Output is produced and both ”consumers” and the superior observe the true utility given by a good (g) or bad (b) outcome (1 or 0 respectively). All ”consumers” receive the same public realization of utility outcome.

3) The superior has to report the cause of the failure in case of a bad outcome, deciding blaming intensities and how much evidence to present against employees (i.e. x, y or z depending on the type of superior and employee).

4) A ”court” decides if the employee was the culprit (e_c) or if the superior was the culprit (s_c) of the failure.

5) With probability λ the superior is replaced by another one, who is competent with a probability θ.

4.2.3 Definition of Equilibrium

Under uncertainty about superior's type, the state variable is just the probability assigned by "consumers" to the superior being competent (i.e. the reputation denoted as ϕ). Before production, a Markov strategy for competents is a mapping $\tau_k : [0,1] \rightarrow [0,1]$, where $\tau_k(\phi)$ is the probability of hiring an expert when reputation is $\phi \in [0,1]$ and the state of nature is $k \in \{B,G\}$. Inepts make no choice, having then a trivial strategy of hiring nonexperts[12].

After a bad result in production, a Markov strategy for competents that hired experts is a mapping $y : [0,1] \rightarrow [0,\bar{y}]$, where $y(\phi)$ is the probability the "court" decides against the employee when reputation is $\phi \in [0,1]$. We will call this strategy just blaming intensities. The same strategy is available for inepts ($x(\phi)$) and competents who hired nonexperts ($z(\phi)$).

The behavior of "consumers" is described by the Markov belief function $p : [0,1] \rightarrow [0,1]$ where $p(\phi)$ is the probability "consumers" assign to receiving a good outcome, given a reputation $\phi \in [0,1]$ (recall utilities from good and bad results have been normalized to 1).

In a Markov perfect equilibrium superiors maximize profits, "consumers"' expectations are correct and "consumers" use a Bayes' rule to update their posterior probabilities.

Since the state variable is the reputation ϕ, the model relies importantly on the updating of beliefs about the competence of the superior. There are two rounds of updating that follow a Bayes rule: The update after production ($\Pr(C|g)$ and $\Pr(C|b)$) and the potential update ONLY after a bad outcome ($\Pr(C|b,e_c)$ and $\Pr(C|b,s_c)$), which is based on the observation of "court"'s decisions after scapegoating.

[12] As noted in Mailath and Samuelson [2001], by restricting attention to strategies that only depend on consumers' posteriors, in equilibrium different superiors will behave identically in identical situations.

As an example, $\Pr(C|b)$ can be written explicitly in terms of parameters and decision rules as

$$\Pr(C|b) = \frac{\Pr(b|C)\phi}{\Pr(b|C)\phi + \Pr(b|I)(1-\phi)} \tag{4.1}$$

where

$$\Pr(b|C) = \gamma[\rho\tau_G + \alpha(1-\tau_G)] + (1-\gamma)[(1-\alpha)\tau_B + (1-\rho)(1-\tau_B)]$$

$$\Pr(b|I) = \gamma\alpha + (1-\gamma)(1-\rho)$$

Before defining the equilibrium, we need to define the value function for the superior as a function of the reputation value ϕ. For competents,

$$V_k(\phi) = \max_{\tau_k,y,z}\{p(\phi) - \tau_k w + \delta(1-\lambda)E[V(\phi')|\tau_k, y, z]\} \tag{4.2}$$

where $k = \{B, G\}$ and expectation is constructed over possible states of nature and possible reputation levels next period (ϕ', which is a function of delegation and blaming decisions).

For inepts

$$V(\phi) = \max_{x}\{p(\phi) + \delta(1-\lambda)E[V(\phi')|x]\} \tag{4.3}$$

Definition 29 *A Markov perfect equilibrium*[13] *is: Probabilities of hiring experts both in good and bad times ($\tau_G(\phi)$ and $\tau_B(\phi)$), blaming intensities ($y(\phi)$, $x(\phi)$ and $z(\phi)$), probabilities "consumers" assign to receiving a good outcome ($p(\phi)$) given a reputation prior $\phi = \Pr(C)$, and posterior beliefs $\varphi = \Pr(C|R, \phi)$ where R are the three possible results $R \in \{g; (b, e_c); (b, s_c)\}$, such that:*

1) Delegation decisions by competent superiors

[13] *We require behavior to be Markov in order to eliminate equilibriums that depend on implausible degrees of coordination between the superior behavior and "consumers" belief's about that superior behavior. (See discussion in Mailath and Samuelson [1998]).*

$\tau_G(\phi)$ (in good times) and $\tau_B(\phi)$ (in bad times) maximize the value function $V(\phi)$ (eq. 4.2) for all possible reputation values ϕ

2) Blaming intensities by superiors

$x(\phi)$, $y(\phi)$ and $z(\phi)$ maximize the value function $V(\phi)$ (eq. 4.2 and 4.3) for all feasible ϕ

3) Expected utility (and payments) of "consumers"

Probabilities "consumers" assign to receiving a good outcome given a reputation prior ϕ (i.e. Profits for the superior)

$$p(\phi) = \Pr(g|\phi) = \Pr(g|C)\phi + Pr(g|I)(1-\phi) \tag{4.4}$$

4) Beliefs about competence (updated using Bayes rule).

***a)** Update after a good outcome (g)*

$$\varphi(\phi|g) = \phi_g = (1-\lambda)\Pr(C|g) + \lambda\theta \tag{4.5}$$

***b)** Update after the "court" considers the employee responsible for a bad outcome (b, e_c)*

$$\varphi(\phi|b,e_c) = \phi_b^{e_c} = (1-\lambda)\Pr(C|b,e_c) + \lambda\theta \tag{4.6}$$

***c)** Update after the "court" considers the superior responsible for a bad outcome (b, s_c)*

$$\varphi(\phi|b,s_c) = \phi_b^{s_c} = (1-\lambda)\Pr(C|b,s_c) + \lambda\theta \tag{4.7}$$

A strategy for superiors uniquely determines the equilibrium updating rule that "consumers" must use if their beliefs are to be correct.

4.3 Efficient Equilibrium, Inefficient Scapegoating

This paper has a fundamental question. Does scapegoating really reduce the probability of achieving an efficient outcome?

With this question in mind we focus on the conditions for an efficient situation to be sustained as an equilibrium[14]. Considering the assumption $(\alpha - \rho) > w > 0$, efficiency is achieved when competents always hire experts, regardless of their current reputation or whether times are good or bad (i.e. $\tau_G(\phi) = \tau_B(\phi) = 1$, for all feasible ϕ).

The condition for this efficient situation to be sustained as an equilibrium is expressed by a cutoff Δ, such that wages w have to be smaller than Δ. This cutoff is obtained both in good and bad times with scapegoating possibilities (Δ_G^S and Δ_B^S) and without scapegoating possibilities (Δ_G^{NS} and Δ_B^{NS}). The last case is used just as a benchmark to see how results differ when superiors are allowed to blame workers with impunity.

Whenever $\bar{y} > \bar{x}$ and the capabilities of competents to blame nonexperts are high enough (specifically, when a sufficient condition $\bar{z} \geq 1 - \frac{\rho}{\alpha}(1 - \bar{y})$ holds),

$$\Delta_G^{NS} = \Delta_B^{NS} \geq \Delta_G^S \geq \Delta_B^S \tag{4.8}$$

These simple inequalities, which are in fact typically strict, summarize the main conclusions of the paper. Given wages in the economy, the first inequality says that scapegoating makes the condition for an efficient equilibrium $\Delta \geq w > 0$ more difficult to hold. Furthermore it will be shown that $\Delta_G^{NS} = \Delta_B^{NS} > 0$, which means that without

[14]This model has multiple equilibria, including a very inefficient one that may arise without conditions, in which competents only hire nonexperts. Intuitively, if ”consumers” think competents will hire nonexperts they will not update beliefs and competents will optimally prefer never to hire experts, who charge higher wages and, given beliefs, do not represent any additional benefit in terms of reputation.

scapegoating it is always possible to find a positive wage differential that sustains efficiency, which is not necessarily the case with scapegoating.

The second inequality says that it is even more difficult to achieve efficiency with scapegoating in bad times than in good times. This will be called ”Machiavellian Effect”, a feature consistent with many real examples.

4.3.1 Conditions for Efficient Equilibrium

As a first step we present the condition for the existence of an efficient equilibrium without scapegoating, which is not only easier to interpret but also helps to build on intuition.

In this case there are only two possible states (g and b) since there is no blaming activity allowed after a failure (*nobody asks why things went wrong!*). The reputation after a bad draw would be ϕ_b directly. In a similar vein, the reputation after two consecutive bad results ($\varphi(\varphi(\phi|b)|b)$) will be denoted as ϕ_{bb}. The proof is in the Appendix.

