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Essays on the Tragedy of the Anticommons in Complementary-Good Markets

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Matteo Alvisi

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To my family

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Publications

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Abstract of the Dissertation

Essays on the Tragedy of the Anticommons in Complementary-Good Markets

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Professor Hugo Hopenhayn, Chair

Recently, a considerable amount of attention has been devoted to a specific class of market distortions, known as "the tragedy of the anticommons". Based on Cournot's "complementary monopoly", such literature argues that social welfare might be better served by policies favoring integration. In fact, when complementary goods are sold by different firms, prices are higher than those set by a monopoly selling all the complementary goods. A merger would then yield a higher consumer surplus. While the resulting social welfare may fall short of the perfectly competitive one, a merger might represent a second best solution. Strictly speaking, this literature is applicable only to situations in which the markets for all complementary goods are monopolies. This dissertation verifies the robustness of such result under different market structures and degrees of product differentiation. Particularly, it concludes that the extent to which public policies should combat an anticommons is actually quite limited when the broader market environment in which industries exist is taken into account.

In Chapter 1, we consider two complementary goods forming a system and we introduce oligopolistic competition first for one and then for both complements.

Particularly, we show that competition in only one of the two markets may be welfare superior to an integrated monopoly if and only if the substitutes differ in their quality so that, as their number increases, average quality and/or quality variance increases. Then, absent an adequate level of product differentiation, favoring competition in some sectors while leaving monopolies in others may be detrimental for consumers and producers alike. Instead, competition in both markets may be welfare superior if goods are close substitutes and their number in each market is sufficiently high, no matter the degree of product differentiation.

In Chapter 2, we discuss the implicit suggestion of the "tragedy", according to which producers of complementary goods should always integrate themselves. In fact, recent decisions by antitrust authorities rather indicate that the tradeoff between the "tragedy" and the lack of competition characterizing an integrated market structure should be more carefully analyzed, and that integration should be allowed only when the former becomes a more serious problem than the latter. We analyze such tradeoff in oligopolistic complementary markets, when products are vertically differentiated. We show that quality leadership plays a crucial role. When there is a quality leader, forcing divestitures or prohibiting mergers, thus increasing competition, lowers prices and enhances consumer surplus. However, when quality leadership is shared, "disintegrating" firms may indeed lead to higher prices. Then, only in this second case concerns about the tragedy of the anticommons seem to be well posed in antitrust decisions.

In Chapter 3, we analyze the impact of the "tragedy" on entry decisions. Particularly, we show that allowing firms to enter a complementary-good market and then sell all components of a composite good may be both welfare-enhancing and pro-competitive. In fact, such strategy may favor the entry of new firms producing lower-quality components in the original market. In other terms, "selling the whole package" may increase consumer surplus, even when the composite good is sold as a bundle only. Interestingly, notwithstanding the subsequent increase in competition, it is always optimal for firms to enter a complementary-good market. By discouraging such practices, then, antitrust authorities may harm both consumers and low-quality firms, at the same time undermining market stability.

CHAPTER 1

Imperfect Substitutes for Perfect Complements. A Welfare Analysis*

1.1 Introduction

A complementary monopoly is characterized by the presence of multiple sellers, each producing a complementary good. It has been known for quite some time in the literature that such market structure is worse than an integrated monopoly, in which a single firm offers all complements (Cournot, 1838). In fact, a firm producing a single good takes into account only the impact of a price raise on its own profits, without considering the negative externality imposed on the sellers of other complementary goods[1]. As a consequence, prices will be higher with separate producers than with an integrated monopolist, generating a lower consumer surplus.[2]

*This chapter is extensively based on the paper "Imperfect Substitutes for Perfect Complements: a Solution to the Tragedy of the Anticommons "(with E. Carbonara), *Bulletin of Economic Research*, 2011, , Wiley-Blackwell, accepted, forthcoming. We thank Paola Bortot, Giuseppe Dari-Mattiacci, Vincenzo Denicolò,Francesco Parisi and participants to the EALE (European Association of Law and Economics) Conference, University of Pathon-Assas, Paris II, September 2010, and to the Workshop "Industrial Organization: Theory, Empirics and Experiments", University of Salento, Otranto, June 2011 for fruitful suggestions.

[1]The quantity demanded would be reduced for everyone, but each seller benefits fully of an increase in its own price.

[2]Complementary monopoly is similar to the problem of double marginalization in bilateral monopoly, with the important difference that here each monopolist competes "side by side", possibly without direct contacts with the others. In bilateral monopoly, the "upstream" monopolist produces an input that will be used by the "downstream" firm, who is then a monopsonist

The complementary monopoly problem is also known as "the tragedy of the anticommons", in analogy with its mirror case, the more famous "tragedy of the commons" and has been applied in the legal literature to issues related to the fragmentation of physical and intellectual property rights.[3] Strictly speaking, such literature is applicable only to situations in which the markets for all complementary goods are monopolies. However, pure monopolies are quite rare in the real world. More often, each complement is produced in an oligopolistic setting. Consider, for instance, software markets, where each component of a system is produced by many competing firms, such as Microsoft, Apple, Unix and Linux for operating systems; Microsoft, Google, Apple, Mozilla for Internet browsers, and so on. Similarly, consider the market for photographic equipment, in which both camera bodies and lenses are produced by many competing companies (Nikon, Canon, Olympus, Pentax, etc.), some of which are active only in the market for lenses (Tamron, Sigma, Vivitar). In such cases, an integrated market structure may reduce the extent of the tragedy on the one hand, while lowering welfare because of reduced competition on the other. The case of software markets is particularly relevant in this respect. In the last ten years, some important antitrust cases, both in the United States and in Europe, have brought to the attention of the economics profession the potential tradeoff between competition and the tragedy of the anticommons. For instance, in the Microsoft case, the American Court of Appeals ordered the firm to divest branches of its business other than operating systems, creating a new company dedicated to application development. The break-up (later abandoned) would have created two firms producing complementary goods, with the likely result of increasing prices in the market. However, far from being unaware of the potential tragedy of the an-

for that specific input (see Machlup and Taber, 1960).

[3]For an application to property rights, see Heller (1998), Buchanan and Yoon (2000) and Parisi (2002).

ticommons, Judge Jackson motivated his decision with the need to reduce the possibility for Microsoft to engage in limit pricing, thus deterring entry. Separation would have facilitated entry and favored competition, possibly driving prices below pre-separation levels.[4] A similar economic argument motivated the European Commission's Decision over the merger between General Electric and Honeywell.[5] In such case, the EC indicated that the post-merger prices would be so low as to injure new entrants, so that a merger would reduce the number of potential and actual competitors in both markets.[6]

Both these decisions indicate that separation may not be an issue (and may even be welfare improving) if the post-separation market configuration is not a complementary monopoly in the Cournot's sense, i.e., the market for each complement is characterized by competition. The initially higher prices due to the tragedy may in fact encourage entry in the market and, if competition increases sufficiently, the resulting market structure may yield lower prices and higher welfare than in the initial integrated monopoly. The question then is how much competition is needed in the supply of each complement in order to obtain at least the same welfare as in the original monopoly.

Investigating the impact of competition on welfare when complementary goods are involved, Dari-Mattiacci and Parisi (2007) note that, when n perfect complements are bought together by consumers and firms compete *à la* Bertrand, two perfect substitutes for $n-1$ complements are sufficient to guarantee the

[4]United States v. Microsoft Corp., 97 F. Supp. 2d. 59 (D.D.C. 2000). See Gilbert and Katz (2001) for a thorough analysis of the Microsoft case.

[5]See European Commission Decision of 03/07/2001, declaring a concentration to be incompatible with the common market and the EEA Agreement Case, No. COMP/M.2220 - General Electric/Honeywell. The "efficiency offense "argument used by the EC is analyzed by Motta and Vasconcelos (2005), which considers the impact of such Antitrust decision in a dynamic setting.

[6]On the possibility that an integrated monopolist engages in limit pricing to deter entry, see Fudenberg and Tirole (2000).

same social welfare experienced when an integrated monopolist sells all n complements. In fact, all competitors in the $n-1$ markets price at marginal cost, thus allowing the monopolist in the n-th market to extract the whole surplus, fixing its price equal to the one that would be set by an integrated monopolist for the composite good. Therefore, the negative externality characterizing a complementary monopoly disappears and the tragedy of the anticommons is solved by competition.

Our analysis maintains this framework when it considers perfect complements but then extends it in several directions. First, differently from previous literature, the competing goods are both imperfect substitutes[7] and vertically differentiated. Second, we consider the presence of substitutes in all components' markets.

Particularly, we consider two perfect complements, proving first that, if one complementary good is still produced in a monopolistic setting and if competition for the other complement does not alter the average quality in the market, an integrated monopoly remains welfare superior to more competitive market settings. In fact, with imperfect substitutability the competing firms retain enough market power as to price above their marginal cost, and the monopolist in the first market is not able to fully extract the surplus enjoyed by consumers. As a result, the equilibrium prices of the composite goods remain higher than in an integrated monopoly, implying that favoring competition in some sectors while leaving monopolies in others may actually be detrimental for consumers. A competitive setting may still be welfare superior, but only if the substitutes of the complementary good produced competitively differ in their quality, so that average quality and/or quality variance increase as their number increases.

[7] Imperfect substitutability in this case means that the cross-price elasticity is lower than own-price elasticity.

Results change when competition is introduced in the supply of both components. In this case we find that the tragedy may be solved for a relatively small number of competing firms in each sector provided that goods are sufficiently close substitutes. Not surprisingly, the higher the degree of substitutability and the number of competitors in one sector, the more concentrated the remaining sector can be and still yield a higher consumer surplus.

The welfare loss attached to a complementary monopoly has been analyzed, among others, by Economides and Salop (1992)who present a generalized version of the Cournot complementary monopoly in a duopolistic setting. Differently from our contribution, however, their model does not consider quality differentiation (as implied by the assumption of symmetric demands for the composite goods), so that the tragedy always prevails whenever goods are not close substitutes. Moreover, they don't study if and how the tragedy can be solved when the number of substitutes for each complement increases.[8] McHardy (2006) demonstrates that ignoring demand complementarities when breaking up firms that produce complementary goods may lead to substantial welfare losses. However, if the break-up stops limit-pricing practices by the previously merged firm, even a relatively modest degree of post-separation entry may lead to higher welfare than an integrated monopoly. He assumes a setting in which firms producing the same component compete *à la* Cournot among them, whereas competition is *à la* Bertrand among complements (i.e., among sectors). Differently from McHardy (2006), we analyze the impact of complementarities and entry in a model where

[8]Gaudet and Salant (1992) study price competition in an industry producing perfect complements and prove that welfare-improving mergers may fail to occur endogenously. Tan and Yuan (2003) are concerned with the opposite issue, i.e., they consider a market in which two firms sell imperfectly substitutable composite goods consisting of several complementors. They show that firms have the incentive to divest along complementary lines, because the price raise due to competition among producers of complements counters the downward pressure on prices due to Bertrand competition in the market for imperfect substitutes.

all firms choose prices when competing both intra and inter layer and in such framework we also study the impact of product differentiation and imperfect substitutability.[9]

The chapter is organized as follows. Section 2 introduces the model when one sector is a monopoly and presents the benchmark cases of complementary and integrated monopoly. Section 3 analyzes the impact of competition on welfare when one complement is produced by a monopolist while Section 4 extends the model considering competition in the markets for all complements. Section 5 concludes. Appendix A contains some technical material while Appendix B contains the proofs of the Lemmas and Propositions.

1.2 The Model

Consider a composite good (a system) consisting of two components, A and B. The two components are perfect complements and are purchased in a fixed proportion (one to one for simplicity). Initially, we assume that complement A is produced by a monopolist, whereas complement B is produced by n oligopolistic firms.[10] Marginal costs are the same for all firms and are normalized to zero.[11] Firms compete by setting prices. We also assume full compatibility among components, meaning that the complement produced by the monopolist in sector A

[9]Previous literature on the relationship between complementary goods and market structure is scanty and deals mostly with bundling practices. Matutes and Regibeau (1988) study compatibility and bundling in oligopolistic markets in which complementary goods have to be assembled into a system. Anderson and Leruth (1993) study bundling choices under different market structures. Denicolò (2000) analyzes compatibility and bundling choices when an integrated firm selling all complements in a system competes with non-integrated firms, each producing a single, different complement. Nalebuff (2004) analyzes the incentives to bundle by oligopolistic firms, showing that bundling is a particularly effective entry-deterrent strategy.

[10]We will remove this assumption later and consider a market configuration in which n_1 firms produce complement A, whereas n_2 firms produce complement B.

[11]This assumption is with no loss of generality, because results would not change for positive, constant marginal costs (see Economides and Salop, 1992).

can be purchased by consumers in combination with any of the n versions of complement B. This assumption is made because we are interested in the effect of competition on the pricing strategies of the firms operating in the various complementary markets. If we let firms decide to restrict compatibility, competition may be limited endogenously (for instance, the monopolist could allow combination with a subset of producers in sector B only) and the purpose of our analysis would be thwarted.[12] Finally, we assume that the n systems of complementary goods have different qualities and that consumers perceive them as imperfect substitutes.[13]

More specifically, the representative consumer has preferences represented by the following utility function, quadratic in the consumption of the n available systems and linear in the consumption of all the other goods (as in Dixit, 1979, Beggs, 1994):

$$U(q, I) = \sum_{j=1}^{n} \alpha_{1j} q_{1j} - \frac{1}{2} \left[\beta \sum_{j=1}^{n} q_{1j}^2 + \gamma \sum_{j=1}^{n} q_{1j} \left(\sum_{s \neq j} q_{1s} \right) \right] + I \qquad (1.1)$$

where I is the total expenditure on other goods different from the n systems, $q = [q_{11}, q_{12}, .., q_{1n}]$ is the vector of the quantities consumed of each system and q_{1j} represents the quantity of system $1j$, $(j = 1, ..., n)$, obtained by combining q_{1j} units of component A purchased from the monopolist, indexed by the number 1 (component $A1$), and $q_{Bj} = q_{1j}$ units of component B purchased from the j-th firm in sector B (component Bj).[14] Also, $\alpha = (\alpha_{11}, \alpha_{12}, .., \alpha_{1n})$ is the vector of the qualities of each system (with α_{1j} representing the quality of system

[12] The assumption of perfect compatibility is common to many contributions in the literature on complementary markets, see Economides and Salop (1992), McHardy (2006), Dari-Mattiacci and Parisi (2007).

[13] This implies that the consumption possibility set consists of n imperfectly substitutable systems. Later on, when we consider n_1 components in sector A, consumers will have the opportunity to combine each of these components with any of the n_2 complements produced in Sector B. We would then have $n_1 \times n_2$ imperfectly substitutable systems in the market.

[14] Note that when referring to a particular system, we use a couple of numbers indicating the two firms in sector A and B, respectively, selling each component of such system. When

$1j$, $j = 1, ..., n$), γ measures the degree of substitutability between any couple of systems, $\gamma \in [0, 1]$, and β is a positive parameter. The representative consumer maximizes the utility function (1.1) subject to a linear budget constraint of the form $\sum_{j=1}^{n} p_{1j} q_{1j} + I \leq M$, where

$$p_{1j} = p_{A1} + p_{Bj}, \quad j = 1, ..., n \tag{1.2}$$

is the price of system $1j$ (expressed as the sum of the prices of the single components set by firm 1 in sector A and firm j in sector B, respectively) and M is income.

1.2.1 Equilibrium Prices and Demand

The first order condition determining the optimal consumption of system $1k$ is[15]

$$\frac{\partial U}{\partial q_{1k}} = \alpha_{1k} - \beta q_{1k} - \gamma \sum_{j \neq k} q_{1j} - p_{1k} = 0 \tag{1.3}$$

Summing (1.3) over all firms in sector B, we obtain the demand for system $1k$

$$q_{1k} = \frac{(\beta + \gamma(n-2))(\alpha_{1k} - p_{A1} - p_{Bk}) - \gamma \left(\sum_{j \neq k} \alpha_{1j} - (n-1) p_{A1} - \sum_{j \neq k} p_{Bj} \right)}{(\beta - \gamma)(\beta + \gamma(n-1))} \tag{1.4}$$

Using (1.4), we sum the demands of all firms in sector B to obtain the total market size

$$Q = \sum_{j=1}^{n} q_{1j} = \frac{\sum_{j=1}^{n} (\alpha_{1j} - p_{Bj}) - n p_{A1}}{\beta + \gamma (n-1)} \tag{1.5}$$

referring instead to separate components, we use a couple of one letter and one number, the first indicating the sector (the component) and the second the particular firm selling it. This might appear redundant for $A1$ when component A is sold by a monopolist, but it will become useful when we introduce competition in sector A.

[15] The second order conditions for the maximization of $U(q, I)$ require $\gamma \leq \beta$, i.e., $\gamma < 1$.

Following Shubik and Levitan (1980), we set

$$\beta = n - \gamma(n-1) > 0 \tag{1.6}$$

to prevent changes in γ and n to affect Q, so that, substituting such expression into (1.5), the normalized market size becomes

$$Q = \bar{\alpha} - \bar{p}_B - p_{A1} \tag{1.7}$$

where $\bar{\alpha} = \frac{\sum_{j=1}^{n} \alpha_{1j}}{n}$ is the average quality of the n available systems and $\bar{p}_B = \frac{\sum_{j=1}^{n} p_{Bj}}{n}$ is the average price in the market for the second component.

Note that component $A1$ is part of all the n systems, so that (1.7) also represents the demand function for the monopolist in sector A. Its profit can then be written as $\Pi_{A1} = p_{A1}Q = (\bar{\alpha} - \bar{p}_B)\, p_{A1} - p_{A1}^2$, whereas the profit of a single producer of component B is $\Pi_{Bk} = p_{Bk} \cdot q_{Bk}$, where $q_{Bk} = q_{1k}$ is given in (1.4). Bertrand equilibrium prices for the monopolist $A1$ and for the k-th oligopolist are, respectively

$$p_{A1}^{M} = \frac{\bar{\alpha}(n-\gamma)}{n(3-\gamma) - 2\gamma} \tag{1.8}$$

$$p_{Bk}^{M} = \frac{\bar{\alpha} n(1-\gamma)}{n(3-\gamma) - 2\gamma} + \frac{n(\alpha_{1k} - \bar{\alpha})}{2n - \gamma} \tag{1.9}$$

where the superscript M stands for "monopoly in sector A". Note first, not surprisingly, that p_{A1}^{M} is increasing in $\bar{\alpha}$. In fact, $A1$ is part of all systems, so that an increase in their average quality increases the representative consumer's willingness to pay for them and allows the monopolist to set an higher price p_{A1}^{M} [16], which also depends positively on the number of systems sold, n, and on the

[16]This result is not surprising in our setting because an increase in average quality comes at no cost, particularly in sector B. If we assume instead that a system's quality can be increased only through a costly investment by the firm producing component B for that system, then conclusions might be less obvious. In fact, even in the symmetric case, where $\alpha$is the same

degree of substitutability between any couple of systems, γ. As we will show below, the increase in competition in the market of the second component (either because of a greater number of firms or of an higher degree of substitutability among systems) reduces all oligopolistic prices. Then, as γ or n increases, *ceteris paribus*, the monopolist in sector A is able to extract a bigger share of consumer surplus through an higher p_{A1}^M because of the fiercer competition in sector B.[17] Not surprisingly, from (1.9), producers of below-average quality charge lower than average prices (since $(\alpha_{1k} - \bar{\alpha}) < 0$), whereas the opposite is true for producers of above-average quality. However, quality "premiums and discounts" cancel out on average. In fact, the average price in the market for the second component is

$$\bar{p}_B = \frac{\sum_{k=1}^{n} p_{Bk}^M}{n} = \frac{\bar{\alpha}n(1-\gamma)}{n(3-\gamma)-2\gamma} \tag{1.10}$$

Combining (1.8) and (1.9), the equilibrium price of system $1k$ is

$$p_{1k}^M = p_{A1}^M + p_{Bk}^M = \frac{(n(2-\gamma)-\gamma)\bar{\alpha}}{n(3-\gamma)-2\gamma} + \frac{n\left(\alpha_{1k}-\bar{\alpha}\right)}{2n-\gamma} \tag{1.11}$$

so that, the average system price becomes

$$\bar{p}_{1k}^M = p_{A1}^M + \bar{p}_B = \frac{(n(2-\gamma)-\gamma)\bar{\alpha}}{n(3-\gamma)-2\gamma} \tag{1.12}$$

Finally, using (1.4), (1.8) and (1.9), we derive the equilibrium quantities

$$q_{1k}^M = \frac{\bar{\alpha}(n-\gamma)}{n(n(3-\gamma)-2\gamma))} + \frac{(\alpha_{1k}-\bar{\alpha})(n-\gamma)}{n(2n-\gamma)(1-\gamma)} \tag{1.13}$$

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for all systems, increasing average quality implies higher costs for the whole set of n firms in sector B, so that prices will need to be higher to cover that investment. In other terms, quality investment might be considered a way to relax price competition in sector B and then to lower the ability of the monopolist in sector A to extract consumer surplus through the joint sale of its component with those produced in the other sector. In conclusion, p_{A1}^M could in principle decrease with $\bar{\alpha}$ if this effect counterbalances the increased willingness-to-pay of the representative consumer when systems' qualities are higher.

[17] It should be noted that the impact of an increase in n on p_{A1}^M is analyzed assuming a constant $\bar{\alpha}$, which implies that we are concentrating on mean-preserving distributions of quality across firms.

We are now ready to compute profits and consumer welfare. Given (1.8) and (1.4), the monopolist's profits in sector A are equal to

$$\Pi_{A1}^{M} = p_{A1}^{M} \sum_{j=1}^{n} q_{1j}^{M} = \frac{\bar{\alpha}^2 (n-\gamma)^2}{(n(\gamma-3)+2\gamma)^2} \tag{1.14}$$

As for the $k-$th oligopolist's profit, note first that

$$p_{Bk}^{M} = t \cdot q_{1k}^{M} \tag{1.15}$$

where $t = \frac{n^2(1-\gamma)}{(n-\gamma)}$. Hence

$$\Pi_{Bk}^{M} = t\left(q_{1k}^{M}\right)^2 = \frac{n^2(1-\gamma)}{(n-\gamma)} \left(\frac{\bar{\alpha}(n-\gamma)}{n(n(3-\gamma)-2\gamma)} + \frac{(\alpha_{1k}-\bar{\alpha})(n-\gamma)}{n(2n-\gamma)(1-\gamma)} \right)^2, \tag{1.16}$$

so that aggregate profits in sector B are equal to $\Pi_{B}^{M} = \sum_{j=1}^{n} \Pi_{Bj}^{M} = t \sum_{j=1}^{n} \left(q_{1j}^{M}\right)^2$, that is to

$$\Pi_{B}^{M} = n(1-\gamma)(n-\gamma) \left(\frac{\bar{\alpha}^2}{n(n(3-\gamma)-2\gamma)^2} + \frac{\sigma_{\alpha}^2}{n(2n-\gamma)^2(1-\gamma)^2} \right) \tag{1.17}$$

where $\sigma_{\alpha}^2 = \frac{\sum_{j=1}^{n}(\alpha_{1j}-\bar{\alpha})^2}{n}$ represents the variance of the qualities of the n available systems.

Given the utility function in (1.1), consumer surplus is defined as

$$CS = U(q,I) - \left(\sum_{j=1}^{n} p_{1j} q_{1j} + I \right) = \frac{n(1-\gamma)}{2} \sum_{j=1}^{n} q_{1j}^2 + \frac{\gamma}{2} \left(\sum_{j=1}^{n} q_{1j} \right)^2 \tag{1.18}$$

and, after some rearrangements, can be rewritten as

$$CS_M = \frac{n^2(1-\gamma)}{2} \tilde{B}^2 \sigma_{\alpha}^2 + \frac{n^2}{2} \tilde{A}^2 \bar{\alpha}^2 \tag{1.19}$$

where $\widetilde{A} = \frac{(n-\gamma)}{n(n(3-\gamma)-2\gamma)}$ and $\widetilde{B} = \frac{(n-\gamma)}{n(1-\gamma)(2n-\gamma)}$.[18] In the next Section we will compare equilibrium prices, profits and welfare of our model with those obtained

[18]See Appendix A.

under both an integrated and a complementary monopoly,*ceteris paribus*. In this respect we report here the main findings in these two alternative regimes. Particularly, a profit-maximizing integrated monopoly producing both complements would set its system price at $p_{IM} = \frac{\alpha_{IM}}{2}$, selling $Q_{IM} = \frac{\alpha_{IM}}{2}$ systems, so that profits and consumer surplus would amount to

$$\Pi_{IM} = \frac{\alpha_{IM}^2}{4}; \qquad CS_{IM} = \frac{\alpha_{IM}^2}{8}. \tag{1.20}$$

In a complementary monopoly, two independent firms $A1$ and $B1$ produce one component each of the composite good (i.e., $n = 1$) and, in equilibrium, they set their prices at $p_{CM}^i = \frac{\alpha_{CM}}{3}$, $i = A, B$ (where CM stands for "complementary monopoly"). Hence, consumers pay a system price $p_{CM} = \frac{2\alpha_{CM}}{3}$ and purchase $Q_{CM} = \frac{\alpha_{CM}}{3}$ units of the system. Profits and consumer surplus are:

$$\Pi_{CM}^i = \frac{\alpha_{CM}^2}{9}, \;\; i = A, B; \;\; CS_{CM} = \frac{\alpha_{CM}^2}{18}, \tag{1.21}$$

where $CS_{CM} < CS_{IM}$, obviously.

1.3 Monopoly in Sector A: Competition and Welfare

In this section we verify the impact of changes in the number of firms in Sector B, n, in the degree of substitutability among systems,γ, and in the distribution of the quality parameters (the α_{1k}'s) on equilibrium prices and welfare.

Along the way, we will verify how the assumption of imperfect substitutability changes the impact of n on the extent of the tragedy of the anticommons when compared to the case studied by Dari-Mattiacci and Parisi (2007).[19] First of all, comparing prices and quantities when sector A is a monopoly with those obtained

[19] One should recall that, in their simple model, two firms competing in the market for the second component would be enough to guarantee a surplus equal to that attained in the presence of a single, integrated firm.

in an integrated monopoly, it can be noticed immediately that, when $\sigma_\alpha^2 = 0$ and $\alpha_{1k} = \bar{\alpha} = \alpha_{IM} = \alpha^*$, $k = 1, ..., n$, individual component prices in sector B are lower than p_{IM}, while system prices are higher. In fact,

$$p_{Bk}^M - p_{IM} = -\frac{\alpha^*(n-\gamma)}{(3n - \gamma(2+n))} < 0 \tag{1.22}$$

$$p_{1k}^M - p_{IM} = \frac{\alpha^*(n-\gamma(4-n))}{2(3n - \gamma(2+n))} > 0 \tag{1.23}$$

for all $\gamma \in [0,1]$. Thus, while competition certainly lowers prices in the oligopolistic sector, the monopolist in sector A optimally reacts by extracting more surplus and setting higher prices, so that overall $p_{1k}^M > p_{IM}$. This has a negative impact on the number of systems sold in the market. In fact, it is immediate to check that $Q_M = nq_{1k}^M < Q_{IM}$. Similarly,when σ_α^2=0 and the common quality level among all systems coincides with that of a complementary monopoly (again, when $\alpha_{1k} = \bar{\alpha} = \alpha_{CM}$, $k = 1, ..., n$), component and system prices are lower with competition than with a complementary monopoly (i.e. $p_{Bk}^M < p_{CM}^B$and $p_{1k}^M < p_{CM}$, respectively). This implies that $Q_M > Q_{CM}$, even if each oligopolist sells less than a complementary monopolist ($q_{1k}^M < Q_{CM}$).

The following Lemma illustrates first the relationship between olipolistic prices p_{Bk}^M, substitutability γ, and competition in sector B, given by n.

Lemma 1. *Oligopolistic prices decrease with n and γ.*

Proof: See Appendix B.

The negative relationship between p_{Bk}^M, γ and n is intuitive. The higher the number of firms in sector B and the degree of substitutability among systems, the fiercer the competition for the second component and the lower the Bertrand equilibrium prices. Similarly, it is immediate to verify from (1.10) that the impact of a change in n and γ on $\bar{p}_B$ is the usual and negative one.[20]

When checking instead the relationship between n, γ and system prices p_{1k}^M, we notice from (1.11) that it is influenced by opposite forces. On the one hand, p_{A1}^M increases as either n or γ increase, whereas p_{Bk}^M decreases. The following Proposition indicates however that the first effect is always dominated by the second, so that, overall, p_{1k}^M decreases with n and γ.

Proposition 1. *The equilibrium system prices decrease with n and γ. Then, consumer surplus increases with n and γ.*

Proof: See Appendix B.

As stated in Lemma 1, when the number of firms in sector B increases, then p_{Bk}^M decreases. The monopolist's best response would be to increase p_{A1}, given the complementarity between goods $A1$ and Bk. However, such an increase would negatively affect the demand of all the n systems. Then, the monopolist internalizes such negative externality and limits the increase in p_{A1}. As a result, the equilibrium system prices decrease with n and the same applies to the degree of substitutability γ.

As for welfare comparisons, we notice first from (1.23) that, with a common quality value, no matter the extent of competition in sector B (i.e., no matter n), "separating" the two components of the system produced by an integrated monopolist and having them sold by two independent firms, always leads to higher prices. This clearly indicates that, when goods are not perfect substitutes, the

[20] p_{Bk}^M is also decreasing with $\bar{\alpha}$. In fact, it is defined for a given α_{1k}, so that if $\bar{\alpha}$ increases it is because the quality of some systems other than $1k$ has increased. In such circumstance, the ratio $\frac{\alpha_{1k}}{\bar{\alpha}}$ actually decreases, reducing the price that firm k can charge. On the other hand, the average price $\bar{p}_B$ is positively affected by $\bar{\alpha}$: as the average quality of the available systems increases, their average price also increases.

tragedy of the anticommons is never solved by introducing competition in sector B, contrarily to what happens with perfect substitutes (Dari-Mattiacci and Parisi, 2007).[21] In order to confirm such prediction, we now compare consumer surplus when sector B is an oligopoly with the one enjoyed under an integrated monopoly, establishing the following result

Proposition 2. *When sector A is a monopoly and n firms compete in sector B,*

1) if $\alpha_{1k} = \alpha_{IM} = \alpha_{CM}$ $(k = 1, ..., n)$, consumer surplus with an oligopoly in sector B is always lower than with an integrated monopoly but higher than with a complementary monopoly ($CS_{CM} < CS_M < CS_{IM}$).

2) if systems differ in quality, consumer surplus is higher with an oligopoly in sector B than with an integrated monopoly if and only if

$$\sigma_\alpha^2 > \sigma_{CS}^2 = \frac{1}{(1-\gamma)\widetilde{B}^2}\left[\frac{\alpha_{IM}^2}{4n^2} - \tilde{A}^2\bar{\alpha}^2\right] \tag{1.24}$$

where σ_{CS}^2 is decreasing in γ and n. If quality variance is sufficiently high, competition may be preferred even if $\bar{\alpha} < \alpha_{IM}$.

Proof: See Appendix B.

When goods are imperfect substitutes and quality is the same across systems and market structures, then, competition in one sector can certainly improve consumer welfare with respect to a complementary monopoly, but it is never enough to solve the anticommons problem ($CS_M < CS_{IM}$).[22] Competition can effectively increase consumer surplus above CS_{IM} only if both average quality

[21] Our conclusion seem to contradict also the results obtained by McHardy (2006). In his paper, a very low number of competitors selling imperfect substitutes is sufficient to attain the level of social welfare of a complementary monopoly, even if the other sector remains monopolistic.