Proposition 30 ***Efficient Equilibrium without Scapegoating***

Suppose $\lambda \in (0,1)$, $\phi_0 \in [\lambda\theta, 1-\lambda(1-\theta)]$, $\delta \in (0,1)$ *and* $\theta \in (0,1)$ *In case the report about the causes of the failure is not allowed (no ”blaming” stage), then both in good and bad times there exists a positive cutoff*

$$\Delta^{NS} = \Delta_G^{NS} = \Delta_B^{NS} = \min_{\phi \in [\lambda\theta, 1-\lambda(1-\theta)]} \left\{ \delta(1-\lambda)[X + \delta(1-\lambda)V_f] \right\} > 0 \tag{4.9}$$

such that, for all wages $\Delta^{NS} \geq w > 0$*, the efficient pure strategy profile in which competents always hire experts is a Markov perfect equilibrium.*

where

$$V_f = \Pr(g|E)Y_g + \Pr(b|E)Y_b$$

$$X = (\alpha - \rho)[p(\phi_g) - p(\phi_b)]$$

$$Y_i = (\alpha - \rho)[V(\phi_{gi}) - V(\phi_{bi})] \qquad \textit{for } i \in \{g, b\}$$

with $V(\phi)$ *defined in equation (4.2)*

Since our objective is to compare this benchmark with the extended model that allows for blaming activities, the next proposition presents the conditions to have an efficient equilibrium when scapegoating is a possibility (*people ask why things went wrong!*). The proof is in the Appendix.

Proposition 31 ***Efficient Equilibrium with Scapegoating***

Suppose $\lambda \in (0,1)$, $\phi_0 \in [\lambda\theta, 1-\lambda(1-\theta)]$, $\delta \in (0,1)$ *and* $\theta \in (0,1)$. *In case the report about the causes of the failure is allowed (scapegoating),*

a) If $\bar{y} \leq \bar{x}$, *conditions for an efficient equilibrium are exactly the same as the case without scapegoating (Proposition 30).*

b) If $\bar{y} > \bar{x}$, *there exists a, not necessarily positive, cutoff for each state of the world* $k \in \{B, G\}$

$$\Delta_k^S = \min_{\phi \in [\lambda\theta, 1-\lambda(1-\theta)]} \left\{ \delta(1-\lambda)[X^k + \delta(1-\lambda)V_f^k] \right\} \tag{4.10}$$

such that, for all wages $\Delta_k^S \geq w > 0$, *the efficient pure strategy profile in which competents always hire experts is a Markov perfect equilibrium.*

where

$$V_f^k = \Pr(g|E)Y_g^k + \Pr(b|E)Y_b^k$$

$$X^B = (\alpha - \rho)p(\phi_g) + (1-\alpha)p(\phi_{b,E}) - (1-\rho)p(\phi_{b,N})$$

$$X^G = (\alpha - \rho)p(\phi_g) + \rho p(\phi_{b,E}) - \alpha p(\phi_{b,N})$$

being

$$\phi_{b,E} = \bar{y}\phi_b^{e_c} + (1-\bar{y})\phi_b^{s_c} \tag{4.11}$$

$$\phi_{b,N} = \bar{z}\phi_b^{e_c} + (1-\bar{z})\phi_b^{s_c} \tag{4.12}$$

and, for $i \in \{g, b\}$

$$Y_i^B = (\alpha-\rho)V(\phi_{gi}) + (1-\alpha)[\bar{y}V(\phi_{bi}^{e_c}) + (1-\bar{y})V(\phi_{bi}^{s_c})] - (1-\rho)[\bar{z}V(\phi_{bi}^{e_c}) + (1-\bar{z})V(\phi_{bi}^{s_c})]$$

$$Y_i^G = (\alpha-\rho)V(\phi_{gi}) + \rho[\bar{y}V(\phi_{bi}^{e_c}) + (1-\bar{y})V(\phi_{bi}^{s_c})] - \alpha[\bar{z}V(\phi_{bi}^{e_c}) + (1-\bar{z})V(\phi_{bi}^{s_c})]$$

A couple of features are worth noting before going to the main proposition of the paper. First, it's necessary to emphasize that the non-scapegoating case is just a particular example of the scapegoating case. When $\bar{y} \leq \bar{x}$ both cases are in fact exactly the same. When $\bar{y} > \bar{x}$, as $\bar{x}$, $\bar{y}$, $\bar{z} \to 0$ (maintaining the relation $\bar{z} \geq \bar{y} > \bar{x}$) always $\phi_{b,N} \to \phi_{b,E} \to \phi_b$ (as can be checked easily from equations (4.6), (4.7), (4.11) and (4.12)). Hence $X^B \to X^G \to X = (\alpha-\rho)[p(\phi_g) - p(\phi_b)]$, and $V_f^B \to V_f^G \to V_f$. This is the same as saying that cutoffs in all situations approach each other ($\Delta_B^S \to \Delta_G^S \to \Delta^{NS}$), or that Proposition 31 approaches Proposition 30 as the importance of blaming disappears.

Second, in the differing case ($\bar{y} > \bar{x}$), since w is positive by assumption and there is no way to know the sign of Δ_k^S, it can only be said that whenever $\Delta_k^S < 0$, no wage can possibly support an efficient equilibrium. Even when in the absence of scapegoating there is always a positive wage that supports an efficient equilibrium, this is not necessarily true under scapegoating possibilities. This naturally goes in the proposed direction that scapegoating is harmful for efficiency, which will be formalized in the next subsections.

In the remainder of the paper, and unless stated otherwise, when referring to the scapegoating case we will be referring specifically to the case where $\bar{y} > \bar{x}$, the only interesting case in which scapegoating is a problem.

4.3.2 Scapegoating Inefficiency

Here, the most important conclusion of the paper, the negative impact of scapegoating in achieving efficiency, is derived. The strategy is to prove that conditions for efficiency with scapegoating (when $\bar{y} > \bar{x}$ from Proposition 31) are more difficult to hold than conditions for efficiency without scapegoating (from Proposition 30).

Proposition 32 ***Scapegoating Inefficiency***

Suppose $\lambda \in (0,1)$, $\phi_0 \in [\lambda\theta, 1-\lambda(1-\theta)]$, $\delta \in (0,1)$, $\theta \in (0,1)$ *and competents have better blaming capabilities than inepts (*$\bar{y} > \bar{x}$*). It is always possible to find a* $\bar{z} \geq z^* = 1 - \frac{\rho}{\alpha}(1-\bar{y})$ *such that the range of wages* $w > 0$ *that supports an efficient situation is smaller with scapegoating than without it.*

Proof We need to prove that $\Delta^{NS} \geq \Delta^S_G$ for all $\phi \in (0,1)$. This is enough since the "Machiavellian Effect" Theorem (Proposition 35) ahead will show that always $\Delta^S_G \geq \Delta^S_B$. This proof is based on the simpler case in which scapegoating is not a possibility in the future, only in the current period. The conclusion for the more general case does not varies but it is characterized by awkward statements (shown in the Appendix). We consider only the relevant case in which $\bar{y} > \bar{x}$ and there is a separating blaming equilibrium such that $\phi_b^{e_c} > \phi_b > \phi_b^{s_c}$

We will proceed in three steps. First we will show that $\phi_{b,N} \geq \phi_{b,E}$, second that $\phi_{b,E} \geq \phi_b$ (as defined in Proposition 31) and finally that $\Delta^{NS} \geq \Delta^S_G$ by proving that $X + \delta(1-\lambda)V_f \geq X^G + \delta(1-\lambda)V_f^G$ for all feasible ϕ.

Step 1: *(*$\phi_{b,N} \geq \phi_{b,E}$*)*

Consider beliefs about decision rules in the efficient equilibrium ($\tau_G(\phi) = \tau_B(\phi) = 1$), from equations (4.12), (4.6) and (4.7),

$$\phi_{b,N} = \bar{z}\phi_b^{e_c} + (1-\bar{z})\phi_b^{s_c}$$

$$\phi_{b,N} = (1-\lambda)\left[\bar{z}\frac{\bar{y}\phi_b^{pb}}{\bar{y}\phi_b^{pb} + \bar{x}(1-\phi_b^{pb})} + (1-\bar{z})\frac{(1-\bar{y})\phi_b^{pb}}{(1-\bar{y})\phi_b^{pb} + (1-\bar{x})(1-\phi_b^{pb})}\right] + \lambda\theta$$

and,from equations (4.11), (4.6) and (4.7),

$$\phi_{b,E} = \bar{y}\phi_b^{e_c} + (1-\bar{y})\phi_b^{s_c}$$

$$\phi_{b,E} = (1-\lambda)\left[\bar{y}\frac{\bar{y}\phi_b^{pb}}{\bar{y}\phi_b^{pb} + \bar{x}(1-\phi_b^{pb})} + (1-\bar{y})\frac{(1-\bar{y})\phi_b^{pb}}{(1-\bar{y})\phi_b^{pb} + (1-\bar{x})(1-\phi_b^{pb})}\right] + \lambda\theta$$

where $\phi_b^{pb} = \Pr(C|b)$[15].

Subtracting both expressions.

$$\phi_{b,N} - \phi_{b,E} = \frac{(1-\lambda)\phi_b^{pb}(1-\phi_b^{pb})(\bar{z}-\bar{y})(\bar{y}-\bar{x})}{\bar{x}(1-\bar{x}) + \phi_b^{pb}(\bar{y}-\bar{x})[1-2\bar{x}-\phi_b^{pb}(\bar{y}-\bar{x})]} \tag{4.13}$$

which cannot be negative since $\bar{z} \geq \bar{y} > \bar{x}$ and $\phi_b^{pb} \in [0,1]$.

Step 2: *($\phi_{b,E} \geq \phi_b$)*

Subtracting $\phi_b = (1-\lambda)\Pr(C|b) + \lambda\theta$ from equation (4.11).

$$\phi_{b,E} - \phi_b = (1-\lambda)\left[\bar{y}\frac{\bar{y}\phi_b^{pb}}{\bar{y}\phi_b^{pb} + \bar{x}(1-\phi_b^{pb})} + (1-\bar{y})\frac{(1-\bar{y})\phi_b^{pb}}{(1-\bar{y})\phi_b^{pb} + (1-\bar{x})(1-\phi_b^{pb})} - \phi_b^{pb}\right]$$

which implies

$$\phi_{b,E} - \phi_b = \frac{(1-\lambda)\phi_b^{pb}(1-\phi_b^{pb})^2(\bar{y}-\bar{x})^2}{\bar{x}(1-\bar{x}) + \phi_b^{pb}(\bar{y}-\bar{x})[1-2\bar{x}-\phi_b^{pb}(\bar{y}-\bar{x})]} \tag{4.14}$$

which cannot be negative since $\phi_b^{pb} \in [0,1]$.

Step 3: *($\Delta^{NS} \geq \Delta_G^S$ for all $\phi \in (0,1)$)*

By equations (4.9) and (4.10), it is sufficient to show the following two claims.

[15] Recall $\phi_b^{pb} = \Pr(C|b)$ represents the standard Bayes updating after a bad outcome and before any blaming activity, (superscrip *pb* denotes ”pre blaming”). This is an update not adjusted by λ because it happens before the period ends and a replacement occurs.

Claim 1) $X \geq X^G$ *for all* ϕ.

Subtracting these expressions

$$X - X^G = \alpha[p(\phi_{b,N}) - p(\phi_b)] - \rho[p(\phi_{b,E}) - p(\phi_b)]$$

which is non-negative since $\alpha > \rho$ by assumption and $p(\phi_{b,N}) \geq p(\phi_{b,E})$ for all feasible ϕ by step 1 (equation 4.13) and monotonicity of $p(\phi)$.

Claim 2) $V_f \geq V_f^G$ *for all* ϕ.

Subtracting these expressions

$$V_f - V_f^G = \Pr(g|E)[Y_g - Y_g^G] + \Pr(b|E)[Y_b - Y_b^G]$$

is non-negative if, for $i \in \{g, b\}$

$$Y_i - Y_i^G = (\alpha\bar{z} - \rho\bar{y})[V(\phi_{bi}^{e_c}) - V(\phi_{bi}^{s_c})] - (\alpha - \rho)[V(\phi_{bi}) - V(\phi_{bi}^{s_c})] \geq 0$$

Because of the monotonicity of $V(\phi)$ in ϕ and since, by equation (4.6), $\phi_b^{e_c} \geq \phi_b$, then $V(\phi_{bi}^{e_c}) \geq V(\phi_{bi})$. A sufficient condition for non-negativity is then $(\alpha\bar{z} - \rho\bar{y}) \geq (\alpha - \rho)$, or which is the same,

$$\bar{z} \geq z^* = 1 - \frac{\rho}{\alpha}(1 - \bar{y}) \tag{4.15}$$

where $\frac{\rho}{\alpha}$ is a measure of the relative capability of experts to achieve good production results when compared to nonexperts.

Hence, whenever the sufficient condition $\bar{z} \geq z^*$ holds, regardless of the value function, the likelihood of having an efficient situation reaches its maximum without scapegoating. Q.E.D.

The intuition behind this general result is that reports after a failure when $\bar{y} > \bar{x}$ represent a way for competents to further signal their competence. If this is the case, competents can exploit differences in the blaming capacity as an additional channel to distinguish themselves from inepts.

This can be done in two ways. First by the difference between competents and inepts in maximum blaming capabilities ($\bar{y} - \bar{x}$), which reduces reputation losses after bad results. Second, by the assumed difference between blaming experts and nonexperts ($\bar{z} - \bar{y}$), which may introduce an additional gain from hiring nonexperts (additional to low wages), by increasing the probability that the "court" assigns the responsibility of the failure to the employee.

Now, it is important to put into context the sufficient condition for the inefficiency of scapegoating, $z \geq z^*$.[16] As can be seen, z^* depends on $\frac{\rho}{\alpha}$, a measure of the relative capability of experts to achieve good results in production when compared to nonexperts. If hiring experts almost guarantees success in good times ($\rho \to 0$), then $z^* \to 1$. If hiring experts does not add much to the probability of success ($\rho \to \alpha$), then $z^* \to \bar{y}$.

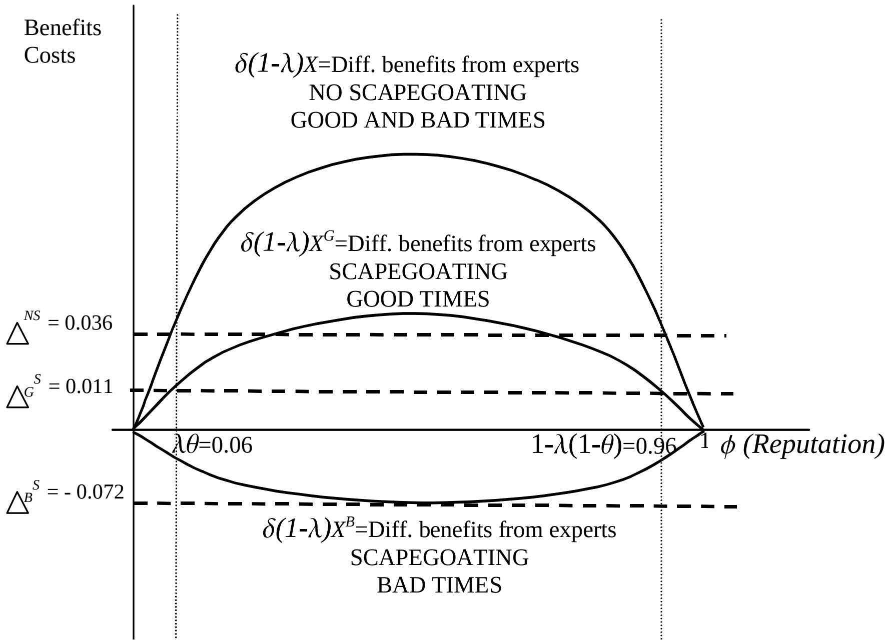
This implies that the sufficient condition $\bar{z} \geq z^*$ is more difficult to hold when hiring experts is really beneficial from a productive point of view, which means superiors can signal their competence directly in the first stage, without the need to go to "court". On the other hand, when hiring experts does not make an important difference in production, competents tend to rely more on the use of scapegoating to signal competence, leading heavily towards inefficiency.

Conditions for an efficient equilibrium in the three cases discussed previously, without scapegoating (both in good and bad times, given by Δ^{NS}), with scapegoating in good times (given by Δ^S_G) and with scapegoating in bad times (given by Δ^S_B) can be easily seen in Figure 4.1. In this case we assumed that $V_f = V_f^G = V_f^B = 0$, which easy computations conservatively biasing results in favor of hiring experts. Even in

[16]This is relevant because we do not know the behavior of the value function. But, for example, if the value function were linear, both Y_g and Y_b would behave exactly as X and scapegoating would always imply inefficiency, regardless of the specific value of $\bar{z}$.

this conservative situation, not allowing for scapegoating (*not asking why things went wrong!*) increases the incentives to efficiently hire experts.[17].

Figure 4.1: Example of conditions for Efficient Equilibrium



4.3.3 The intuition Behind the Inefficiency of Scapegoating

The importance of the blaming report for efficiency resides both in its value to competents to further signal their competence and in its irrelevance to ”consumers”. Since by assumption superiors only care about reputation, decisions will react more to activities that better help them to signal competence. If those activities only occur after particular situations, such as scapegoating only happens after failures, superiors will make decisions trying to achieve these nodes, even when detrimental to activities that really matter to society. This effect will be even more important in the case superiors value reputation beyond the impact on profits (for example, reputation as a career booster).