[22] Interestingly this result still holds even if the number of competitors in sector B is endogenized. In the absence of barriers to entry in sector B and for a common quality level ($\sigma_\alpha^2 = 0$), the equilibrium number of firms will tend to be infinitely large. In fact, by Lemma

and variance play a role. Particularly, while it is not surprising that competition increases consumer welfare when it also increases average quality, from (1.19) it can be verified that quality variance has a positive effect, as well. In other words, our representative consumer benefits from variety (*varietas delectat*). Moreover, both parameters n and γ have a negative effect on σ^2_{CS}. This is because an increase in n and γ decreases equilibrium prices under competition, thus raising consumer surplus, *ceteris paribus*.[23]

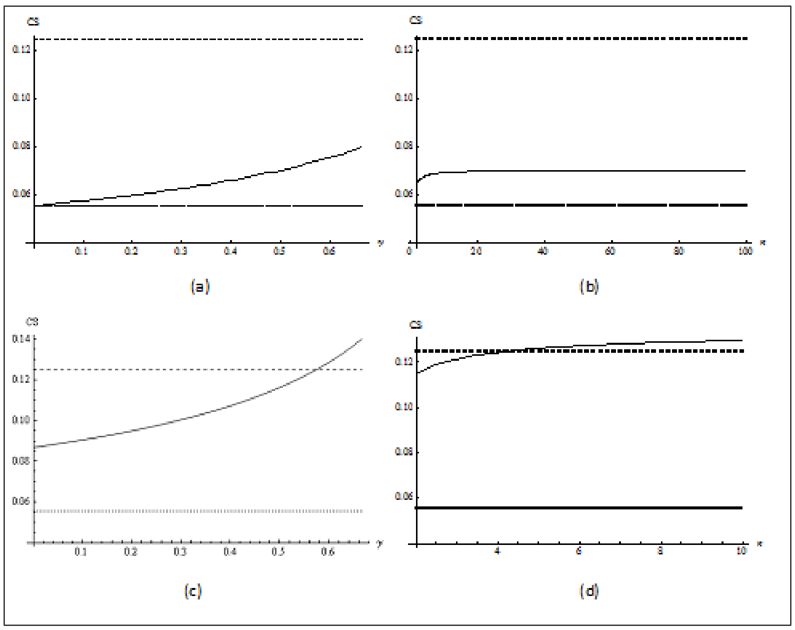

Figure 1.1: Comparing consumer surplus under three regimes - competition, integrated and complementary monopoly (— CS_M,- - - CS_{IM}, ...CS_{CM})

The results in Proposition 2 are shown graphically in Figure 1.1, presenting

1, p_{Bk} decreases with n, but it stays above marginal cost. Particularly, using l'Hôpital's Rule, $lim_{n\to\infty} p^M_{Bk} = \frac{\bar{\alpha}(1-\gamma)}{3-\gamma} > 0$ for $\gamma < 1$. Moreover, $lim_{n\to\infty} p^M_{A1} = \frac{\bar{\alpha}}{3-\gamma}$, so that $lim_{n\to\infty}(p^M_{A1} + p_{Bk}) = \frac{\bar{\alpha}(2-\gamma)}{3-\gamma} > \frac{\bar{\alpha}}{2} = p_{IM}$ and $lim_{n\to\infty} CS_M = \frac{\bar{\alpha}^2}{2(3-\gamma)^2} < CS_{IM}$. Then, contrarily to Dari-Mattiacci and Parisi (2007), under imperfect substitutability, the tragedy is never solved as long as sector A is a monopoly.

[23]Obviously, when $\alpha_{IM} > \bar{\alpha}$, the greater the gap between α_{IM} and $\bar{\alpha}$, the greater the value of σ^2_{CS} needed to compensate for lower quality.

simulations for different parameter values. Panel a) illustrates a case in which $n = 2$, $\bar{\alpha} = \alpha_{IM} = \alpha_{CM} = 1$ and $\sigma_\alpha^2 = 0$. Panel b) represents the same case, this time letting the number of firms n vary and setting $\gamma = \frac{1}{3}$. Both panels clearly show that $CS_M < CS_{IM}$. Panel c) considers a case in which $\sigma_\alpha^2 = 0.25$. and again $\bar{\alpha} = \alpha_{IM} = \alpha_{CM} = 1$. It is possible to verify that now $CS_M > CS_{IM}$ for a sufficiently high value of γ. Finally, panel d) depicts the case in which average quality under competition is slightly lower than the quality of an integrated monopoly ($\bar{\alpha} = 0.95$ and $\alpha_{IM} = 1$). Here, $\gamma = \frac{1}{3}$ and variance is set sufficiently high ($\sigma_\alpha^2 = 0.37$), so that, for $n > 4$, the representative consumer prefers an oligopoly in sector B to an integrated monopoly.[24] When turning to equilibrium profits and producer surplus in the various market configurations, we establish first the following results regarding equilibrium quantities.

Lemma 2. *(a) Q_M is increasing in n and γ; (b) q_{1k}^M is decreasing in n, $k = 1, ..., n$; (c) There exists $\hat{\alpha}_{1k} < \bar{\alpha}$, such that q_{1k}^M is increasing in γ for $\alpha_{1k} > \hat{\alpha}_{1k}$ and is decreasing in γ for $\alpha_{1k} < \hat{\alpha}_{1k}$.*

Proof: See Appendix B.

As for part (a), note that when n increases, both oligopolistic prices and total system prices in (1.9) and (1.11) decrease due to enhanced competition. Moreover, as assumed, such increase in the number of competing firms takes place leaving average quality $\bar{\alpha}$ unchanged, so that the difference $\alpha_{1k} - \bar{\alpha}$ is not affected by the entry of a new available system. Thus, overall, demands for all systems raise proportionately. The case in which γ changes is more complex.

[24]In the simulations presented here, consumer surplus in complementary monopoly (CS_{CM}), is always lower than CS_M whenever $\alpha_{IM} = \alpha_{CM}$. This is due to the assumption that $\bar{\alpha}$ is only slightly smaller than or equal to α_{IM}. If $\bar{\alpha}$ were smaller enough, we might have $CS_M < CS_{CM}$, at least for low values of γ and n.

As γ increases, systems become closer substitutes and their prices decrease (see Lemma 1). However, this does not necessarily translate into a greater demand for each of them. In fact, as implied by the utility function (1.1), consumers have a taste for quality so that, *ceteris paribus*, they prefer systems characterized by a higher α_{1k}. Then, as systems become closer substitutes, consumers will demand more high-quality systems at the detriment of low-quality ones. Consequently, the demand for some of the latter ones (those with $\alpha_{1k} < \hat{\alpha}_{1k}$) decreases as γ increases. This has immediate repercussions on profits, as we will see below. The following Corollary and Proposition use Lemmas 1 and 2 to discuss and compare equilibrium profits.

Corollary 1. Π^M_{A1} *is increasing in* n *and* γ*. Both* Π^M_{Bk} *and* Π^M_B *are decreasing in* n*.*

Corollary 1 states that the monopolist in sector A always benefits from an increase in competition in sector B. This is because both the monopolist's equilibrium price p^M_{A1} and total demand Q_M (from Lemma 2) increase in n and γ. The Corollary also establishes a clear negative relationship between the number of firms in sector B and their profits: as n increases, competition gets fiercer and each firm sets a lower price, sells a lower quantity and obtains lower profits (see Lemmas 1 and 2). This implies that also aggregate profits in sector B decrease with n, "counterbalancing" the growth in the monopolist's profit level in sector A. Regarding the relationship between γ and Π^M_{Bk}, we know from Lemmas 1 and 2 that both p^M_{Bk} and q^M_{1k} decrease with γ for low-quality systems, but that q^M_{1k} increases with γ when the quality of system $1k$ is sufficiently high, i.e. $\alpha_{1k} > \hat{\alpha}_{1k} > \bar{\alpha}$. Then for high-quality systems such positive impact of γ on quantities might prevail and Π^M_{Bk} can be increasing with γ. Such possibility also influences the relationship between γ and Π^M_B, as the following Proposition shows. Particularly, this is more likely to happen when quality variance is high and then the chance of having firms in sector B with $\alpha_{1k} > \hat{\alpha}_{1k}$ is greater, *ceteris paribus*.

Proposition 3. *If* $\bar{\alpha} = \alpha_{IM} = \alpha_{CM}$,

(a)$\Pi_{IM} > \Pi_{A1}^{M} > \Pi_{CM}^{A}$ *for any* $n \geq 2$ *and* $\gamma \in [0, 1]$. *When systems are perfect substitutes (*$\gamma = 1$*),* $\Pi_{A1}^{M} = \Pi_{IM} > \Pi_{CM}^{A}$*;*

(b)Π_{B}^{M} *is increasing in* γ *if and only if* σ_{α}^{2} *is sufficiently high;*

(c) If $\sigma_{\alpha}^{2} = 0$ *then* Π_{B}^{M} *is lower than* Π_{CM}^{B} *and* Π_{IM}. *Also, Producer Surplus (*$PS \equiv \Pi_{B}^{M} + \Pi_{A1}^{M}$*) is such that* $\Pi_{IM} > PS > \Pi_{CM}^{A} + \Pi_{CM}^{B}$.

(d) If $\sigma_{\alpha}^{2} > 0$*,* $n = 2$*, then* $\Pi_{B}^{M} < \Pi_{IM}$. *If* $\sigma_{\alpha}^{2} > 0$*,* $n \geq 3$*, then*$\Pi_{B}^{M} \geq \Pi_{IM}$ *for sufficiently high* σ_{α}^{2}. *Also, for* $n \geq 2$*,* $PS \geq \Pi_{IM}$ *if and only* σ_{α}^{2} *is sufficiently high.*

Proof: See Appendix B.

The positive relationship between Π_{A1}^{M} and n illustrated in Corollary 1 explains why, as indicated in part (a) of Proposition 3, the monopolist's profits are higher when sector B is an oligopoly than when the market is a complementary monopoly. However, whenever $\gamma < 1$ the monopolist's profits are always lower than Π_{IM} so that, even an infinite number of competitors would not allow the monopolist to obtain the same profits of an integrated monopolist. This is because systems are not perfect substitutes, so that prices in the oligopolistic sector remain, on average, above marginal cost and the negative externality of the tragedy is not fully overcome.[25] As for part (b) of Proposition 3, it con-

[25]Only in the limit case in which $\gamma = 1$, the monopolist in sector A is able to extract the whole surplus from sector B, thus behaving like an integrated monopolist. One should notice the analogy between this case and the results in Dari-Mattiacci and Parisi (2007).

firms the previous intuition that industry profits in sector B are increasing with γ when quality variance in sufficiently high. In such case, the increase in profits of high-quality producers more than compensates the decrease in the profits of low-quality ones.

In the remaining two parts, Proposition 3 compares both industry profits in sector B and producer surplus with their respective values under a complementary and an integrated monopoly. In the simple case of a common quality level (part (c)), industry profits in sector B (and then *a fortiori* individual profits) are smaller than both the profits of a complementary and of an integrated monopolist producing the same quality level. The relationship between Π_B^M and Π_{IM} is not surprising and is a direct implication of the results at the beginning of this Section, according to which $p_{Bk}^M < p_{IM}$ and $Q_M < Q_{IM}$, no matter the number of competing firms. Once more, when quality variance is zero, increasing the number of competitors in one sector only is not enough to eliminate the tragedy of the anticommons. Note that we also established that both q_{Bk}^M and p_{Bk}^M are lower than q_{CM} and p_{CM}^B, respectively, so that $\Pi_{Bk}^M < \Pi_{CM}^B$. Part (c) now states that this result holds in aggregate, as well, and that $\Pi_B^M < \Pi_{CM}^B$: introducing competition in sector B unambiguously lowers industry profits, no matter the degree of substitutability.

As for producer surplus, results are ambivalent. On one side, the idea that post-separation entry of new firms in sector B is never able to overcome the tragedy is supported also in terms of the sum of all firms' profits in the economy (so that $\Pi_B^M + \Pi_{A1}^M < \Pi_{IM}$). On the other, we verify that competition in sector B increases the profits of the monopolist in sector A in a way that more than compensates the losses in industry profits in sector B, so that overall producer surplus under competition is greater than under a complementary monopoly ($\Pi_B^M + \Pi_{A1}^M > \Pi_{CM}^A + \Pi_{CM}^B$).

Finally, in part (d) we establish that industry profits in sector B can actually be larger than those of an integrated monopolist (and then *a fortiori*, of a complementary monopolist) when variance is positive. As indicated by equation (1.17), the higher the quality variance, the larger the value of aggregate profits in sector B, so that it may happen indeed that $\Pi_B^M \geq \Pi_{IM}$. Then, provided a sufficiently large value for σ_α^2, producer surplus under competition might also be greater than with an integrated monopoly.[26] In conclusion, quality variance is an indicator of product differentiation and *varietas delectat* not only for consumers, but for sector B as a whole as well. Then, joining the results in Propositions 2 and 3, the following Corollary holds

Corollary 2. *(a) Total Surplus increases with quality variance. (b) When* $\sigma_\alpha^2 = 0$ *and* $\alpha_{1k} = \alpha_{IM} = \alpha_{CM}$ $(k = 1, ..., n)$*, total surplus is greater than with a complementary monopoly but lower than with an integrated monopoly. (c) When* $\sigma_\alpha^2 > 0$, $\bar{\alpha} = \alpha_{IM}$*, there exists a value for* σ_α^2 *such that total surplus is greater than with an integrated monopoly.*

Summing up, consumers are always worse off in a complementary monopoly. Moreover, they might sometimes prefer competition in sector B to an integrated monopoly if quality variance is very high. In fact, in such case they would fully enjoy the benefits of product differentation. Similarly, when variance is large enough, producers in sector B might earn greater industry profits than those obtained by an integrated monopolist. In such circumstance, as indicated in

[26] Note that, for a given average quality, variance is obviously weakly increasing in the number of firms in sector B. In other terms, the higher n, the higher the maximum value that quality variance can take while still satisfying the constraints of the model (that is non-negative prices). This is the reason why this result holds only if $n \geq 3$. Two firms only in sector B are not enough to generate a sufficiently high quality variance (or equivalently a sufficiently high value of the parameter α_{1k}^{max} introduced in the proof of Lemma 1) such that $\Pi_B^M \geq \Pi_{IM}$.

Proposition 3, some very high-quality firms are able to earn sufficiently high profits to compensate both for the low profits of their low-quality competitors and for the loss in market power due to competition compared to an integrated monopoly. Moerover, when quality variance is high, such possibility is actually *favored* by an high degree of substitutability, given that in such instance Π_B^M increases with γ.

Total surplus follows a similar trend. As long as quality is uniform across systems, the tragedy prevails and competition in sector B is never able to raise social welfare above the corresponding integrated monopoly level. However, this does not necessarily hold with a sufficiently high product differentiation, with important implications for antitrust regulation of complementary-good markets. In fact, according to such results the break-up of an integrated firm into independent units producing one component each can be welfare improving if this generates competition for at least one component and if the competing systems in the market exhibit enough quality differentiation. Note that in Proposition 3 we assumed that $\bar{\alpha} = \alpha_{IM} = \alpha_{CM,}$ but our result would be qualitatively the same for $\bar{\alpha} \neq \alpha_{IM}$. Particularly, competition in one sector can still be welfare enhancing even if post-separation entry in such sector reduces average quality, provided a sufficiently high value for quality variance.[27] In the next Section, we extend the model to consider competition in Sector A, too.

1.4 Oligopolies in the markets for both complements

In this Section we assume that both complements A and B are produced in oligopolistic markets. Particularly, component A is produced by n_1 different

[27] In this respect, our paper integrates the main conclusion in Economides (1999), according to which separation of the monopolized production of complementary goods may damage quality.

firms, whereas component B is produced by n_2 firms. Again, firms compete by setting prices.

Since consumers can "mix and match" components at their own convenience, there are $n_1 \times n_2$ systems in the market and the utility function in (1.1) becomes

$$U(q,I) = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \alpha_{ij} q_{ij} - \frac{1}{2}\left[\beta \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} q_{ij}^2 + \gamma \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left(q_{ij}\sum_{z=1}^{n_1}\sum_{s=1}^{n_2} q_{zs} - q_{ij}^2\right)\right] + I \tag{1.25}$$

where q_{ij} represents the quantity of system ij, $(i = 1, ..., n_1;\ j = 1, ..., n_2)$, obtained by combining q_{ij} units of component A purchased from the ith firm in sector A (component Ai), and q_{ij} units of component B purchased from the jth firm in sector B (component Bj). Also in this case, $\alpha_{ij} > 0$ $(i = 1, ..., n_1;\ j = 1, ..., n_2)$, $\gamma \in [0, 1]$. The budget constraint now takes the form $\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} p_{ij} q_{ij} + I \leq M$, where $p_{ij} = p_{Ai} + p_{Bj}$ $(i = 1, ..., n_1;\ j = 1, ..., n_2)$ is the price of system ij.

The first order condition determining the optimal consumption of system tk is

$$\frac{\partial U}{\partial q_{tk}} = \alpha_{tk} - (\beta - \gamma)\, q_{tk} - \gamma \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} q_{ij} - p_{tk} = 0 \tag{1.26}$$

After some tedious algebra, we obtain the demand function for system tk

$$q_{tk} = \frac{b\left(\alpha_{tk} - p_{At} - p_{Bk}\right) - \gamma\left[\sum_{j \neq k}\left(\alpha_{tj} - p_{Bj}\right) - p_{At}\left(n_2 - 1\right)\right] - \gamma \sum_{i \neq t}\sum_{j=1}^{n_2}\left(\alpha_{ij} - p_{ij}\right)}{(\beta - \gamma)\left[\beta + \gamma\left(n_1 n_2 - 1\right)\right]} \tag{1.27}$$

where $b = \beta + \gamma\left(n_1 n_2 - 2\right)$.

As before, to prevent total market size to change with γ, n_1 and n_2 we normalize β as follows[28]

$$\beta = n_1 n_2 - \gamma(n_1 n_2 - 1) \tag{1.28}$$

[28] Again, the second-order condition requires $\gamma \leq 1$.

Given that component At is possibly bought in combination with all n_2 components produced in sector B, total demand and then profits for firm t in sector A are obtained summing q_{tk} in (1.27) over all possible values of k, i.e., $\Pi_{At} = p_{At}D_{At} = p_{At}\sum_{j=1}^{n_2} q_{tj}$. Similarly, profits for firm k in sector B are $\Pi = p_{Bk}D_{Bk} = p_{Bk}\sum_{i=1}^{n_1} q_{ik}$. Then, equilibrium prices p_{At}^O and p_{Bk}^O (the superscript "O" stands for "oligopoly in both sectors") are, respectively

$$p_{At}^O = A\bar{\alpha} + B\left(\bar{\alpha}_t - \bar{\alpha}\right) \tag{1.29}$$

$$p_{Bk}^O = C\bar{\alpha} + D\left(\bar{\alpha}_k - \bar{\alpha}\right) \tag{1.30}$$

where $\bar{\alpha} = \frac{\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\alpha_{ij}}{n_1 n_2}$ is the average quality of all systems available in the market, $\bar{\alpha}_t = \frac{\sum_{j=1}^{n_2}\alpha_{tj}}{n_2}$ is the average quality of the systems containing component t, and $\bar{\alpha}_k = \frac{\sum_{i=1}^{n_1}\alpha_{ik}}{n_1}$ is the average quality of systems containing component k. Parameters A, B, C and D are defined as follows: $A = \frac{n_1(1-\gamma)(n_2-\gamma)}{n_1 n_2(3-2\gamma)+\gamma^2(1+n_1+n_2)-2\gamma(n_1+n_2)}$ $B = \frac{n_1}{2n_1-\gamma}$, $C = \frac{n_2(1-\gamma)(n_1-\gamma)}{n_1 n_2(3-2\gamma)+\gamma^2(1+n_1+n_2)-2\gamma(n_1+n_2)}$ and $D = \frac{n_2}{2n_2-\gamma}$. The equilibrium price of system tk, $p_{tk}^O = p_{At}^O + p_{Bk}^O$, is therefore

$$p_{tk}^O = (A+C)\,\bar{\alpha} + B\left(\bar{\alpha}_t - \bar{\alpha}\right) + D\left(\bar{\alpha}_k - \bar{\alpha}\right) \tag{1.31}$$

Equilibrium quantities are

$$q_{tk}^O = z\bar{\alpha} + \frac{\alpha_{tk} - \bar{\alpha}}{n_1 n_2(1-\gamma)} + \frac{\bar{\alpha}_t - \bar{\alpha}}{n_2(2n_1-\gamma)(1-\gamma)} + \frac{\bar{\alpha}_k - \bar{\alpha}}{n_1(2n_2-\gamma)(1-\gamma)} \tag{1.32}$$

where

$$z = \frac{(n_1-\gamma)(n_2-\gamma)}{n_1 n_2(n_1 n_2(3-2\gamma)+\gamma^2(1+n_1+n_2)-2\gamma(n_1+n_2))} \tag{1.33}$$

Note that component At $(t = 1, ..., n_1)$ is sold in combination with all its n_2 complements, so that each firm's profits in sector A are equal to $\Pi_{At}^O = p_{At}^O \cdot q_{At}^O = p_{At}^O\sum_{k=1}^{n_2} q_{tk}^O$. Similarly, total profits from the sale of complement Bk $(k = 1, ..., n_2)$ amount to $\Pi_{Bk}^O = p_{Bk}^O \cdot q_{Bk}^O = p_{Bk}^O\sum_{t=1}^{n_1} q_{tk}^O$.

As for consumer surplus in this $n_1 \times n_2$ model, we adopt the same procedure followed in section 2.2 to obtain

$$CS = \frac{n_1^2 n_2^2}{2} \left(z^2 \bar{\alpha}^2 + (1-\gamma) Var(q) \right) \tag{1.34}$$

where

$$Var(q) = \frac{\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \left(q_{ij}^O - \bar{q} \right)^2}{n_1 n_2} \tag{1.35}$$

is the variance of the systems' quantities sold in equilibrium in the whole market.[29]

In the remainder of this section we want to investigate the impact that the introduction of competition in sector A has on consumer surplus and on profits, compared to less competitive options, particularly, complementary or integrated monopoly or a situation in which sector A is a monopoly ($n_1 = 1$). The comparison is rather straightforward when all systems produced in oligopoly have the same quality (so that, by symmetry, $Var(q) = 0$). For more general cases, however, the complexity of the expressions for prices, quantities and profits renders the algebraic analysis rather difficult. We will therefore perform numerical simulations.

First, we assume that $Var(q) = 0$, with $\alpha_{tk} = \alpha_{IM} = \alpha_{CM} = \alpha^*$, ($t = 1, ..., n_1$; $k = 1, ..., n_2$) and we establish the following results.

Proposition 4. *When both sectors are oligopolies and* $Var(q) = 0$, $\alpha_{tk} = \alpha_{IM} = \alpha_{CM} = \alpha^*$ *(*$t = 1, ..., n_1$; $k = 1, ..., n_2$*),*

(a) $CS_O > CS_{CM}$*;*

(b) $CS_O > CS_{IM}$ *if and only if*

$$n_1 > n_1^* = \frac{(n_2 - 1)\gamma^2}{n_2(2\gamma - 1) - \gamma^2} \tag{1.36}$$

[29]See Appendix A

where n_1^* decreases both with n_2 and γ.

(c) Oligopolistic profits Π_{At} and Π_{Bk} are always smaller than Π^i_{CM}, hence than Π_{IM}.

Proof: See Appendix B.

The threshold n_1^* is decreasing in n_2, indicating quite intuitively that when the number of firms in one of the two sectors is high (and then competition there is particularly aggressive, benefiting consumers), the tragedy can be solved also for a relatively low number of firms in the other sector.[30] Moreover, a closer look to the expressions for CS_O and CS_{IM} makes us conclude that a competitive industry may be preferred to an integrated monopoly even when both n_1 and n_2 are relatively low. Particularly, two firms in both sectors may be already enough to solve the tragedy when γ is sufficiently high, as shown in Figure 1.2.

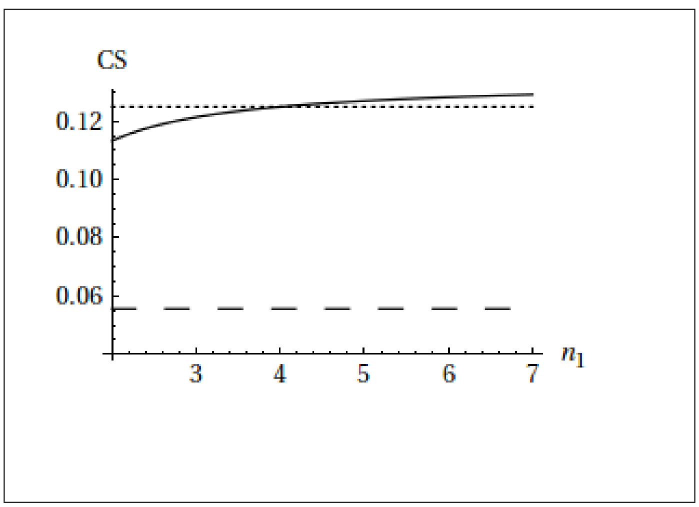

Figure 1.2: Comparing consumer surplus under three regimes when both sectors are olipopolies (— CS_O,- - - CS_{IM}, ...CS_{CM})

[30]Of course, it would be possible to establish a symmetric threshold for n_2, which would then be decreasing in n_1.

Figure 1.2 is obtained assuming $Var(q) = 0$, $\alpha^* = 1$, $n_1 = 2$ and $\gamma = 0.62$. As it can be readily verified, consumer surplus is always higher under competition than in a complementary monopoly. Moreover, it increases with n_1, lying below CS_{IM} for low n_1 and becoming larger than CS_{IM} for $n_1 > 4$ ($n_1^* = 4.021$). Part (b) of Proposition 4 also suggests that the degree of competition required in one sector (say, sector A) to increase consumer surplus above CS_{IM} decreases as either the number of firms in the other sector or the degree of substitutability increase (in fact, n_1^* is decreasing in both n_2 and γ). This happens because an increase in n_2 and/or in γ not only reduces the prices of each single component sold in sector B but also the prices of all systems, thus increasing consumer welfare.[31] Finally, part (c) confirms the relationships among profits found in the $n \times 1$ case, with oligopolists always earning the lowest profits and an integrated monopolist the highest.

If firms produce different qualities and $Var(q) > 0$, the number of competing firms required to make consumer surplus under competition preferred to that obtained in an integrated monopoly decreases. In fact, a positive $Var(q)$ increases CS_O in (1.34), thus increasing the range of the parameters for which $CS_O > CS_{IM}$.[32] The exact changes in prices, quantities, profits and welfare as the number of firms and the degree of substitutability between systems vary are analyzed in the following two simulations.

In both, we assume that the two sectors A and B are characterized by different quality distributions which get reflected on systems' qualities. Specifically, in the first simulation the entry of new firms in one sector allows the composition of ever better systems, so that competition increases average quality in the market.

[31] As we will also see in the simulations below, oligopolists in sector A react to a decrease in the prices in the complementary sector B by increasing their own price. Such increase is however limited, and total system prices overall decrease.

[32] Clearly, a fortiori, $CS_O > CS_{CM}$ always when quality variance is positive.

We set α_{tk} $(t = 1, ..., n_1;\ k = 1, ..., n_2)$ as follows

$\alpha_{11} = 8$	$\alpha_{12} = 8.5$	$\alpha_{13} = 9$	$\alpha_{14} = 9.5$	$\alpha_{15} = 10$
$\alpha_{21} = 7.5$	$\alpha_{22} = 8$	$\alpha_{23} = 8.5$	$\alpha_{24} = 9$	$\alpha_{25} = 9.5$

Due to our chosen values, the set of systems $\{1k\}$ $(k = 1, ..., 5)$ has high average quality than the set $\{2k\}$ and systems denoted by higher k are better in quality. Table 1.1 reports equilibrium prices, quantities and welfare when competition increases in sector B. It can be verified that quantity q_{11} still decreases with n_2, and that quality variance increases demand.

Moreover, prices in sector A increase with n_2, whereas prices in sector B decrease. System prices however decrease in n_2. Unsurprisingly, prices are higher with $\gamma = 0.2$ than with $\gamma = 0.62$, since competition is fiercer in the second case. When $\gamma = 0.2$, consumer and producer surplus are higher under integrated monopoly. Things change when $\gamma = 0.62$; now fiercer competition among closer substitutes leads to substantially lower system prices, thus benefiting consumers (for $n_2 \geq 3$). This more than compensates for the lower producer surplus, so that total surplus in oligopoly is the highest. Complementary monopoly yields the lowest surplus, both for consumers and producers. Individual profits decrease in sector B as n_2 increases, whereas sector A takes advantage of this by increasing its own prices and profits.[33]

In the second simulation, we assume that competition worsens average quality in the market, so that, the larger the number of active firms, the lower $\bar{\alpha}$, $\bar{\alpha}_t$ and $\bar{\alpha}_k$. Again, with no loss of generality, we assume that competition increases in sector B, whereas $n_1 = 2$ throughout the simulation. To obtain the effect of

[33] In Table 1.1 both consumer surplus and profits under monopolistic configurations increase in n_2. This happens because each oligopoly structure (for each n_2) is compared with both types of monopoly at the same average quality and here, by assumption, $\bar{\alpha}$ increases with n_2.

Table 1.1: Impact of competition when firms are heterogeneous and competition decreases quality.

	$\gamma = 0.2$				$\gamma = 0.62$			
	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$
p_{A1}	3.17	3.12	3.06	2.99	2.64	2.66	2.62	-
p_{B1}	3.17	3.14	3.16	3.19	2.64	2.46	2.43	-
p_{11}	6.34	6.26	6.22	6.18	5.28	5.12	5.05	-
q_{11}	1.1	0.8	0.65	0.56	1.65	1.27	1.07	-
CS_O	6.03	6	5.97	5.96	10.63	11.12	11.3	-
CS_{IM}	11.3	10.7	10.12	9.57	11.28	10.7	10.12	-
CS_{CM}	5	4.75	4.5	4.25	5	4.75	4.5	-
CS_M	5.70	5.55	5.34	5.12	6.32	6.41	6.25	-
Π_{A1}	5.65	5.5	5.27	5.02	7.32	7.39	7.21	-
Π_{B1}	5.65	3.84	2.96	2.44	6.32	4.22	3.28	-
Π_O	20.83	19.88	18.91	17.97	22.6	21.5	20.5	-
Π_{IM}	22.56	21.39	20.25	19.14	22.6	21.4	20.25	-
Π_{CM}	20.05	19.01	18	17	20	19	18	-
TS_O	26.86	27.13	24.88	23.94	33.22	32.66	31.8	-
TS_{IM}	33.86	32.08	30.37	28.71	33.84	32.08	30.04	-
TS_{CM}	25.07	23.77	22.5	21.27	27.07	23.77	22.5	-

a decreasing quality level as competition gets fiercer, we set α_{tk} ($t = 1, ..., n_1$; $k = 1, ..., n_2$) as follows[34]

$\alpha_{11} = 10$	$\alpha_{12} = 9.5$	$\alpha_{13} = 9$	$\alpha_{14} = 8.5$	$\alpha_{15} = 8$
$\alpha_{21} = 9.5$	$\alpha_{22} = 9$	$\alpha_{23} = 8.5$	$\alpha_{24} = 8$	$\alpha_{25} = 7.5$

When $\gamma = 0.2$, Table 1.2 shows that individual firms' and system prices decrease with competition. Interestingly, prices are declining and lower in sector A. This reverts the trend observed in the previous simulation, in which the sector not affected by competition was able to limit the impact or even to take advantage of the increased competition in the complementary sector. Such change is indeed driven by the decline in quality. Moreover, demand decreases with competition. (in Table 1.2 we report q_{11}).[35] Even at declining prices and quantities, firms in sector A enjoy however higher profits than firms in sector B and are able to extract a higher surplus than their complementors operating in the more competitive sector. Overall, producer surplus is lower than in an integrated monopoly but higher than in a complementary monopoly. As for consumer surplus, it decreases with competition: lower prices and increased variance are in fact not enough to compensate for the decline in quality. Symmetrically to producer surplus, consumer surplus is highest in integrated monopoly and lowest in complementary monopoly.[36]

When $\gamma = 0.62$, a fifth firm in sector B obtains no demand because of a too low quality level. This is why the most competitive feasible market structure

[34]It should be noticed that the coefficients α_{tk} are the same as in Simulation 2, but in reversed order.

[35]At $n_2 = 6$ the quantity of the lowest quality system becomes negative, implying that increased competition is not sustainable in such market configuration. That's why simulation 2 considers n_2 only up to 5.

[36]Here consumer surplus and profits under monopolistic configurations decrease in n_2 since $\bar{\alpha}$ decreases with higher n_2.

Table 1.2: Impact of competition when firms are heterogeneous and competition decreases quality.