[17]Parameters used: $\lambda = 0.1$, $\theta = 0.6$, $\delta = 0.99$, $\rho = 0.1$, $\alpha = 0.4$, $\gamma = 0.5$, $\bar{x} = 0.15$, $\bar{y} = 0.3$ and $\bar{z} = 0.85$. The sufficient condition from equation (4.15) holds because in this case $\bar{z} > z^* = 0.825$

Assume for example the extreme case $\bar{x} = 0$ and $\bar{y} = \bar{z} = 1$ (i.e. inepts cannot convince anybody about the blame of employees while competents can always blame convincingly). In this situation, if ”consumers” see that after a failure the ”court” decides against subordinates, they learn for sure the superior is competent, increasing immediately the reputation. Here blaming is better than production for competents to signal their competence. In fact they will prefer to have a failure in order to show their capabilities more effectively through ”court” rather than through production performance, a possibility clearly not allowed after a success. In this very extreme example, competents will never hire experts since nonexperts are not only less expensive but also increase the probability of going to ”court”.

Naturally, the previous extreme example is consistent with the case $\bar{y} > \bar{x}$. But what happens if $\bar{y} \leq \bar{x}$? As shown formally, in this case blaming is useless to signal competence and competents will not behave differently than without scapegoating. This result is a version of a cheap talk game. Inepts always want to pool with competents' strategies, adjusting his blaming efforts downwards (say not presenting proofs, burning evidence against employees, etc.)[18]. Potentially, competents can always be imitated by inepts, not being an equilibrium the use of the blaming report to impose a new updating round after production.

The whole action in previous propositions and proofs comes from the comparison of reputation competents expect to obtain from hiring experts as opposed to hiring nonexperts.

Without scapegoating, the reputation conditional on first round's results is known and given by ϕ_g after a success and ϕ_b after a failure.

With scapegoating, while the expected reputation after a good outcome is also

[18]Blaming is an activity, as are many others, where the success probability can be easily adjusted downwards (just being lazy) but not upwards.

independent of the hiring decision, ϕ_g (because the game ends after a success), the expected reputation after bad results depends on the hiring decision (since it's easier to blame nonexperts).

The expected reputation after a failure when hiring experts is $E(\Pr(C|b)|E) = \phi_{b,E} = \Pr(e_c|E)\varphi(\phi|b,e_c) + \Pr(s_c|E)\varphi(\phi|b,s_c)$ which is the expression in equation (4.11). Similarly, the expected reputation after a failure when hiring nonexperts, $\phi_{b,N}$, was defined in equation (4.12).

The differential gains in reputation expected from good results in production can be seen as a measure of the incentives to hire experts, since this decision increases the probabilities of success. These gains can be represented by $(\phi_g - \phi_b)$ without scapegoating (regardless of the hiring decision), by $(\phi_g - \phi_{b,E})$ with scapegoating if the decision is to hire experts and by $(\phi_g - \phi_{b,N})$ with scapegoating if the decision is to hire nonexperts.

Hence, to understand how incentives to efficiently hire experts behave we need to understand how $\phi_{b,N}$, $\phi_{b,E}$ and ϕ_b relate to each other.

As was shown in steps 1 and 2 of Proposition 35's proof, there is a clear ordering between these expressions when $\bar{z} > \bar{y} > \bar{x}$.

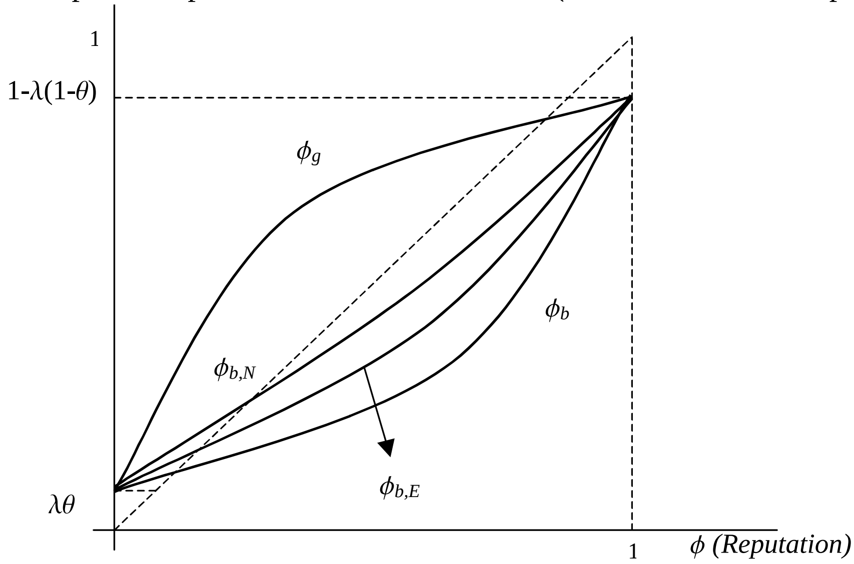
$$\phi_g > \phi_{b,N} > \phi_{b,E} > \phi_b \tag{4.16}$$

Graphically, Figure 4.2 shows the updated reputation value for each case and for each possible reputation prior ϕ.

It is straightforward to check that all possible beliefs' updates are equal when ϕ is either zero or one. If the prior is $\phi = 0$, the update in all cases is $\lambda\theta$. If the prior is $\phi = 1$, the update is $1 - \lambda(1-\theta)$. For all other values $\phi \in (0,1)$, reputation updates have the ordering shown by relation (4.16) and Figure 4.2.

The difference $(\phi_{b,E} - \phi_b)$ in equation (4.13) can be interpreted as the reduction in

Figure 4.2: Expected reputation after the first round (with and without scapegoating)



the incentives to hire experts and the difference $(\phi_{b,N} - \phi_{b,E})$ in equation (4.14) can be seen as the increase in the incentives to hire nonexperts.

While the expression $(\bar{y} - \bar{x})$ in the numerator of equations (4.13) and (4.14) shows the magnitude of reputation maintenance due to scapegoating, the expression $(\bar{z} - \bar{y})$ on the numerator of (4.13) shows the additional benefits from hiring nonexperts by taking advantage of scapegoating after a failure.

The following lemmas show that both $(\phi_{b,N} - \phi_{b,E})$ and $(\phi_{b,E} - \phi_b)$, not only are positive (as shown in steps 1 and 2 of Proposition 32's proof) but also depend positively on the blaming abilities' gaps.

Lemma 33 *The difference between expected reputation after a failure from hiring experts versus hiring nonexperts $(\phi_{b,N} - \phi_{b,E})$ is non-decreasing in $(\bar{z} - \bar{y})$ nor in $(\bar{y} - \bar{x})$*

Proof Taking the derivative of expression $\left(\phi_{b,N} - \phi_{b,E}\right)$ in equation (4.13) with respect to $(\bar{z}-\bar{y})$ and with respect to $(\bar{y}-\bar{x})$.

$$\frac{\partial[\phi_{b,N}-\phi_{b,E}]}{\partial(\bar{z}-\bar{y})}=\frac{(1-\lambda)\phi_b^{pb}(1-\phi_b^{pb})(\bar{y}-\bar{x})}{\bar{x}(1-\bar{x})+\phi_b^{pb}(\bar{y}-\bar{x})[1-2\bar{x}-\phi_b^{pb}(\bar{y}-\bar{x})]^2}\geq 0$$

$$\frac{\partial[\phi_{b,N}-\phi_{b,E}]}{\partial(\bar{y}-\bar{x})}=\frac{(1-\lambda)\phi_b^{pb}(1-\phi_b^{pb})(\bar{z}-\bar{y})[\bar{x}(1-\bar{x})+\phi_b^{pb}(\bar{y}-\bar{x})^2]}{\left[\bar{x}(1-\bar{x})+\phi_b^{pb}(\bar{y}-\bar{x})[1-2\bar{x}-\phi_b^{pb}(\bar{y}-\bar{x})]^2\right]^2}\geq 0$$

The two expressions are strictly positive when $\bar{z}>\bar{y}>\bar{x}$ and $\phi_b^{pb}\in(0,1)$. Q.E.D.

Lemma 34 *The difference between expected reputation after a failure in cases with and without scapegoating, ($\phi_{b,E}-\phi_b$) is non-decreasing in ($\bar{y}-\bar{x}$)*

Proof For this proof just consider the difference $\phi_{b,E}-\phi_b$ in equation (4.14) since, as shown in Lemma 33, $\frac{\partial[\phi_{b,N}-\phi_{b,E}]}{\partial(\bar{y}-\bar{x})}\geq 0$. Taking derivatives of $(\phi_{b,E}-\phi_b)$ with respect to $(\bar{y}-\bar{x})$

$$\frac{\partial[\phi_{b,E}-\phi_b]}{\partial(\bar{y}-\bar{x})}=\frac{(1-\lambda)\phi_b^{pb}(1-\phi_b^{pb})^2(\bar{y}-\bar{x})[2\bar{x}(1-\bar{x})+(1-2\bar{x})\phi_b^{pb}(\bar{y}-\bar{x})]}{\left[\bar{x}[(1-\bar{x})+\phi_b^{pb}(\bar{y}-\bar{x})[1-2\bar{x}-\phi_b^{pb}(\bar{y}-\bar{x})]^2\right]^2}\geq 0$$

which is non-negative because in the numerator, $(1-\bar{x})\geq(\bar{y}-\bar{x})\geq\phi_b^{pb}(\bar{y}-\bar{x})$. This is also strictly positive whenever $\bar{z}>\bar{y}>\bar{x}$ and $\phi_b^{pb}\in(0,1)$. Q.E.D.

The difference in the blaming abilities between competents and inepts $(\bar{y}-\bar{x})$ basically measures the drop in expected reputation that, thanks to scapegoating, does NOT occur after a failure. Hence, an increase in $(\bar{y}-\bar{x})$ not only reduces the incentives to hire experts (by increasing $\phi_{b,E}-\phi_b$) but also makes more beneficial to hire nonexperts (by increasing $\phi_{b,N}-\phi_{b,E}$).

Similarly, the difference in the abilities between blaming experts and nonexperts $(\bar{z}-\bar{y})$ measures the greater probability of having a positive "court" decision against employees from hiring nonexperts. Hence an increase in $(\bar{z}-\bar{y})$ makes even more beneficial to hire nonexperts (by further increasing $\phi_{b,N}-\phi_{b,E}$).

4.3.4 Machiavellian Effect

The next proposition shows that in bad times an efficient outcome is more difficult to arise than in good times.

Proposition 35 ***"Machiavellian Effect"***

Suppose $\lambda \in (0,1)$, $\phi_0 \in [\lambda\theta, 1-\lambda(1-\theta)]$, $\delta \in (0,1)$ *and* $\theta \in (0,1)$, *blaming reports are allowed and competents have better blaming capabilities than inepts (*$\bar{y} > \bar{x}$*). If there exist some* $w > 0$ *such that competents decide to hire experts in bad times, then they also decide to hire experts in good times, while the contrary is not necessarily true.*

Proof We need to show that $\Delta_G^S \geq \Delta_B^S$ by proving $X^G + \delta(1-\lambda)V_f^G \geq X^B + \delta(1-\lambda)V_f^B$ for all $\phi \in (0,1)$. Considering equation (4.10) it suffices to show the following two claims,

Claim 1) $X^G \geq X^B$ *for all* ϕ.

Subtracting these expressions

$$X^G - X^B = (1-\alpha-\rho)[p(\phi_{b,N}) - p(\phi_{b,E})]$$

which is non-negative since $\alpha + \rho < 1$ by assumption; $p(\phi)$ is monotonic in ϕ and by equation (4.13) $\phi_{b,N} \geq \phi_{b,E}$.

Claim 2) $V_f^G \geq V_f^B$ *for all* ϕ

Subtracting these expressions

$$V_f^G - V_f^B = \Pr(g|E)[Y_g^G - Y_g^B] + \Pr(b|E)[Y_b^G - Y_b^B]$$

which is non-negative since $Y_i^G - Y_i^B = (1-\alpha-\rho)(\bar{z}-\bar{y})[V(\phi_{bi}^{e_c}) - V(\phi_{bi}^{s_c})] \geq 0$ for $i \in \{g, b\}$. This is because $\alpha + \rho < 1$ and $\bar{z} \geq \bar{y}$ by assumption, $V(\phi)$ is monotonic in ϕ and $\phi_b^{e_c} \geq \phi_b^{s_c}$.

Assuming scapegoating also in the future does not change the conclusion.

Hence, in good times experts are hired for a wider range of wages w than in bad times. This does not imply a positive cutoff Δ_k^S, but it does imply it is more likely to have $\Delta_G^S > 0$ rather than $\Delta_B^S > 0$ and then, that efficiency be achieved in good times but not in bad times. This is what we called "Machiavellian Effect". Q.E.D.

Reputation concerns and scapegoating rationalize this "Machiavellian Effect" since in bad times superiors are more worried about potential reputation losses rather than potential reputation gains. The intuition behind the Proposition is that, even when in good and bad times differences in probabilities to obtain a good outcome $(\alpha - \rho)$ are the same[19], the probability of having a bad outcome is greater in bad times than in good times (in our model, greater than half). Furthermore, "consumers" do not know the state of the nature Under scapegoating this is important because of the possibility to avoid a big reduction in reputation if a failure in fact occurs, hence making more attractive the hiring of nonexperts who can be blamed easily, exactly as proposed by Machiavelli.

4.4 Ways to increase the likelihood of an efficient equilibrium (without spending more money!)

Up to this point we conclude that scapegoating generates inefficiencies because of its location after bad results in the nested reputation game. Then, a natural question arises. What happens with activities nested after successes?

Many examples of this kind of situations can be found in real life. In the sporting arena, All-Star Games, national teams and international championships (such as the

[19] This is just an assumption to clarify the "Machiavellian Effect" as much as possible. To assume otherwise does not change the main conclusion.

Soccer World Cup) are organized for the best players to participate. In organizations, corporations and public offices, additional funds and responsibilities are assigned to divisions that outperform. In academic environments, round tables and plenary sessions at professional meetings are held by top researchers. In the show business, TV shows invite successful music and movie stars to exhibit their charisma or being funny.

All these situations share the characteristic that persons who are successful at their main professions gain access to additional activities in order to signal their competence even further. Even when society cares more on their main activities and not so much on these additional events, activities nested after successes may be very important to introduce the right incentives, increasing the likelihood to reach efficiency.

4.4.1 Efficiency of nested activities after successes.

The model can be easily reinterpreted and modified to introduce nested activities after successes. Assume that instead of an irrelevant activity nested after a failure, such as scapegoating, the game is characterized by an irrelevant activity nested after successes (think about any of the previous examples). The structure of probabilities, timing and parameters for the production stage have the same interpretation as before.

The difference appears in the second stage. After bad results the game ends but after good results there is a nested activity, which at the time can be a success (g,s) or a failure (g,f) (these have basically the same spirit than (b,e_c) and (b,s_c) in the original model). Using the same notation as before we can write $\Pr(s|I,g,N) = x_g \in [0,\bar{x}_g]$, $\Pr(s|C,g,E) = y_g \in [0,\bar{y}_g]$, $\Pr(s|C,g,N) = z_g \in [0,\bar{z}_g]$, which means some differences in the capabilities of being successful at the nested activity may exist[20].

[20]Many of the examples discussed do not need delegation. In fact, in a better description competents would decide between exerting high or low efforts while inepts would only be able to exert low efforts. This alternative environment, even the same in spirit, is different in that superiors would need to choose the effort level both before the first and second rounds and pay twice the effort costs. Introducing this modification does not substantially change the main conclusion, though.

For example, if $\bar{y}_g > \bar{z}_g$, hiring experts not only increases the probability of being successful at producing but also at the additional nested activity. Superiors may choose the probabilities of being successful at the nested activity by eventually boycotting worker's efforts. If there is no boycott, the probability of success would be the maximum achievable (say $y_g = \bar{y}_g$ if the employee is an expert) and a maximum boycott intensity by the superior would eliminate the probability of success in the nested activity ($y_g = 0$). This decision about the boycotting intensity has the same logic that decisions about blaming intensities in the original model with scapegoating.

The equilibrium definition in this environment is the same as before with the difference that the three possible updating (in place of equations (4.5)-(4.7)) are now ϕ_b (after a bad outcome), ϕ_g^s (after successes both in the first and second rounds) and ϕ_g^f (after success in production and failure in the nested unproductive activity).

After a good outcome, two updates can occur, either $\Pr(C|g,s)$ or $\Pr(C|g,f)$. It is straightforward to show that in the efficient equilibrium, for all ϕ, after the first round $\Pr(C|g) > \Pr(C) > \Pr(C|b)$ and after the potential second round $\Pr(C|g,s) > (<)\Pr(C|g) > (<)\Pr(C|g,f)$ if $y_g > (<) x_g$

As in the scapegoating situation, only when $\bar{y}_g > \bar{x}_g$ may the nested stage generate a new reputation updating and affect efficiency[21]. In what follows, unless stated otherwise, we consider only the relevant case $\bar{y}_g > \bar{x}_g$.