	$\gamma = 0.2$				$\gamma = 0.62$			
	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$
p_{A1}	3.17	3.12	3.06	2.99	2.64	2.66	2.62	-
p_{B1}	3.17	3.14	3.16	3.19	2.64	2.46	2.43	-
p_{11}	6.34	6.26	6.22	6.18	5.28	5.12	5.05	-
q_{11}	1.1	0.8	0.65	0.56	1.65	1.27	1.07	-
CS_O	6.03	6	5.97	5.96	10.63	11.12	11.3	-
CS_{IM}	11.3	10.7	10.12	9.57	11.28	10.7	10.12	-
CS_{CM}	5	4.75	4.5	4.25	5	4.75	4.5	-
CS_M	5.70	5.55	5.34	5.12	6.32	6.41	6.25	-
Π_{A1}	5.65	5.5	5.27	5.02	7.32	7.39	7.21	-
Π_{B1}	5.65	3.84	2.96	2.44	6.32	4.22	3.28	-
Π_O	20.83	19.88	18.91	17.97	22.6	21.5	20.5	-
Π_{IM}	22.56	21.39	20.25	19.14	22.6	21.4	20.25	-
Π_{CM}	20.05	19.01	18	17	20	19	18	-
TS_O	26.86	27.13	24.88	23.94	33.22	32.66	31.8	-
TS_{IM}	33.86	32.08	30.37	28.71	33.84	32.08	30.04	-
TS_{CM}	25.07	23.77	22.5	21.27	27.07	23.77	22.5	-

is at $n_2 = 4$. System prices and quantities decrease as n_2 increases (and prices are lower than in the $\gamma = 0.2$ case, whereas quantities are higher). Interestingly, comparing consumer surplus across market configurations, it can be noticed that $CS_O < CS_{IM}$ for $n_2 = 2$ but $CS_O > CS_{IM}$ for $n_2 \geq 3$. This happens because the comparison is performed for the same quality level (α_{IM} is set equal to $\bar{\alpha}$ for each value of n_2), but quality variance is increasing. Similarly to the $n \times 1$ case, then, as variance increases, consumer welfare might be greater in competition than with an integrated monopoly. Finally, although p_{B1} has the usual pattern (as competition increases in sector B, p_{B1} decreases), p_{A1} has a non-monotonic behavior. First, it increases when n_2 increases from $n_2 = 2$ to $n_2 = 3$. When $n_2 = 4$, however, p_{A1} gets significantly lower than before: average quality is getting so low that firms in sector A are forced to reduce their prices. The initial positive relationship with n_2 was caused by the high degree of substitutability γ that rendered competition especially fierce in sector B. When a further increase of n_2 takes quality to very low levels, however, this does not hold anymore. Profits follow the same pattern: they increase in sector A when n_2 goes from 2 to 3 but then decrease. In other terms, the fiercer competition due to high substitutability does not allow firms in sector A to counteract the decline in demand due to lower average quality with a profit-enhancing price reduction, as it happened when $\gamma = 0.2$. As for profits in sector B, they always decrease and so do total profits. However, $\Pi_O > \Pi_{IM} > \Pi_{CM}$ because of the high quality variance exogenously produced in the simulation, and this result, combined with the trend observed for consumer surplus, produces an increasing trend for social welfare. In fact, as n_2 increases, total surplus increases as well, surpassing the corresponding integrated monopoly value for $n_2 \geq 3$.

Finally, from Table 1.1 and 1.2 it is also immediate to check the positive effect that the increase in competition in sector A has on consumer surplus. In fact, no matter the degree of substitutability γ, $CS_O > CS_M$. Then, even when either γ or n_2 are low (so that they yield lower consumer surplus than an integrated monopoly) and an integrated monopoly is not a viable solution, introducing some competition in sector A is desirable.

1.5 Conclusions

Complementary monopoly is tipically dominated in welfare terms by an integrated monopoly, in which all such complementary goods are offered by a single firm. This is "the tragedy of the anticommons". We have considered the possibility of competition in the market for each complement, presenting a model in which n imperfect substitutes for each perfect complement are produced. We have proved that, if at least one complementary good is produced in a monopoly, an integrated monopoly is always welfare superior to a more competitive market setting. Consequently, favoring competition in some sectors, leaving monopolies in others may be detrimental for consumers. Competition may be welfare enhancing if and only if the goods produced by competitors differ in quality, so that also average quality and variance become important factors to consider.

We have also proved that, when competition is introduced in each sector, the tragedy may be solved for relatively small numbers of competing firms in each sector if systems are close substitutes, and this even in the limit case of a common quality level across systems. Unsurprisingly, the higher the degree of substitutability and the level of competition in one sector, the more concentrated the other sector can be, while still producing higher consumer surplus than an integrated monopoly. Throughout the chapter we have assumed that quality is costless and exogenously distributed across systems. It would be interesting to extend our model and explicitly consider quality as a costly investment in complementary-good markets. Particularly, a study of the incentives for the monopolist A to discourage innovation and quality improvements in sector B seems a very promising line of research. Heller and Eisenberg (1998) have already argued that patents may produce an anticommons problem in that holders of a specific patent may hold up potential innovators in complementary sectors. Particularly, they focus on the case of biomedical research, showing how a patent holder on a segment of a gene can block the development of derivative innovations based on the entire gene. Emblematic, in this respect, the case of Myriad Genetics Inc., which held patents on specific applications of the BRCA1 and BRCA2 genes, and blocked the development of cheaper breast-cancer tests (see Van Oderwalle, 2010, particularly for important legal developments regarding patent protection for human genes in the U.S.).

Appendix A

Consumer surplus when A is a monopoly

Following Hsu and Wang (2005), consumer surplus can be written as

$$CS = \frac{n(1-\gamma)}{2}\sum_{j=1}^{n} q_{1j}^2 + \frac{\gamma}{2}\left(\sum_{j=1}^{n} q_{1j}\right)^2 = \frac{n(1-\gamma)}{2}\sum_{j=1}^{n}\left(q_{1j}-\bar{q}\right)^2 + \frac{n^2}{2}\left(\bar{q}\right)^2 \quad (1.37)$$

where $\bar{q} = \frac{\sum_{j=1}^{n} q_{1j}}{n} = \frac{Q}{n}$ is average quantity. Using (1.13), we can write

$$\bar{q} = \widetilde{A}\bar{\alpha} \quad (1.38)$$

and

$$q_{1k} - \bar{q} = \widetilde{B}\left(\alpha_{1k} - \bar{\alpha}\right) \quad (1.39)$$

where $\widetilde{A} = \frac{(n-\gamma)}{n(n(3-\gamma)-2\gamma)}$ and $\widetilde{B} = \frac{(n-\gamma)}{n(1-\gamma)(2n-\gamma)}$. Also, using (1.39),

$$\sum_{j=1}^{n}\left(q_{1j}-\bar{q}\right)^2 = \widetilde{B}^2 n\sigma_{\alpha}^2 \quad (1.40)$$

Finally, substituting (1.38) and (1.40) into (1.37), we obtain

$$CS^M = \frac{n^2(1-\gamma)}{2}\tilde{B}^2\sigma_\alpha^2 + \frac{n^2}{2}\tilde{A}^2\bar{\alpha}^2 \tag{1.41}$$

Profits and Consumer Surplus in the $n_1 \times n_2$ case

It is immediate to obtain the total amount of component At ($t = 1, ..., n_1$) sold in equilibrium if we sum q_{tk}^O over the n_2 complements which At is sold with (that is $q_{At}^O = \sum_{k=1}^{n_2} q_{tk}^O$). Then,

$$q_{At}^O = \frac{(n_1-\gamma)(\gamma-n_2)\overline{\alpha}}{n_1(\gamma(n_2(2-\gamma)-\gamma)+n_1((2-\gamma)\gamma+n_2(2\gamma-3)))} + \frac{(n_1-\gamma)(\bar{\alpha}_t-\bar{\alpha})}{n_1(2n_1-\gamma)(1-\gamma)} \tag{1.42}$$

Similarly,

$$q_{Bk}^O = \frac{(n_2-\gamma)(\gamma-n_1)\overline{\alpha}}{n_2(\gamma(n_1(2-\gamma)-\gamma)+n_2((2-\gamma)\gamma+n_1(2\gamma-3)))} + \frac{(n_2-\gamma)(\bar{\alpha}_k-\bar{\alpha})}{n_2(2n_2-\gamma)(1-\gamma)} \tag{1.43}$$

As for consumer surplus, we generalize Hsu and Wang (2005) and rewrite it as

$$CS = \frac{n_1n_2(1-\gamma)}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}(q_{ij}-\bar{q})^2 + \frac{n_1^2n_2^2}{2}\bar{q}^2 \tag{1.44}$$

Using (1.32), we find that

$$\bar{q} = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} q_{ij}^O = z\bar{\alpha} \tag{1.45}$$

so that we can define $Var(q)$in equation (1.35). Finally, substituting (1.45) and (1.35) into (1.44), we obtain equation (1.34).

Appendix B

Proof of Lemma 1

In order to prove that $\frac{\partial p_{Bk}^M}{\partial \gamma} < 0$ we note first that this is always true if $\alpha_{1k} < \bar{\alpha}$. In fact, $\frac{\partial}{\partial \gamma}\frac{n(1-\gamma)}{n(\gamma-3)-2\gamma} = -\frac{2(n-1)n}{(n(3-\gamma)-2\gamma)^2} < 0$ and $\frac{\partial}{\partial \gamma}\frac{n}{2n-\gamma} = \frac{n}{(2n-\gamma)^2} > 0$. If $\alpha_{1k} > \bar{\alpha}$, it may be that $\frac{\partial p_{Bk}^M}{\partial \gamma} > 0$ for a sufficiently high value of α_{1k}, and in particular for $\alpha_{1k} > \tilde{\alpha}_{1k}$, where $\tilde{\alpha_{1k}}$ is obtained solving $\frac{\partial p_{Bk}^M}{\partial \gamma} = 0$ with respect to α_{1k}. We then check whether $\tilde{\alpha}_{1k}$ is a feasible value for an above-average quality. To do that, we compute first the highest α_{1k} compatible with a given average $\bar{\alpha}$, α_{1k}^{max}, which is obtained when the remaining $n-1$ firms produce systems of such low quality $\alpha_{1s}^{min} < \bar{\alpha}$, $s \neq k$ as to optimally set their price equal to marginal cost (so that they remain active in sector B), that is $p_{Bs}^M = 0$. From (1.9), we obtain:

$$\alpha_{1s}^{min} = \frac{\bar{\alpha}(n-\gamma)(1+\gamma)}{n(3-\gamma)-2\gamma} \tag{1.46}$$

Setting $\alpha_{1s} = \alpha_{1s}^{min}$ for all firms $s \neq k$, we obtain α_{1k}^{max} solving

$$\frac{(n-1)\,\alpha_{1s}^{min} + \alpha_{1k}^{max}}{n} = \bar{\alpha} \tag{1.47}$$

i.e., $\alpha_{1k}^{max} = n\bar{\alpha} - (n-1)\,\alpha_{1s}^{min}$. Substituting such value into $\frac{\partial p_{Bk}^M}{\partial \gamma}$, we have

$$\left.\frac{\partial p_{Bk}^M}{\partial \gamma}\right|_{\alpha_{1k}=\alpha_{1k}^{max}} = \frac{(n-1)n\bar{\alpha}\left[2\gamma^2 - n\left(1+4\gamma-\gamma^2\right)\right]}{(2n-\gamma)\left[n(\gamma-3)+2\right]^2} < 0.$$

Hence, $\alpha_{1k}^{max} < \tilde{\alpha}_{1k}$ always and $\frac{\partial p_{Bk}^M}{\partial \gamma} < 0$ for all $\gamma \in [0,\, 1]$.

Similarly, in order to prove that $\frac{\partial p_{Bk}^M}{\partial n} < 0$ for all $n \geq 2$, we note from (1.9) that $\frac{\partial p_{Bk}^M}{\partial n} < 0$ always if $\alpha_{1k} > \bar{\alpha}$, since $\frac{\partial}{\partial n}\frac{n(1-\gamma)}{n(3-\gamma)-2\gamma} = -\frac{2(1-\gamma)}{(n(3-\gamma)-2\gamma)^2} < 0$ and $\frac{\partial}{\partial n}\frac{n}{2n-\gamma} = -\frac{\gamma}{(2n-\gamma)^2} < 0$. If $\alpha_{1k} < \bar{\alpha}$, it may be that $\frac{\partial p_{Bk}^M}{\partial n} > 0$ for a sufficiently low value of α_{1k}, but substituting to α_{1k} in (1.46) its minimun value, α_{1k}^{min}, we obtain $\left.\frac{\partial p_{Bk}^M}{\partial n}\right|_{\alpha_{1k}=\alpha_{1k}^{min}} = \frac{n\bar{\alpha}\gamma(\gamma^2-1)}{(2n-\gamma)(n(3-\gamma)-2\gamma)^2}$, which is negative for the whole parameters' range. □

Proof of Proposition 1

Note first that $\frac{\partial p_{1k}^M}{\partial \bar{\alpha}} = -\frac{(2\gamma-1)[1+(n-2)\gamma]}{[3+\gamma(2n-5)][2+\gamma(2n-3)]} > 0$ for all $\gamma < 1$.

We now prove that $\bar{p}_{1k}^M$ decreases with n. From (1.8) it can be readily verified that $\frac{\partial p_{A1}^M}{\partial n} > 0$, whereas Lemma 1 demonstrates that $\frac{\partial p_{Bk}^M}{\partial n} < 0$. It is then sufficient to prove that $\frac{\partial p_{A1}^M}{\partial n} < \left|\frac{\partial p_{Bk}^M}{\partial n}\right|$ when $\left|\frac{\partial p_{Bk}^M}{\partial n}\right|$ takes its minimum value with respect to α_{1k}, *ceteris paribus*. Note first that $\frac{\partial p_{Bk}^M}{\partial n} = -\frac{\gamma(\alpha_{1k}-\bar{\alpha})}{[2+(2n-3)\gamma]^2} - \frac{2(1-\gamma)\bar{\alpha}}{[3+(2n-5)\gamma]^2}$, which reaches its minimum value when $\alpha_{1k} = \alpha_{1k}^{min}$ (α_{1k}^{min} is defined in the proof of Lemma 1), since $-\frac{\gamma(\alpha_{1k}-\bar{\alpha})}{[2+(2n-3)\gamma]^2}$ is positive and maximum at α_{1k}^{min}. It is then easy to verify that $\frac{\partial p_{A1}^M}{\partial n} - \left|\frac{\partial p_{Bk}^M}{\partial n}\right|_{\alpha_{ik}=\alpha_{ik}^{min}} = -\frac{\alpha(1-\gamma)(2+4\gamma(n-\bar{2})+(8-7n+2n^2)\gamma^2)}{(1+\gamma(n-1))(3+\gamma(2n-5))^2(2+\gamma(2n-3))} < 0$ for all γ and n.

A similar proof works for $\frac{\partial p_{1k}^M}{\partial \gamma}$.

The effect on CS_M is a direct consequence of the influence of γ and n on system prices. □

Proof of Proposition 2

Part 1). In this case $\alpha_{1k} = \bar{\alpha}$, $(k = 1, ..., n)$ and $\sigma_\alpha^2 = 0$. From (1.19), $CS_M = \frac{n^2}{2}\tilde{A}^2\bar{\alpha}^2$. Comparing such expression with consumer surplus under integrated and complementary monopoly (given by (1.20) and (1.21), respectively), we note immediately that the difference $CS_M - CS_{IM} = \frac{\bar{\alpha}^2 n(1-\gamma)(n(\gamma-5)+4\gamma)}{8(n(3-\gamma)-2\gamma)^2}$ is negative, while the difference $CS_M - CS_{CM} = \frac{\bar{\alpha}^2\left(n(6\gamma(n-1)+\gamma)+5\gamma^2\right)}{18(n(3-\gamma)-2\gamma)^2}$ is positive, for all $n \geq 2$ and $\gamma \in [0, 1]$.

Part 2). When $\sigma_\alpha^2 > 0$, subtracting CS_{IM} from CS_M and solving for σ_α^2, we obtain σ_{CS}^2 in expression (1.24). Note that $\sigma_{CS}^2 > 0$ iff $\bar{\alpha} < \frac{\alpha_{IM}}{2\tilde{A}n}$. It can be verified that $\alpha_{IM} < \frac{\alpha_{IM}}{2\tilde{A}n}$, so that it is possible to have a case in which $\bar{\alpha} < \alpha_{IM}$ and $CS_M > CS_{IM}$.

Finally, given that CS_M is increasing in n and γ, the minimum value of σ_α^2 required to have $CS_M \geq CS_{IM}$, σ_{CS}^2, must be decreasing in n and γ. $\square$

Proof of Lemma 2

Differentiating q_{ik}^M in (1.13) with respect to γ we get

$$\frac{\partial q_{ik}^M}{\partial \gamma} = \frac{(n-1)\bar{\alpha}}{(n(3-\gamma)-2\gamma)^2} + \frac{(2n^2+\gamma^2-n(1+2\gamma))(\alpha_{1k}-\bar{\alpha})}{n(1-\gamma)^2(2n-\gamma)^2} \tag{1.48}$$

When $n \geq 2$ and $\gamma \in [0,1]$, the first term on the right-hand side of (1.48) is positive. The second term is positive if $\alpha_{1k} > \bar{\alpha}$ and negative otherwise. Thus, $\frac{\partial q_{ik}^M}{\partial \gamma} > 0$ always if $\alpha_{1k} > \bar{\alpha}$. If $\alpha_{1k} < \bar{\alpha}$, the maximum negative value of the second term in (1.48) is obtained when α_{1k} reaches its minimum feasible value, α_{1s}^{min} (see equation (1.46) in the proof of Lemma 1). Evaluating $\frac{\partial q_{ik}^M}{\partial \gamma}$ at $\alpha_{1k} = \alpha_{1s}^{min}$ we obtain $\left.\frac{\partial q_{ik}^M}{\partial \gamma}\right|_{\alpha_{1k}=\alpha_{1s}^{min}} = -\frac{(n(4n-6\gamma-1)+\gamma^2(2+n))(n-\gamma)\bar{\alpha}}{n(2n-\gamma)(1-\gamma)(n(3-\gamma)+2\gamma^2}<0$. Thus, given that $\frac{\partial q_{ik}^M}{\partial \gamma}$ is continuous in α_{1k}, there exists $\hat{\alpha}_{1k} < \bar{\alpha}$ such that $\frac{\partial q_{ik}^M}{\partial \gamma} \geq 0$ for $\alpha_{1k} \geq \hat{\alpha}_{1k}$ and negative otherwise.

Differentiating q_{ik}^M in (1.13) with respect to n we get

$$\frac{\partial q_{ik}^M}{\partial n} = \frac{((3-\gamma)n(2-n)-2\gamma^2)\bar{\alpha}}{n^2(n(3-\gamma)-2\gamma)^2} + \frac{(2n(n-2\gamma)+\gamma^2)(\alpha_{1k}-\bar{\alpha})}{n^2(\gamma-1)(2n-\gamma)^2} \tag{1.49}$$

When $n \geq 2$ and $\gamma \in [0,1]$, the first term on the right-hand side of (1.49) is negative. The second term is negative if $\alpha_{1k} > \bar{\alpha}$ and positive otherwise. Thus, $\frac{\partial q_{ik}^M}{\partial n} < 0$ always if $\alpha_{1k} > \bar{\alpha}$. If $\alpha_{1k} < \bar{\alpha}$, the maximum positive value for the second term of (1.49) occurs when $\alpha_{1k} = \alpha_{1s}^{min}$. Evaluating $\frac{\partial q_{ik}^M}{\partial n}$ at this value we obtain $\left.\frac{\partial q_{ik}^M}{\partial n}\right|_{\alpha_{1k}=\alpha_{1s}^{min}} = -\frac{(n-\gamma)\gamma(1-\gamma)\bar{\alpha}}{n(2n-\gamma)(n(3-\gamma)-2\gamma)^2} < 0$. Thus, $\frac{\partial q_{ik}^M}{\partial n} < 0$.

Finally, define total quantity as

$$Q^M \equiv \sum_{k=1}^{n} q_{ik}^M = \frac{\bar{\alpha}(n-\gamma)}{n(3-\gamma)-2\gamma} \tag{1.50}$$

Differentiating (1.50) with respect to γ and n we obtain $\frac{\partial Q^M}{\partial \gamma} = \frac{n(n-1)\bar{\alpha}}{(n(3-\gamma)-2\gamma)^2} > 0$ and $\frac{\partial Q^M}{\partial n} = \frac{\gamma(1-\gamma)\bar{\alpha}}{(n(3-\gamma)-2\gamma)^2} > 0$ in the admissible range of the parameters. $\square$

Proof of Proposition 3

Part (a). Comparing Π^M_{A1} in (1.14) and Π^A_{CM} in (1.21), we obtain $\Pi^M_{A1} - \Pi^A_{CM} = \frac{(n-1)\bar{\alpha}^2\gamma(n(6-\gamma)-5\gamma)}{9(n(3-\gamma)-2\gamma)^2} > 0$ in the relevant parameters' range. Similarly, $\Pi^M_{A1} - \Pi_{IM} = -\frac{n\bar{\alpha}(1-\gamma)(n(5-\gamma)-4\gamma)}{4(n(3-\gamma)-2\gamma)^2} < 0$. Note that $lim_{n\to\infty}\Pi^M_{A1} = \frac{\bar{\alpha}^2}{(3-\gamma)^2}$, which is in any case smaller than Π_{IM} when $\gamma \in [0,1]$. Only at $\gamma = 1$ we would have $\Pi^M_{A1} = \Pi_{IM}$.

Part (b). From Lemmas 1 and 2, both p^M_{Bk} and q^M_{1k} decrease with n. Then both Π^M_{Bk} and Π^M_B also decrease with n.

To prove the impact of γ on Π^M_B, let us differentiate expression (1.17) with respect to γ. We find:

$$\frac{\partial \Pi^M_B}{\partial \gamma} = \frac{n(n(2n-3\gamma)+\gamma(2-\gamma))}{(2n-\gamma)^3(1-\gamma)^2}\sigma^2_\alpha - \frac{n(n-1)(n+\gamma(n-2))}{(n(3-\gamma)-2\gamma)^3}\bar{\alpha} \tag{1.51}$$

It might then happen that $\frac{\partial \Pi^M_B}{\partial \gamma} > 0$ if σ^2_α is high enough for given $\bar{\alpha}$. It is a well-known result in statistics that the maximum variance $\sigma^2_{\alpha max}$ in a discrete distribution is attained when $\frac{n}{2}$ firms have quality equal to the minimum value in the range and $\frac{n}{2}$ firms have quality equal to the maximum value in the range (see Plackett, 1947). In our specific case, the minimum value in the range is given by α^{min}_{1s}, whereas the maximum value can be computed given the average $\bar{\alpha}$ and the fact than $\frac{n}{2}$ firms produce α^{min}_{1s}. Define such maximum $\check{\alpha} = \bar{\alpha}\left(n - \frac{(n-\gamma)(1+\gamma)}{n(3-\gamma)-2\gamma}\right)$. Then maximum variance would be $\sigma^2_{max} = \frac{1}{2}\bar{\alpha}^2\left(\frac{(1-\gamma)^2(2n-\gamma)^2}{(n(3-\gamma)-2\gamma)^2}\right) + \left(n-1+\frac{(n-\gamma)(1+\gamma)}{n(3-\gamma)-2\gamma}\right)^2$. By differentiating Π^M_B with respect to γ and solving the derivative with respect to σ^2_α, it is possible to verify that $\frac{\partial \Pi^M_B}{\partial \gamma} \geq 0$ iff $\sigma^2_\alpha \geq \sigma^2_0 = \frac{(n-1)\bar{\alpha}^2(2n-\gamma)^3(1-\gamma)^2(n-2\gamma+n\gamma)}{(n(3-\gamma)-2\gamma)^3(2n^2-3n\gamma-\gamma(1-2\gamma))}$. To compare σ^2_0 with σ^2_{max}, we evaluate the expression $\sigma^2_{max} - \sigma^2_0$ numerically for all admissible values of γ and we find that $\sigma^2_{max} > \sigma^2_0$ for all $n \geq 2$, implying that

$\frac{\partial \Pi_B^M}{\partial \gamma} > 0$ when σ_α^2 is sufficiently high.

Part (c). If $\sigma_\alpha^2 = 0$, all systems have the same quality level α_{1k}, $k = 1, ..., n$. Moreover, if $\alpha_{1k} = \alpha_{IM} = \alpha_{CM}$, the difference $\Pi_B^M - \Pi_{CM}^B = -\frac{(n-1)\bar{\alpha}^2\gamma(3n+\gamma(n-4))}{9(n(3-\gamma)-2\gamma)^2}$ is always negative in the admissible parameters' range. (We know already that $\Pi_{CM}^B < \Pi_{IM}$. Hence, *a fortiori*, $\Pi_B^M - \Pi_{IM} < 0$). As for Producer Surplus, $PS \equiv \Pi_{A1}^M + \Pi_B^M = \frac{\bar{\alpha}^2\left[n^2(2-\gamma)-n(3-\gamma)\gamma+\gamma^2\right]}{[n(3-\gamma)-2\gamma]^2}$. It is easy to verify that $PS - \Pi_{IM} = -\frac{n\bar{\alpha}^2(1-\gamma)^2}{4(n(3-\gamma)-2\gamma)^2} < 0$. Also, $\Pi_{A1}^M + \Pi_B^M - \Pi_{CM}^A - \Pi_{CM}^B = PS - 2\Pi_{CM}^i = \frac{(n-1)\bar{\alpha}^2\gamma(n(3-2\gamma)-\gamma)}{9(n(3-\gamma)-2\gamma)^2}$ which is always positive in the relevant parameters' range.

Part (d). The final result is immediate and is obtained solving $\Pi_{Bk}^M = \Pi_{IM}$ with respect to σ_α^2. Then $\Pi_{Bk}^M \geq \Pi_{IM}$ iff $\sigma_\alpha^2 \geq \sigma_{\Pi_B}^2 = \frac{(n-1)\bar{\alpha}^2(1-\gamma)\gamma(2n-\gamma)^2(n(3+\gamma)-4\gamma)}{9n(n-\gamma)(n(3-\gamma)-2\gamma)^2}$, where $\sigma_{\Pi_B}^2 < \sigma_{max}^2$ for all $n \geq 3$ (numerical evaluation for all admissible values of γ). For $n = 2$, $\sigma_{\Pi_B}^2 > \sigma_{max}^2$, implying that $\Pi_{Bk}^M < \Pi_{IM}$. As for Producer Surplus, the result is obtained solving $\Pi_{A1}^M + \Pi_B^M = \Pi_{IM}$ with respect to σ_α^2. Then $\Pi_{A1}^M + \Pi_B^M \geq \Pi_{IM}$ iff $\sigma_\alpha^2 \geq \sigma_{PS}^2 = \frac{n\bar{\alpha}^2(1-\gamma)^2(2n-\gamma)^2}{4(n-\gamma)(n(3-\gamma)-2\gamma)^2}$. Also, it is possible to establish (through numerical evaluation) that $\sigma_{PS}^2 < \sigma_{max}^2$ for all $n \geq 2$. □

Proof of Proposition 4

Part (a). The proof is immediate, setting $Var(q) = 0$ in (1.34) and comparing the resulting expression with CS_{CM}.

Part (b). Solving $CS_O - CS_{IM} = 0$ with respect to n_1, i.e. $\frac{n_1^2 n_2^2}{2} z^2 q^2 - \frac{\alpha^{*2}}{8} = 0$, yields two solutions, $n_{11} = \frac{(n_2-1)\gamma^2}{n_2(2\gamma-1)-\gamma^2}$ and $n_{12} = \frac{\gamma(n_2(4-\gamma)-3\gamma)}{n_2(5-2\gamma)-(4-\gamma)\gamma}$, so that $CS_O > CS_{IM}$ iff either $n_1 < n_{12}$ or $n_1 > n_{11}$. It is possible to verify, however, that $n_{12} < 1$ for all γ and n_2 in the admissible range of the parameters. Therefore, $CS^O \geq CS_{IM}$ iff $n_1 \geq n_{11}$ and $n_{11} = n_1^*$ in (1.36). Finally, differentiating (1.36) with respect to γ yields $\frac{\partial n_1^*}{\partial \gamma} = -\frac{2(n_2-1)n_2(1-\gamma)\gamma}{(n_2(1-2\gamma)+\gamma^2)^2} < 0$, whereas differentiating it with respect to n_2 yields $\frac{\partial n_1^*}{\partial n_2} = -\frac{(1-\gamma)^2\gamma^2}{(n_2(1-2\gamma)+\gamma^2)^2} < 0$.

Part (c). For this part, it suffices to prove that either Π_{At} or Π_{Bk} is smaller than Π^i_{CM}. The remaining inequality would be implied by the clear symmetry. Moreover, being $\Pi^i_{CM} < \Pi_{IM}$, in such case Π_{At} and Π_{Bk} would also be smaller than Π_{IM}. By comparing Π_{At} with Π^A_{CM}, we find that

$$\Pi_{At} - \Pi^A_{CM} = \frac{1}{9}\alpha^{*2}\left(\frac{9(n_1-\gamma)(n_2-\gamma)^2(1-\gamma)}{n_2(\gamma(n_2(\gamma-2)+\gamma)+n_1(n_2(3-2\gamma)+(\gamma-2)\gamma))^2}-1\right) \tag{1.52}$$

Numerically solving (1.52) with respect to n_1 for given values of n_2 and considering all the admissible values for γ, it is possible to check that (1.52) admits two solutions $\tilde{n}_a$ and $\tilde{n}_b$ and that both are always lower than 1 when not imaginary. Simulations show that $\Pi_{At} - \Pi^A_{CM} \geq 0$ for $\tilde{n}_a \leq n_1 \leq \tilde{n}_b$ (when $\tilde{n}_a$ and $\tilde{n}_b$ are real) and $\Pi_{At} - \Pi^A_{CM} < 0$ when $\tilde{n}_a$ and $\tilde{n}_b$ are imaginary. This implies that $\Pi_{At} - \Pi^A_{CM} < 0$ in the relevant range of the parameters. The same proof can be applied to Π_{Bk}. □

CHAPTER 2

The Effects of Mergers and Quality Leadership on the Tragedy of the Anticommons *

2.1 Introduction

Recently, a considerable amount of attention has been devoted to a specific class of market distortions, known as "the tragedy of the anticommons" (Heller 1998, Buchanan and Yoon 2000, Parisi et al. 2005). Based on Cournot (1838)'s "complementary oligopoly", such literature argues that social welfare might be better served by policies favoring integration. In fact, when complementary goods are sold by different firms, prices are higher than those set by a monopoly selling all the complementary goods and a merger would yield a higher consumer surplus. While the resulting social welfare may fall short of the perfectly competitive one, a merger might represent a second best solution.

In Chapter 1 we established that this result might not hold when the complementary goods are sold in oligopolistic markets. In particular, we showed that when the products sold in each market are imperfect substitutes, the tragedy can be overcome when competition in "sufficiently high", that is for a sufficiently

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high number of firms operating in each market. In this second Chapter we keep the assumption of imperfect substitutability among the competitors of each complement, but now their number is maintained fixed. What varies instead is the type of quality leadership emerging in the market. In fact, what we claim is that this is another crucial variable behind the determination of the relative strength of the anticommons problem and of the lack of competition. To define quality leadership, consider a setting in which competing integrated firms produce all components of a system (e.g., an operating system plus a word processor or a camera body plus lenses). We have a "quality leader" when a single firm produces better quality components than all other competitors. For instance, the same firm produces both the high-quality operating system and the high-quality word processor. We have "shared quality leadership" in the opposite situation, in which one firm manufactures the best operating system and another firm manufactures the best word processor.

When the market is characterized by the presence of a quality leader, either "disintegrating" (i.e. "breaking up") a firm producing complementary goods or prohibiting a merger leads to lower prices, lower profits and higher consumer surplus. On the contrary, if a market is characterized by shared leadership, integration may be welfare superior for consumers, since disintegrating (or not allowing mergers) could create an anticommons problem. In other words, while the negative effects of lack of competition always overcome the anticommons problem in the presence of a quality leader, the tragedy might prevail in case of shared leadership. One of the key insights of our analysis is that, with full-quality leadership, complements produced by the same firm are in fact perceived as substitutes, so that an increase in the price of one good increases the demand for the complements produced by the same firm. In the case of shared leadership, on the other hand, cross-price effects among complements produced by the same firm have the usual impact in that an increase in the price of one good decreases the demand for all complements. Now, it's the cross-price effect among complements produced by different firms that has instead an opposite impact, with an increase in the price of a good inducing an increase in the demand of complements produced by other firms.