As in equations (4.13) and (4.14) it's also possible to define expected reputation after good results in case of hiring experts and in case of hiring nonexperts. Equations (4.13) and (4.14) could be restated as,

$$\phi_{g,E} - \phi_{g,N} = \frac{(1-\lambda)\phi_g^{pb}(1-\phi_g^{pb})(\bar{y}_g - \bar{z}_g)(\bar{y}_g - \bar{x}_g)}{\bar{x}_g(1-\bar{x}_g) + \phi_g^{pb}(\bar{y}_g - \bar{x}_g)[1 - 2\bar{x}_g - \phi_g^{pb}(\bar{y}_g - \bar{x}_g)]} > 0 \qquad (4.17)$$

[21] If $\bar{y}_g \leq \bar{x}_g$ inepts prefer to boycott the probability of success at the nested activity, imitating competents in order to be confused with them. In this way inepts would not signal their own ineptitude (this is the same logic explained in Section 4.3.3).

$$\phi_{g,E} - \phi_g = \frac{(1-\lambda)\phi_g^{pb}(1-\phi_g^{pb})^2(\bar{y}_g - \bar{x}_g)^2}{\bar{x}_g(1-\bar{x}_g) + \phi_g^{pb}(\bar{y}_g - \bar{x}_g)[1 - 2\bar{x}_g - \phi_g^{pb}(\bar{y}_g - \bar{x}_g)]} > 0 \tag{4.18}$$

where $\phi_g^{pb} = \Pr(C|g)$.

In this case it's also informative to obtain the difference $\phi_{g,N} - \phi_g$

$$\phi_{g,N} - \phi_g = \frac{(1-\lambda)(\phi_g^{pb})(1-\phi_g^{pb})(\bar{y}_g - \bar{x}_g)[(\bar{z}_g - \bar{x}_g) - \phi_g^{pb}(\bar{y}_g - \bar{x}_g)]}{\bar{x}_g(1-\bar{x}_g) + \phi_g^{pb}(\bar{y}_g - \bar{x}_g)[1 - 2\bar{x}_g - \phi_g^{pb}(\bar{y}_g - \bar{x}_g)]} \tag{4.19}$$

which is positive only for $\phi_g^{pb} < \frac{\bar{z}_g - \bar{x}_g}{\bar{y}_g - \bar{x}_g}$.

A clear ordering exists among these expressions when $\bar{y}_g > \bar{x}_g$, as drawn also in Figure 4.3 (similar to Figure 4.2).

$$\phi_{g,E} > \phi_{g,N} > \phi_b \quad \text{and} \quad \phi_{g,E} > \phi_g > \phi_b \tag{4.20}$$

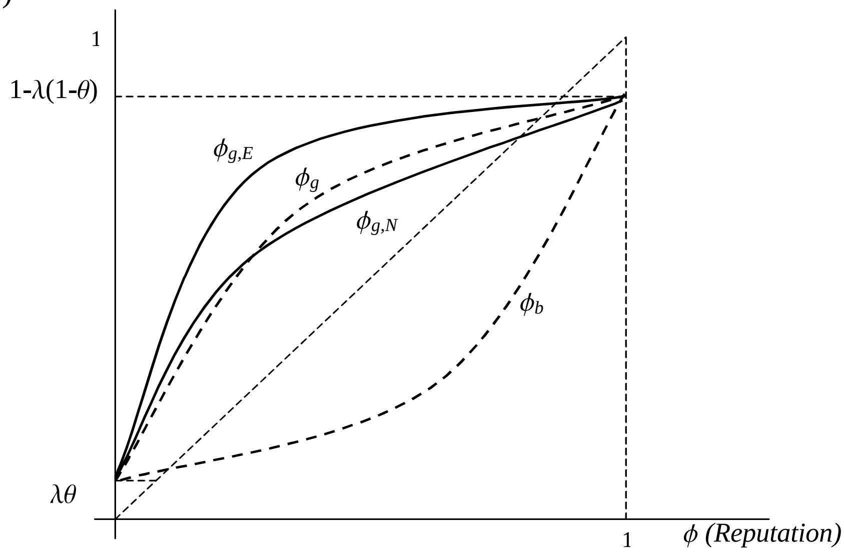
and, even when not relevant for our results,

$$\phi_{g,N} > \phi_g \quad \text{when} \quad \phi_g^{pb} < \frac{\bar{z}_g - \bar{x}_g}{\bar{y}_g - \bar{x}_g}$$

Hence hiring experts increases expected reputation after good results ((4.18) is positive). Furthermore, hiring nonexperts decreases expected reputation after good results ((4.19) is negative) for relative high reputation levels. The final effect is always an increase in gains from hiring experts.

The obvious difference between nested activities after failures (such as scapegoating) and the situation explained here is that, while the former reduces the expected reputation gains from successes (from $(\phi_g - \phi_b)$ to $(\phi_g - \phi_{b,E})$) and decreases the incentives to hire experts, the later increases the expected reputation gains from successes (from $(\phi_g - \phi_b)$ to $(\phi_{g,E} - \phi_b)$) and increases the incentives to efficiently hire experts. Even more, while the former increases the incentives to hire nonexperts (by $(\phi_{b,N} - \phi_{b,E})$), the later decreases them (by $(\phi_{g,E} - \phi_{g,N})$).

Figure 4.3: Expected reputation after the first round (with and without activities after successes)



While Figure 4.3 delivers the basic intuition that sustains efficiency from nested activities after good results when $\bar{y}_g > \bar{x}_g$, formal proofs are very similar to Propositions 30 and 31's proofs. Since, contrary to the scapegoating case, here it makes more sense to assume $\bar{y}_g > \bar{z}_g$, the unproductive stage after a good outcome will always increase the likelihood of achieving an efficient equilibrium, without requiring a sufficient condition.

Finally, it is very interesting to note that "Machiavellian Effects" persist also in this case, meaning there are more incentives to hire experts in good times than in bad times. The probability of being successful and reaching the nested activity is, in absolute terms, greater in good times than in bad times, increasing even more the incentives to hire experts.

In these sections we have shown the importance of nested activities in influencing incentives, to understand superiors' decisions and to explain efficiency consequences. In general, the introduction of nested activities after successes seems to be better than after failures in order to promote efficient decisions. The only condition is a positive correlation between abilities to be successful, both in the productive and not so productive nested activities.

Many real life situations can be rationalized from this point of view, which means it is not only a theoretical curiosity. Furthermore it is possible to think about extremely irrelevant activities that may be useful to promote efficient results just by exploiting reputation concerns, without requiring costly incentives or monetary resources.[22]

4.5 Conclusions

Scapegoating is a common behavior in public institutions, firms, sports and even in the Army. The problem is not only the redistribution effects unfair blaming may generate but also the inefficiencies in the performance of organizations it may introduce. In fact, this conventional wisdom has been the main argument for last decade's reforms designed to assign more responsibility to superiors, reducing their chances to blame subordinates.

Inefficiencies may arise because of imperfect information. Reputation concerns may align interests, making superiors to decide actions that are preferred by the society, achieving the best possible outcome at the lowest possible cost.

However, scapegoating corrupt the nice features of reputation, making harder to achieve efficiency since it attenuates potential losses of reputation after failures, reducing the incentives to make costly decisions conducive to obtaining good results, such as hiring experts. Furthermore, scapegoating in fact increases the incentives to hire nonexperts in order to blame them easily if something goes wrong.

[22] An extreme but funny example is the following one. Suppose that hiring experts is efficient but incentives from production are not enough for superiors to do it. Assume also experts heavily outperform nonexperts at playing chess. A cheap way to achieve efficiency would be to introduce a chess game right after successes in production !

To formalize this idea, we defined scapegoating as a non-productive blaming activity ”nested” after a bad result and introduced it as an extension of a reputation model. This ”nested” activity may represent an additional way for superiors to signal their competence. Depending on whether production or blaming is a better reputation builder, incentives to hire efficient workers will be affected. If blaming is a more secure way to build reputation, scapegoating reduces the incentives to hire experts, making the efficient situation more difficult to be sustained as an equilibrium.

Exploiting this nested reasoning, it may be better to locate activities after successes rather than after failures. If society only cares about the results in the first stage, it can be a good idea to introduce, right after positive results, activities in which competents outperform inepts. This will give superiors more incentives to achieve good results in the first stage in order to obtain the right to access next stages and signal their competence even better.

The model also delivers an interesting feature observed in reality, named here as ”Machiavellian Effect”, in which competents tend to hire experts more in good times than in bad times.

Since our interest in this model is to present the signaling features of scapegoating by the interaction between superiors and consumers, we abstract from the more subtle interaction between superior and employee. Even when out of the scope of this paper, we think it is important to fully understand the scapegoating phenomena.

Obviously these conclusions should be taken carefully. This model just focuses on one particular delegation motive, which is reputation, leaving out other important reasons such as specialization and scale. This is why conclusions are biased towards the assignment of complete responsibility to superiors. A more comprehensive model, considering all determinants, would be necessary to obtain the optimal allocation of responsibility and accountability to superiors.

4.6 Appendix

4.6.1 Proof Proposition 30

Fix ϕ and suppose an efficient situation (i.e., competent superiors always choose to hire experts both in good and bad times ($\tau_G(\phi) = \tau_B(\phi) = 1$)). It is straightforward to show that for all ϕ, after the first round $\Pr(C|g) > \Pr(C) > \Pr(C|b)$.