Similarly to our approach, Economides and Salop (1992) analyze the different effects of competition and integration on the equilibrium prices of complementary components by examining several alternative market configurations. Particularly, they prove that such prices are always lower with integration than with independent firms. As already mentioned in Chapter 1, however, in their model there is no quality differentiation, as implied by the assumption of symmetric demand for systems, so that there is no room for quality leadership of any sort. As a consequence, their results are characterized by traditional cross price effects among same-firm components and disintegration always involves an anticommons problem. In other words, their contribution simply represents a generalized version of the Cournot complementary monopoly.

Our contribution is also related to the economic literature on "mix and match": firms producing all or some components of a system might sell them as a bundle or separately, allowing consumers to fully "mix and match" across firms (Matutes and Regibeau 1988, Einhorn 1992, Denicolò 2000). There are two main differences between such literature and our contribution. First of all, we assume that consumer tastes are distributed across systems and not across single components. In the latter case, in fact, the demand for each component would end up being independent of the prices of other components, i.e. there would be no cross-price effects, at least as long as firms do not engage in "mixed bundling" practices. It is therefore questionable whether such previous literature is fully capable of giving account of the complexities of complementary markets where substitutes exist for each complement. Second, our approach is more policy oriented. Rather than focusing on firms' strategic decisions we analyze the impact of integration and/or mergers on social welfare.

Finally, our results also provide a contribution to the literature studying pricing decisions and welfare effects of mergers in complementary system markets when the merged firm can also engage in mixed bundling (Gans and King 2006, Choi 2008). Our model already offers novel insights on the effects of antitrust policies in complementary markets, even without allowing for such practice. However, when considering mixed bundling, our conclusion that the tragedy of the anticommons may not characterize all markets for complementary products is actually reinforced. In fact, while in full leadership competition continues to overcome the tragedy, now disintegrating firms under shared leadership might not create an anticommons problem at all.[1]

The Chapter is organized as follows. Section 2 introduces the model. Section 3 and 4 analyze the structure of market demand under the two alternative assumptions of full and shared quality leadership. Section 5 presents the main results of the paper, showing how the effects of mergers and disintegration change with different quality leadership. Intermediate cases in which integrated firms compete with independent producers of separate components are analyzed separately in Section 6. Section 7 extends the model to the case of mixed bundling. Section 8 concludes. The Appendix contains the proofs of some Propositions and Lemmas in the text.

[1] Anticipating the main topic of Chapter 3, our analysis of the tradeoff between the tragedy of the anticommons and competition is also related to the literature on vertical differentiation and entry (Nalebuff 2004, Chen and Nalebuff 2006, Casadesus-Masanell et al. 2007, Peitz, 2008, Alvisi et al., 2009). In particular, while Nalebuff (2004) and Peitz (2008) argue that integration tends to generate barriers to entry, we will show in the next Chapter that allowing firms to sell all components of a system may be both welfare enhancing and pro-competitive.

2.2 The Model

Consider two complementary goods, 1 and 2, which are valuable only if purchased together. An example of such a case would be a software package run on a complementary hardware product. Consumers combine 1 and 2 on a one to one basis to form a system. Initially there are two competing firms, A and B, each manufacturing both complements. Components are fully compatible, so that there are four ways to form a system, defined as $AA = \{A_1, A_2\}$, $BB = \{B_1, B_2\}$, $AB = \{A_1, B_2\}$, $BA = \{B_1, A_2\}$.

We consider two distinct producer relationships: "*full quality leadership*", where firm A manufactures a superior version of both components and "*shared quality leadership*", where firm A manufactures a superior version of component 1 (hardware) and firm B a superior version of component 2 (software). In analogy with Einhorn (1992), we also assume that, for all consumers, the incremental value of the hardware to the system is higher than the one provided by the software. As such, a system with good quality hardware is valued by consumers more than a system with good quality software.

Under full quality leadership, the qualities of the four available systems will then be ranked as follows: $q_{AA} > q_{AB} > q_{BA} > q_{BB}$. Analogously, under shared quality leadership, the quality ranking is $q_{AB} > q_{AA} > q_{BB} > q_{BA.}$

Let the price of component i_1 be p_{i1} and the price of component j_2 be p_{j2} ($i = A, B$ and $j = A, B$). Then the system ij is available at a total price of $p_{ij} = p_{i1} + p_{j2}$, ($i = A, B$ and $j = A, B$). Each consumer has the same reservation price V for the worst available system. Let θ represent the consumer taste parameter for the quality of the system, where θ is uniformly distributed in the interval $[0, 1]$. The (indirect) utility function of a consumer purchasing system ij is then $U_{ij} = V + \theta q_{ij} - p_{ij}$. This functional form is similar to that used by Gabszewicz and Thisse (1979) and Economides (1989). However, with respect to their approach, we only consider cases in which all consumers purchase one of the four available systems and all systems have positive demand.[2] Both firms set their prices simultaneously and we assume that all components are produced at zero costs. This assumption is quite common in the above cited literature, and is with no loss of generality under symmetry, that is when the production of all components entails the same unit cost. However, one may expect that the high-quality components and the components that contributes more to the incremental value of the system are also more costly to produce. In such case, under full leadership with integrated firms, the quality leader would suffer higher unit costs for both components, and firms A and B would face very asymmetric costs. Results would however be qualitatively similar to those obtained in the next sections.[3]

We then consider the possibility of breaking up both A and B into two separate entities, each producing one of the two components, leading to four independent producers, A_1, A_2, B_1 and B_2. In such a setting the tragedy of the anticommons may reappear and we could observe higher prices with respect to the integrated market case. This is because firms now do not need to consider the impact of raising their price on the demand of the complementary component. However,

[2]Gabszewicz and Thisse (1979) consider a spectrum of consumers with varying tastes for quality who choose between a low- and a high-quality product. These authors consider some cases that we do not, i.e. some consumers may not purchase any system or some system may not have positive demand.

[3]As already stated, our main focus is in fact on equilibrium market configurations in which all available systems are sold in positive amounts. Through tedious, and then omitted, algebra, it can be shown that under full leadership such equilibrium market configuration continues to emerge. The same logic can also be applied under shared leadership or when firms are disintegrated. In all such alternative cases, though, the costs distribution across firms is always "less asymmetric" than under full leadership, so that such equilibrium market configuration emerges *a fortiori*.

the fact that each firm is now able to control the price of one component only makes competition fiercer, possibly generating lower system prices than in the integrated market case. Depending on the effect that dominates, disintegration may lead to either lower prices and higher consumer welfare or to the opposite result. In the next Sections we will analyze the conditions under which each effect dominates. We will find that the form of the quality leadership will play a crucial role in this analysis. In particular, under full quality leadership, competition leads to lower prices and enhances consumer surplus. On the contrary, when quality leadership is shared, breaking up integrated firms (or, equivalently, prohibiting a merger) may lead to higher prices so that concerns about the tragedy of the anticommons are well posed in antitrust policies.

2.3 Full-quality leadership: complements as substitutes

Under full-quality leadership, A is the high quality producer for both components. Quality takes values in the $[0,1]$ interval, with the least- and highest-quality systems at its boundaries, i.e. $q_{BB} = 0$ and $q_{AA} = 1$. Demand functions for the four systems are obtained in the standard way. Define $\theta_{AB}^{AA} = \frac{p_{A2}-p_{B2}}{1-q_{AB}}$ the parameter value of the marginal consumer who is indifferent between systems AA and AB, and similarly define $\theta_{BA}^{AB} = \frac{p_{A1}+p_{B2}-p_{A2}-p_{B1}}{q_{AB}-q_{BA}}$ and $\theta_{BB}^{BA} = \frac{p_{A2}-p_{B2}}{q_{BA}}$. Given the quality ranking under full leadership, a necessary condition to have a positive demand for all four systems is[4]

$$0 < \theta_{BB}^{BA} < \theta_{BA}^{AB} < \theta_{AB}^{AA} < 1. \tag{2.1}$$

Figure 2.1 illustrates the resulting demands for systems and single components. The latter are given by $D_{A1}^F = 1 - \theta_{BA}^{AB}$, $D_{A2}^F = \left(1 - \theta_{AB}^{AA}\right) + \left(\theta_{BA}^{AB} - \theta_{BB}^{BA}\right)$,

[4] By direct comparison of θ_{AB}^{AA} and θ_{BB}^{BA}, condition (2.1) requires $q_{AB} + q_{BA} > 1$.

$D_{B1}^F = \theta_{BA}^{AB}$ and $D_{B2}^F = \left(\theta_{AB}^{AA} - \theta_{BA}^{AB}\right) + \theta_{BB}^{BA}$, where the superscript "F" stands for "full leadership".

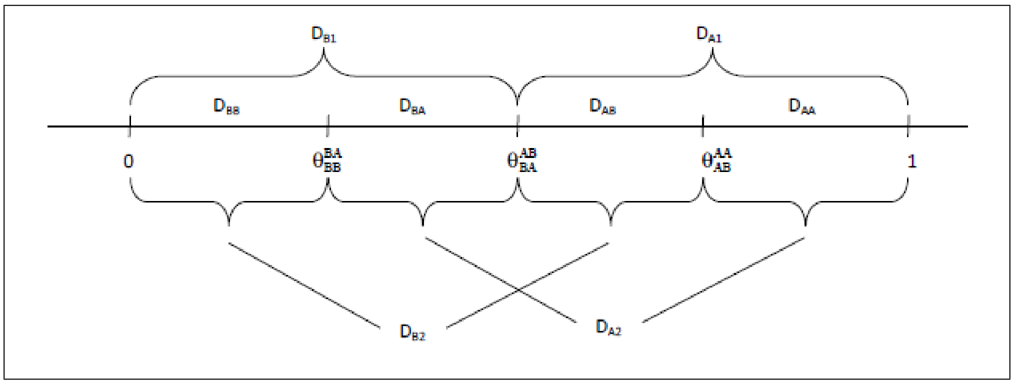

Figure 2.1: Demands for systems and single components under full-quality leadership

Interestingly, the cross-price elasticity of the demand of each component with respect to the price of the same quality complement is positive. For instance, $\eta_{12}^A = \frac{\partial D_{A1}^F}{\partial p_{A2}} \frac{p_{A2}}{D_{A1}} > 0$, since $\frac{\partial D_{A1}^F}{\partial p_{A2}} = \frac{1}{q_{AB} - q_{BA}} > 0$, indicating that A_1 and A_2 are perceived as substitutes notwithstanding their technical complementarity. In fact, as p_{A2} increases, the demand for A_1 does not decrease in the upper part of the market. As indicated in Figure 2.2, some consumers (segment C) might shift from system AA to AB, but they do not vary their demand of A_1. In particular, the consumer who was previously indifferent between systems AA and AB now prefers AB (θ_{AB}^{AA} in Figure 2.2 has moved to the right) but still purchases A_1. On the other hand, the increase in p_{A2} raises the demand for AB, which is now relatively cheaper than BA. In fact, the threshold θ_{BA}^{AB} depends negatively on p_{A2} and thus shifts to the left. Then, overall,the demand for A_1 becomes strictly larger, increasing by the segment D in Figure 2.2.

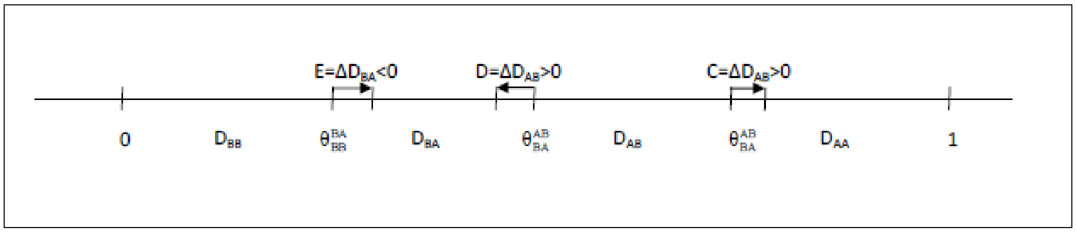

Figure 2.2: Effect of an increase in p_{A2} on demands under full leadership. Demand for *A1* increases: the two complements behave as (gross) substitutes. Demand for *B1* decreases.

This same analysis can be very easily applied to the demand of B_1 with respect to p_{B2}, to D^F_{A2} with respect to p_{A1} and to D^F_{B2} with respect to p_{B1}. This special characteristic of the demand functions under full leadership, in which technical complements produced by the same firm behave as substitutes and exhibit "inverse" cross-price effects, will play a crucial role in the upcoming results and is described more formally in the following Proposition.

Proposition 5. *Under full-quality leadership, consumers perceive same-firm (or equivalently, same-quality) components as gross substitutes.*

It is well-known that when a firm produces substitute goods, breaking it up into independent production processes unambiguously increases the degree of competition in the market, so that even if technically speaking we are studying complementary products, there may be no tragedy of the anticommons. We postpone such analysis, however, to Section 2.5, following the description of the shared leadership case. Now, we define profit functions and equilibrium prices under full leadership. In pursuit of this purpose, in order to simplify algebra, the analysis will be performed assuming that $q_{AB} = 2q_{BA}$.[5]

[5]This assumption is with no loss of generality, since all results would also hold in the more

When firms A and B produce both components, their overall profits amount to $\Pi_A^{FI} = p_{A1}D_{A1}^F + p_{A2}D_{A2}^F$ and $\Pi_B^{FI} = p_{B1}D_{B1}^F + p_{B2}D_{B2}^F$, where "$FI$" stands for "integrated market with full leadership". Differentiating Π_A^{FI} with respect to p_{A1} and p_{A2} and Π_B^{FI} with respect to p_{B1} and p_{B2} and solving the first-order conditions simultaneously yields the following Bertrand equilibrium prices:

$$p_{A1}^{FI} = \frac{2q_{BA}(3-5q_{BA})}{3(1-q_{BA})} \tag{2.2}$$

$$p_{A2}^{FI} = \frac{4q_{BA}(1-2q_{BA})}{3(1-q_{BA})} \tag{2.3}$$

$$p_{B1}^{FI} = \frac{q_{BA}(3-5q_{BA})}{3(1-q_{BA})} \tag{2.4}$$

$$p_{B2}^{FI} = \frac{2q_{BA}(1-2q_{BA})}{3(1-q_{BA})} \tag{2.5}$$

Equilibrium profits are $\Pi_A^{FI} = \frac{4q_{BA}(5-9q_{BA})}{9(1-q_{BA})}$; $\Pi_B^{FI} = \frac{q_{BA}(5-9q_{BA})}{9(1-q_{BA})}$. Hence, the quality leader A earns higher profits than B.[6]

2.4 Shared quality leadership: the tragedy strikes back

Under shared leadership, A manufactures the high-quality component 1, whereas B manufactures the high-quality component 2. As before, quality takes values

general case $q_{AB} > q_{BA}$. Given that, as stated in footnote 7, we require $q_{AB} + q_{BA} > 1$, $q_{AB} = 2q_{BA}$ implies $q_{BA} > \frac{1}{3}$. Moreover, since $q_{AB} < 1$, then it must be that $q_{BA} < \frac{1}{2}$. General proofs for generic q_{AB} and q_{BA} are available upon request.

[6]Notice that prices are all positive. The demands for the four systems are however all positive only if V is sufficiently large. In particular, condition (2.1) is always satisfied whenever $V \geq \frac{1}{3}$. Also, this equilibrium holds only if none of the firms has an incentive to unilaterally deviate to different market configurations involving fewer then four systems being purchased. Following the procedure to assess the existence and stability of equilibria in vertically differentiated markets with complementary goods illustrated in Alvisi et al. (2009) and also extensively used in the next Chapter, it can be shown that such possibility is excluded for a sufficiently large value of $q_{BA} : q_{BA} > 0.42$.

in the $[0, 1]$ interval, with the least- and highest-quality systems at its boundaries, i.e. $q_{BA} = 0$ and $q_{AB} = 1$. Demand functions for the four systems are obtained as usual. Define $\theta_{AA}^{AB} = \frac{p_{B2}-p_{A2}}{1-q_{AA}}$ the parameter value of the marginal consumer who is indifferent between systems AB and BA, and similarly define $\theta_{BB}^{AA} = \frac{p_{A1}+p_{A2}-p_{B2}-p_{B1}}{q_{AA}-q_{BB}}$ and $\theta_{BA}^{BB} = \frac{p_{B2}-p_{A2}}{q_{BB}}$. Given the quality ranking under shared leadership, a necessary condition to have a positive demand for all four systems is[7]

$$0 < \theta_{BA}^{BB} < \theta_{BB}^{AA} < \theta_{AA}^{AB} < 1. \tag{2.6}$$

Figure 2.3 illustrates the resulting demands for systems and single components. The latter are given by $D_{A1}^S = 1 - \theta_{BB}^{AA}$, $D_{A2}^S = \left(\theta_{AA}^{AB} - \theta_{BB}^{AA}\right) + \theta_{BA}^{BB}$, $D_{B1}^S = \theta_{BB}^{AA}$ and $D_{B2}^S = \left(1 - \theta_{AA}^{AB}\right) + \left(\theta_{BB}^{AA} - \theta_{BA}^{BB}\right)$, where the superscript "S" stands for "shared leadership".