Hence, given any state $k \in \{B, G\}$, for all feasible ϕ, $\varphi(\varphi(\phi|g)|g) = \phi_{gg} > \phi_g > \phi > \phi_b > \phi_{bb}$ and $\phi_{gi} > \phi_{bi}$ for $i \in \{g, b\}$

If $k = B$, competent's value function when hiring experts is,

$$V(\phi,E) = p(\phi) - w + \delta(1-\lambda)[\Pr(g|E,B)V(\phi_g) + \Pr(b|E,B)V(\phi_b)]$$

$$V(\phi,E) = p(\phi) - w + \delta(1-\lambda)[\alpha V(\phi_g) + (1-\alpha)V(\phi_b)]$$

The payoff from deviating by hiring a nonexpert and thereafter playing the equilibrium strategy of hiring experts is

$$V(\phi;N) = p(\phi) + \delta(1-\lambda)[\rho V(\phi_g) + (1-\rho)V(\phi_b)]$$

Thus

$$V(\phi,E) - V(\phi;N) = -w + \delta(1-\lambda)[X] + \delta^2(1-\lambda)^2\{\Pr(g|E)Y_g + \Pr(b|E)Y_b\}$$

where

$$X = (\alpha-\rho)[p(\phi_g) - p(\phi_b)]$$

$$Y_i = (\alpha-\rho)[V(\phi_{gi}) - V(\phi_{bi})] \qquad \text{for } i \in \{g,b\}$$

$$\Pr(g|E) = \alpha + \gamma(1-\rho-\alpha) = 1 - \Pr(b|E)$$

In order $V(\phi,E) - V(\phi;N) \geq 0$, it's necessary that

$$w \leq \delta(1-\lambda)[X + \delta(1-\lambda)V_f]; \quad \text{for all } \phi \in [\lambda\theta, 1-\lambda(1-\theta)]$$

where $V_f = \Pr(g|E)Y_g + \Pr(b|E)Y_b$

Then we can define Δ_B^{NS} as the minimum value of the expression $\delta(1-\lambda)[X + \delta(1-\lambda)V_f]$ over the range $\phi \in [\lambda\theta, 1-\lambda(1-\theta)]$

$$\Delta_B^{NS} = \min_{\phi \in [\lambda\theta, 1-\lambda(1-\theta)]} \left\{ \delta(1-\lambda)[X + \delta(1-\lambda)V_f] \right\} \tag{4.21}$$

If $k = G$, the condition for competents to hire experts is the same than (4.21). This is because ϕ_g and ϕ_b do not change and $(\alpha - \rho)$ is by assumption the same. Then, it's always possible to find some $\Delta^{NS} \geq w > 0$ such that $\Delta^{NS} = \Delta_G^{NS} = \Delta_B^{NS}$ and competents hire experts for all ϕ.

Finally it's necessary to show that $\delta(1-\lambda)[X + \delta(1-\lambda)V_f]$ is positive for all $\phi \in (0,1)$, such that $\Delta^{NS} > 0$.

a) $\delta(1-\lambda) > 0$ since $\delta > 0$ and $\lambda < 1$.

b) $X = (\alpha - \rho)[p(\phi_g) - p(\phi_b)] > 0$ since $\alpha > \rho$ and $p(\phi)$ is monotonically increasing in ϕ ($\frac{\partial p(\phi)}{\partial \phi} = \alpha - \rho > 0$).[23]

c) $V_f > 0$ because $Y_g > 0$ and $Y_b > 0$ since the value function V is monotonically increasing in ϕ as well.[24]

Hence, $\Delta^{NS} > 0$ for all $\phi \in [\lambda\theta, 1-\lambda(1-\theta)]$. Q.E.D.

4.6.2 Proof Proposition 31

his proof proceed in two steps. First we solve for superiors' optimal blaming intensities ($x(\phi)$, $y(\phi)$ and $z(\phi)$) that are consistent with "consumers" beliefs in equilibrium. Sec-

[23] More specifically, as in Mailath and Samuelson [2001], suppose F and G are two distributions describing "consumers" beliefs over the delegation decisions by competents in period t. If F first-order stochastically dominates G then superior's revenues in period t under F is higher than under G.

[24] Following Mailath and Samuelson [2001], let $f_t(\phi, \phi_0, t_0)$ be the distribution of "consumer" posteriors ϕ at time $t > t_0$ induced by strategy τ given period-t_0 posteriors ϕ_0. Then, $f_t(\phi, \phi_0, t_0)$ first-order stochastically dominates $f_t(\phi, \phi_0', t_0)$ for all $t > t_0$ and $\phi_0 > \phi_0'$. The same idea is true for the distribution of revenues. Hence, $V(\phi)$ is monotonic.

ond, using these results from the blaming stage, we derive conditions for an efficient equilibrium.

Step 1: *Blaming stage equilibrium*

The strategy we follow in this part of the proof is: First, consider as given the ”consumers” beliefs about blaming intensities and determine optimal decisions by superiors (both competents (y) and inepts(x))[25]. Second, considering the optimal blaming intensities, we check if beliefs are correct and consistent with those strategies.

Blaming decisions are made by superiors knowing their own type, their previous delegation choices and the state of nature. For example, in bad times, when competents hired experts and decide a blaming intensity y.

$$V(\phi,y) = p(\phi) - w + \delta(1-\lambda)[\alpha V(\phi_g) + (1-\alpha)[yV(\phi_b^{e_c}) + (1-y)V(\phi_b^{s_c})]]$$

where $\phi_b^{e_c}$ and $\phi_b^{s_c}$ are given by ”consumers”’ beliefs about y and x.

For any deviation from y, say to y', we can define,

$$VD(y) = V(\phi,y') - V(\phi;y) = \delta(1-\lambda)(1-\alpha)(y'-y)[V(\phi_b^{e_c}) - V(\phi_b^{s_c})]$$

1) Assume ”consumers” believe $y = x$.

By equations (4.6) and (4.7), $\phi_b^{e_c} = \phi_b^{s_c} = \phi_b$. Since $V(\phi_b^{e_c}) - V(\phi_b^{s_c}) = 0$, competents are indifferent choosing any $y' \in [0,\bar{y}]$ because, regardless of $(y'-y)$, always $VD(y) = 0$. Similarly, inepts are indifferent choosing any $x' \in [0,\bar{x}]$ because, regardless of $(x'-x)$, $VD(x) = 0$. Hence, ”consumers” beliefs $y = x$ are correct and consistent with equilibrium strategies, supporting multiple pooling equilibria in which no further reputation update is obtained from the blaming activity.

[25] Recall at this point z is not relevant for ”consumers” to update beliefs since we are focusing only on efficient equilibria in which competents always hire experts.

2) Assume "consumers" believe $y > x$. Then $\phi_b^{e_c} > \phi_b > \phi_b^{s_c}$. Since $V(\phi_b^{e_c}) - V(\phi_b^{s_c}) > 0$, competents choose $y' = \bar{y}$, which maximizes $VD(y)$. Similarly, inepts will choose $x' = \bar{x}$. Only "consumers" beliefs $y = \bar{y}$ and $x = \bar{x}$ will be correct, which are consistent with beliefs $y > x$ solely when $\bar{y} > \bar{x}$ being the only separating equilibrium in which the blaming activity represents an additional reputation updating.

3) Assume "consumers" believe $y < x$. Then $\phi_b^{e_c} < \phi_b < \phi_b^{s_c}$. Since $V(\phi_b^{e_c}) - V(\phi_b^{s_c}) < 0$, competents choose $y' = 0$ and inepts $x' = 0$. Only "consumers" beliefs $y = 0$ and $x = 0$ will be correct, which is not consistent with beliefs in which $y < x$. This case cannot be an equilibrium.[26]

Step 2: *Delegation stage equilibrium*

a) Let $\bar{y} \leq \bar{x}$

Fix ϕ and suppose an efficient situation ($\tau_G(\phi) = \tau_B(\phi) = 1$). Since the only possible equilibrium in the blaming stage is a pooling one where $\phi_b^{e_c} = \phi_b^{s_c} = \phi_b$, we have exactly the same expressions used to obtain equilibrium conditions without scapegoating (in Proposition 2's proof). Hence, under $\bar{y} \leq \bar{x}$ scapegoating does not affect efficiency conditions.

b) Let $\bar{y} > \bar{x}$

Even when pooling equilibria in blaming intensities that do not affect efficiency conditions exist in this situation, we will focus on the unique separating equilibrium in which $y = \bar{y}$, $x = \bar{x}$ and $z = \bar{z}$ such that $\phi_b^{e_c} > \phi_b > \phi_b^{s_c}$.

Fix ϕ and suppose an efficient situation ($\tau_G(\phi) = \tau_B(\phi) = 1$). It is straightforward to show that for all ϕ, after the first round $\Pr(C|g) > \Pr(C) > \Pr(C|b)$ and after the potential second round $\Pr(C|b, e_c) > (<) \Pr(C|g) > (<) \Pr(C|b, s_c)$ if $y > (<) x$. Given

[26] Because we are focusing on efficient equilibria, we checked beliefs for x and y but a competent type that deviated in the first stage hiring nonexperts will also choose any $z \in [0, \bar{z}]$ in 1), $z = \bar{z}$ in 2) and $z = 0$ in 3).

any state $k \in \{B, G\}$, for all feasible ϕ, $\varphi(\varphi(\phi|g)|g) = \phi_{gg} > \phi_g > \phi > \phi_b > \phi_{bb}$ and $\phi_{gi} > \phi_{b_1 i} > \phi_{bi} > \phi_{b_2 i}$ for $i \in \{g, b_1, b_2\}$, with $b_1 = (b, e_c)$ and $b_2 = (b, s_c)$.

If $k = B$, competent's value function when hiring experts is,

$$V(\phi, E) = p(\phi) - w + \delta(1-\lambda)[\alpha V(\phi_g) + (1-\alpha)[\bar{y}V(\phi_b^{e_c}) + (1-\bar{y})V(\phi_b^{s_c})]]$$

and from deviation by delegating to a nonexpert

$$V(\phi, N) = p(\phi) + \delta(1-\lambda)[\rho V(\phi_g) + (1-\rho)[\bar{z}V(\phi_b^{e_c}) + (1-\bar{z})V(\phi_b^{s_c})]]$$

Thus

$$\begin{aligned} V(\phi,E) - V(\phi;N) =& -w + \delta(1-\lambda)[X^B] \\ &+ \delta^2(1-\lambda)^2\{\Pr(g|E)Y_g^B + \Pr(b|E)[\bar{y}Y_{b_1}^B + (1-\bar{y})Y_{b_2}^B]\} \end{aligned}$$

where

$$\begin{aligned} X^B = (\alpha-\rho)p(\phi_g) + (1-\alpha)[\bar{y}p(\phi_b^{e_c}) + (1-\bar{y})p(\phi_b^{s_c})] \\ -(1-\rho)[\bar{z}p(\phi_b^{e_c}) + (1-\bar{z})p(\phi_b^{s_c})] \end{aligned}$$

$$\begin{aligned} Y_i^B = (\alpha-\rho)V(\phi_{gi}) + (1-\alpha)[\bar{y}V(\phi_{bi}^{e_c}) + (1-\bar{y})V(\phi_{bi}^{s_c})] \\ -(1-\rho)[\bar{z}V(\phi_{bi}^{e_c}) + (1-\bar{z})V(\phi_{bi}^{s_c})] \end{aligned}$$

for $i \in \{g, b_1, b_2\}$

and, as in the previous proof, $\Pr(g|E) = \alpha + \gamma(1-\rho-\alpha) = 1 - \Pr(b|E)$

At this point it is useful to express X^B in terms of expected reputation after the hiring decision.

Let's define

$$\phi_{b,E} = \bar{y}\phi_b^{e_c} + (1-\bar{y})\phi_b^{s_c} \tag{4.22}$$

$$\phi_{b,N} = \bar{z}\phi_b^{e_c} + (1-\bar{z})\phi_b^{s_c} \tag{4.23}$$

Hence[27],

$$X^B = (\alpha - \rho)p(\phi_g) + (1-\alpha)p(\phi_{b,E}) - (1-\rho)p(\phi_{b,N})$$

An equilibrium in which competents only hire experts when $k = B$ requires that $V(\phi, E) - V(\phi; N) \geq 0$ for all feasible reputation measures ϕ. A necessary condition is that cost differences w fulfill

$$w \leq \delta(1-\lambda)[X^B + \delta(1-\lambda)V_f^B]; \quad \text{for all } \phi \in [\lambda\theta, 1-\lambda(1-\theta)]$$

where $V_f^B = \Pr(g|E)Y_g^B + \Pr(b|E)[\bar{y}Y_{b_1}^B + (1-\bar{y})Y_{b_2}^B]$

Then we can define Δ_B^S as the minimum value of the expression $\delta(1-\lambda)[X^B + \delta(1-\lambda)V_f^B]$ over the range $\phi \in [\lambda\theta, 1-\lambda(1-\theta)]$

$$\Delta_B^S = \min_{\phi \in [\lambda\theta, 1-\lambda(1-\theta)]} \left\{ \delta(1-\lambda)[X^B + \delta(1-\lambda)V_f^B] \right\} \tag{4.24}$$

To save notation it is possible to assume a case in which the future does not present scapegoating possibilities, so there is just one current shot blaming. In this case, from tomorrow on it would be possible to have only two possible states $i \in \{g, b\}$. It is straightforward to check that (4.24) is simplified to $V_f^B = \Pr(g|E)Y_g^B + \Pr(b|E)Y_b^B$. This last expression is used in Proposition 31.

If $k = G$, the proof is identical to the previous one but having $\Pr(g|E,G) = (1-\rho)$ and $\Pr(g|N,G) = (1-\alpha)$ instead.

Then,

$$X^G = (\alpha - \rho)p(\phi_g) + \rho p(\phi_{b,E}) - \alpha p(\phi_{b,N})$$

[27] It is not possible to do the same for Y_i^B because we do not know the form of the value functions just their monotonicity in ϕ, (recall we are not assuming linearity of $V(\phi)$).

$$Y_i^G = (\alpha-\rho)V(\phi_{gi}) + \rho[\bar{y}V(\phi_{bi}^{e_c}) + (1-\bar{y})V(\phi_{bi}^{s_c})] - \alpha[\bar{z}V(\phi_{bi}^{e_c}) + (1-\bar{z})V(\phi_{bi}^{s_c})]$$

for $i \in \{g, b_1, b_2\}$

Hence the condition for competents to hire experts and to achieve the efficient outcome as an equilibrium is,

$$w \leq \delta(1-\lambda)[X^G + \delta(1-\lambda)V_f^G]; \quad \text{for all } \phi \in [\lambda\theta, 1-\lambda(1-\theta)]$$

where $V_f^G = \Pr(g|E)Y_g^G + \Pr(b|E)[\bar{y}Y_{b_1}^G + (1-\bar{y})Y_{b_2}^G]$

Then we can define Δ_G^S as the minimum value of the expression $\delta(1-\lambda)[X^G + \delta(1-\lambda)V_f^G]$ over the range $\phi \in [\lambda\theta, 1-\lambda(1-\theta)]$

$$\Delta_G^S = \min_{\phi\in[\lambda\theta,1-\lambda(1-\theta)]} \left\{\delta(1-\lambda)[X^G + \delta(1-\lambda)V_f^G]\right\} \tag{4.25}$$

As in the previous case, to save notation $V_f^G = \Pr(g|E)Y_g^G + \Pr(b|E)Y_b^G$ in equation (4.25), which is used in Proposition 3. Q.E.D.

4.6.3 Extension Proof Proposition 32

Considering the existence of blaming activities in current and future periods, the only difference arises in the definition of V_f^G, after equation (4.25)

Hence we need to prove $V_f - V_f^G \geq 0$ where

$V_f^G = \Pr(g|E)Y_g^G + \Pr(b|E)[\bar{y}Y_{b_1}^G + (1-\bar{y})Y_{b_2}^G]$

Then, $V_f - V_f^G = \Pr(g|E)[Y_g - Y_g^G] + \Pr(b|E)[Y_b - \bar{y}Y_{b_1}^G - (1-\bar{y})Y_{b_2}^G]$

This expression will be non-negative whenever $Y_g - Y_g^G \geq 0$ (proved in Proposition 3) and $Y_b - \bar{y}Y_{b_1}^G - (1-\bar{y})Y_{b_2}^G \geq 0$, which happens under two sufficient conditions,

a) $(\alpha-\rho)\left[\bar{y}V(\phi_{gb}) + (1-\bar{y})V(\phi_{gb})\right] \geq (\alpha-\rho)\left[\bar{y}V(\phi_{gb_1}) + (1-\bar{y})V(\phi_{gb_2})\right]$

or, which is the same,

$$\bar{y} \leq \frac{V(\phi_{gb}) - V(\phi_{gb_2})}{V(\phi_{gb_1}) - V(\phi_{gb_2})} \leq 1$$

Since $\phi_{gb_1} \geq \phi_{gb} \geq \phi_{gb_2}$.

b) $(\alpha\bar{z} - \rho\bar{y})[\bar{y}V(\phi^{e_c}_{bb_1}) + (1-\bar{y})V(\phi^{e_c}_{bb_2})] \geq (\alpha - \rho)V(\phi_{bb})$

The sufficient conditions for this to hold are both (as in (4.15))

$$\bar{y} \geq \frac{V(\phi_{bb}) - V(\phi^{e_c}_{bb_2})}{V(\phi^{e_c}_{bb_1}) - V(\phi^{e_c}_{bb_2})} \text{ and } \bar{z} > z^* = 1 - \frac{\rho}{\alpha}(1-\bar{y})$$

Q.E.D.

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