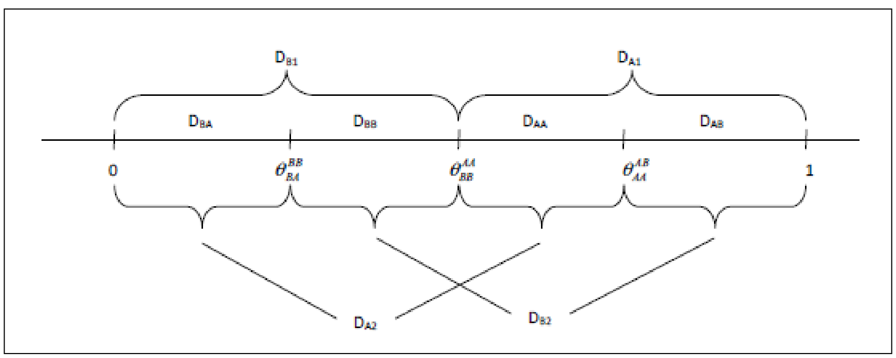

Figure 2.3: Demands for systems and single components under shared-quality leadership

As with the full leadership case, the cross-price elasticity of the demand of each component with respect to the price of the same-quality complement is

[7]By direct comparison of θ_{AA}^{AB}, θ_{BA}^{BB}, condition (2.6) requires $q_{AA} + q_{BB} > 1$.

positive. For instance, $\eta_{12}^{AB} = \frac{\partial D_{A1}^S}{\partial p_{B2}} \frac{p_{B2}}{D_{A1}} > 0$, since $\frac{\partial D_{A1}^S}{\partial p_{B2}} = \frac{1}{q_{AA}-q_{BB}} > 0$, indicating that A_1 and B_2 are perceived as substitutes notwithstanding the fact that they are technically complementary. Notice, however, that such goods are now manufactured by different firms. In contrast with the previous section, then, an integrated firm will produce components that are indeed perceived as complements and their cross-price effects have the "traditional" negative sign. This is indeed the content of the following Proposition

Proposition 6. *Under shared leadership, consumers perceive same-firm components as gross complements and same-quality components as gross substitutes.*

According to this result, we expect that under shared leadership the tragedy of the anticommons will play an important role in assessing the implications of breaking integrated producers into independent firms, and that policy recommendations will be different from those indicated under full leadership. Again, we postpone such analysis to Section 5. Now, we define profit functions and equilibrium prices under shared leadership. In doing this, as in the full leadership case, we simplify the algebra by assuming $q_{AA} = 2q_{BB}$.

When firms A and B produce both components, their overall profits amount to $\Pi_A^{SI} = p_{A1}D_{A1}^S + p_{A2}D_{A2}^S$ and to $\Pi_B^{SI} = p_{B1}D_{B1}^S + p_{B2}D_{B2}^S$, where the superscript SI stands for "integrated market with shared leadership". Differentiating Π_A^{SI} with respect to p_{A1} and p_{A2} and Π_B^{SI} with respect to p_{B1} and p_{B2} and solving the first-order conditions simultaneously, we obtain the Bertrand equilibrium prices

$$p_{A1}^{SI} = \frac{q_{BB}(3 - 4q_{BB})}{3(1 - q_{BB})} \tag{2.7}$$

$$p_{A2}^{SI} = -\frac{q_{BB}(1 - 2q_{BB})}{3(1 - q_{BB})} \tag{2.8}$$

$$p_{B1}^{SI} = \frac{q_{BB}^2}{3(1 - q_{BB})} \tag{2.9}$$

$$p_{B2}^{SI} = \frac{q_{BB}(1-2q_{BB})}{3(1-q_{BB})} \quad (2.10)$$

Equilibrium profits are $\Pi_A^{SI} = \frac{4q_{BB}(5-6q_{BB})}{9(1-q_{BB})}$ and $\Pi_B^{SI} = \frac{q_{BB}(2-3q_{BB})}{9(1-q_{BB})}$. Hence, the producer of the high-quality first component (A) earns higher profits than the producer of the high-quality second component (B). This is reasonable, considering the assumption that component 1 has a higher incremental value than component 2.

It should be noted that $p_{A2}^{SI} < 0$, whereas p_{A1}^{SI}, p_{B1}^{SI} and p_{B2}^{SI} are all positive in the relevant parameters range.[8] Then, in equilibrium, firm A would actually find it optimal to subsidize the consumption of A_2. As indicated in the previous Proposition, while A_1 and A_2 are perceived as complements, B_1 and A_2 are perceived as substitutes. Thus, a decrease in p_{A2} actually *decreases* the demand for component B_1 to the advantage of A_1. This can be seen in Figure 2.4, where, as p_{A2} decreases, the demands for systems AB and BB decrease, enlarging those of AA and BA. Overall, the demands of A_2 and, especially, of A_1 increase, with a positive total effect on profits. Thus, it is perfectly reasonable that firm A finds it profit-maximizing to sell one of its components below marginal cost in order to increase the consumption of the other complement.[9]

In the software industry, for instance, Adobe widely distributes its portable document reader for free. Similarly, in the past, Microsoft and Netscape (now

[8] In analogy with the full quality leadership case, V and q_{BB} have to be sufficiently large to guarantee the existence of this equilibrium (see footnote 9). Also, in analogy with footnote 8, the assumption $q_{AA} = 2q_{BB}$ implies $q_{BB} \in \left[\frac{1}{3}, \frac{1}{2}\right]$. Note that the negative sign of p_{A2}^{SI} does not depend on the restriction we imposed on parameters, rather on the complementarity between A_1 and A_2 and on the assumption of shared leadership. We have in fact obtained the same result also with general quality levels $q_{AA} > q_{BB}$.

[9] If, for some reason, subsidization were not possible, firm A would fix p_{A2} equal to its (zero) marginal cost as a corner solution. We have analyzed such a case but we have found that the main conclusions remain exactly the same, hence we decided not to include it in this paper. Calculations are available upon request.

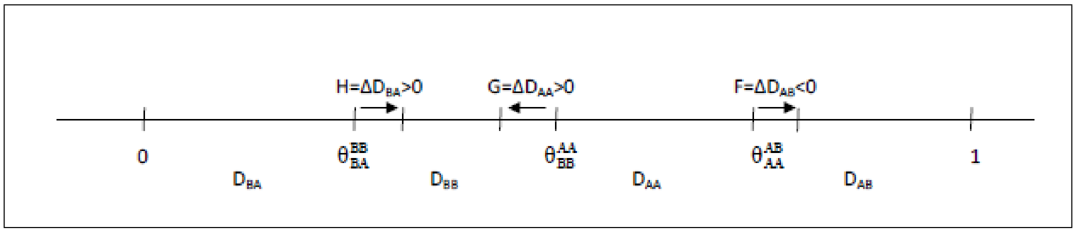

Figure 2.4: Effect of a decrease in p_{A2} on demands under shared leadership. Demand for *A1* increases, restoring the "standard "cross price effect. Demand for *B1* decreases.

part of AOL) have competed by creating new ways to freely distribute their Internet browsers. Finally, Sun Microsystems gives away both its Java virtual machine and Staroffice, the most successful open source office suite.[10]

2.5 The effects of disintegration

Assume now that a decision of the antitrust authority is passed, requiring the breakup of previously integrated firms. We then go from a market configuration where only firms A and B operate to one where four firms, A_1, A_2, B_1 and B_2 are active. Component 1 is manufactured by firms A_1 and B_1, whereas component 2 is manufactured by firms A_2 and B_2. We distinguish the two cases of full and shared quality leadership.

[10] Clearly, these are examples of cross-market subsidies that have been proved to be sustainable by previous literature in the presence of network externalities (see Shapiro and Varian 1999, Parker and Van Alstyne 2005). Here we find that simple complementarity is sufficient to justify such cross-subsidization.

2.5.1 Full quality leadership

In this case firms A_1 and A_2 are still "quality leaders" and produce higher quality goods than firms B_1 and B_2. Profits for each firm amount to $\Pi_{iz}^{FD} = p_{iz}D_{iz}^{F}$, where $i = A, B$; $z = 1, 2$ and the superscript FD stands for "disintegration of an integrated market with full leadership". Differentiating Π_{iz}^{FD} with respect to p_{iz} and solving the first-order conditions simultaneously yields the following Bertrand equilibrium prices:

$$p_{A1}^{FD} = \frac{q_{BA}(11 - 16q_{BA})}{14 - 19q_{BA}} \tag{2.11}$$

$$p_{A2}^{FD} = \frac{q_{BA}(12 - 41q_{BA} + 34q_{BA}^2)}{28 - 80q_{BA} + 57q_{BA}^2} \tag{2.12}$$

$$p_{B1}^{FD} = \frac{3(1 - q_{BA})q_{BA}}{14 - 19q_{BA}} \tag{2.13}$$

$$p_{B2}^{FD} = \frac{2q_{BA}(1 - 3q_{BA} + 2q_{BA}^2)}{28 - 80q_{BA} + 57q_{BA}^2} \tag{2.14}$$

Equilibrium profits are $\Pi_{A1}^{FD} = \frac{(11-16q_{BA})^2 q_{BA}}{(14-19q_{BA})^2}$; $\Pi_{A2}^{FD} = \frac{(12-17q_{BA})^2 q_{BA}(1-2q_{BA})}{(14-19q_{BA})^2(2-3q_{BA})}$; $\Pi_{B1}^{FD} = \frac{9(1-q_{BA})^2 q_{BA}}{(14-19q_{BA})^2}$; $\Pi_{B2}^{FD} = \frac{4(1-q_{BA})^2 q_{BA}(1-2q_{BA})}{(14-19q_{BA})^2(2-3q_{BA})}$. Again, it can be noticed that firms producing high-quality components earn higher profits ($\Pi_{Aj}^{FD} > \Pi_{Bj}^{FD}$, $j = 1, 2$), so that aggregate profits in the high-quality sector are also higher.

The new equilibrium prices can be easily compared to the ones set by two integrated firms, given by equations (2.2) to (2.5). In particular, the following result holds:[11]

Proposition 7. *Under full leadership, breaking up integrated firms involves lower prices for all components and then for all systems* ($p_{iz}^{FD} < p_{iz}^{FI}$, $i = A, B$; $z = 1, 2$, *so that* $p_{ij}^{FD} < p_{ij}^{FI}$, $i = A, B$; $j = A, B$).

[11]Proposition 7 is proved by direct comparison of prices in expressions (2.2) to (2.5) and (2.11) to (2.14). The same procedure is also adopted to prove Proposition 8.

In other terms, breaking up integrated firms or prohibiting mergers when markets are characterized by full quality leadership would be beneficial for each consumer, because all available systems can be purchased at a lower price. Thus, as expected, breaking an integrated duopoly in full leadership does not produce any tragedy of the anticommons. As indicated above, in this setting same-quality technical complements actually behave as substitutes. With full leadership, such complements are produced by the same firm, which then internalizes the negative externality that a decrease in the price of one complement would have on the demand of the other (exactly as it would happen in the case of two substitute goods produced by the same firm). That's why integrated firms set higher prices than independent producers.

The analysis of consumer surplus is thus straightforward. The prices of all components are the highest with a fully integrated market and the lowest with four independent firms, so that $CS^{FD} > CS^{FI}$.

Finally, by comparing individual profits in cases FI and FD we notice immediately then they decrease when firms are broken up.[12] In fact, with no tragedy of the anticommons at play, a lower degree of integration only increases the number of firms in the market and makes competition fiercer. Moreover, if such conclusion holds for each firm, it holds *a fortiori* for the whole industry, and then for producer surplus. Thus, $PS^{FI} > PS^{FD}$, implying that with full leadership the industry as a whole would always be against policies of disintegration.

[12] What happens more precisely is that the sum of the profits of the newly independent firms is always lower than the profit of the pre-break up integrated firms.

2.5.2 Shared quality leadership

Under shared leadership, firms A_1 and B_2 are "quality leaders" and produce higher quality goods than firms B_1 and A_2. Profits for each firm amount to $\Pi_{iz}^{SD} = p_{iz} D_{iz}^{S}$, where $i = A, B$; $z = 1, 2$, and the superscript SD stands for "disintegration of an integrated market with shared leadership". Differentiating Π_{iz}^{SD} with respect to p_{iz} and solving the first-order conditions simultaneously yields the following Bertrand equilibrium prices:

$$p_{A1}^{SD} = \frac{q_{BB}(11 - 16q_{BB})}{14 - 19q_{BB}} \tag{2.15}$$

$$p_{A2}^{SD} = \frac{2q_{BB}(1 - 3q_{BB} + 2q_{BB}^2)}{28 - 80q_{BB} + 57q_{BB}^2} \tag{2.16}$$

$$p_{B1}^{SD} = \frac{3(1 - q_{BB})q_{BB}}{14 - 19q_{BB}} \tag{2.17}$$

$$p_{B2}^{SD} = \frac{q_{BB}(12 - 41q_{BB} + 34q_{BB}^2)}{28 - 80q_{BB} + 57q_{BB}^2} \tag{2.18}$$

Equilibrium profits are $\Pi_{A1}^{SD} = \frac{(11-16q_{BB})^2 q_{BB}}{(14-19q_{BB})^2}$; $\Pi_{A2}^{SD} = \frac{4(1-q_{BB})^2 q_{BB}(1-2q_{BB})}{(14-19q_{BB})^2(2-3q_{BB})}$; $\Pi_{B1}^{SD} = \frac{9(1-q_{BB})^2 q_{BB}}{(14-19q_{BB})^2}$; $\Pi_{B2}^{SD} = \frac{(12-17q_{BB})^2 q_{BB}(1-2q_{BB})}{(14-19q_{BB})^2(2-3q_{BB})}$. Again, it can be noticed that firms manufacturing high-quality components obtain higher profits ($\Pi_{A1}^{SD} > \Pi_{B1}^{SD}$ and $\Pi_{B2}^{SD} > \Pi_{A2}^{SD}$ $i = A, B$). Also in this case, aggregate profits tend to be higher in sector A, which is the sector producing component 1, the one with the highest incremental value.

The equilibrium prices listed in equations (2.15) to (2.18) can be easily compared to the ones in the previous integrated configuration, given by equations (2.7) to (2.10). Particularly, the following result holds.

Proposition 8. *Under shared leadership, breaking up integrated firms involves higher prices for all components with the exception of* p_{A1} ($p_{A2}^{SI} < p_{A2}^{SD}$, $p_{Bz}^{SI} < p_{Bz}^{SD}$,

$z = 1, 2$, but $p^{SI}_{A1} > p^{SD}_{A1}$). All systems' prices are higher ($p^{SI}_{ij} < p^{SD}_{ij}$, $i = A, B$; $j = A, B$).

Notice first that "disintegrating the market" generates a tragedy of the anticommons for all prices but p_{A1}. This asymmetric effect is the result of the interplay of the complex cross-price effects among components, so that, while breaking up firm A would certainly generate the standard tragedy of the anticommons and raise both p_{A1} and p_{A2}, this would not be the whole story. In fact, the break-up would certainly lead the new producer of component A_2 (previously priced below marginal cost) to fix a higher (positive) price. Such substantial increase in p_{A2} brings about three different effects. First, it generates an increase in p_{B2}, the price of the direct competitor (and substitute) of A_2 and this increases the price of A_1 (perceived as substitute of B_2), thus reinforcing the "tragedy effect". Second, the increase in p_{B2} due to the increase of p_{A2} tends to reduce p_{B1}, the complement of B_2. Because of this, the competition in the market for component 1 gets fiercer and p_{A1} tends to decrease. Finally, as usual, an increase in p_{A2} brings about a decrease in the price of its complement A_1. The latter two effects more than compensate the increase in p_{A1} due to the tragedy and to the higher p_{B2}.[13]

In any event, even if $p^{SI}_{A1} > p^{SD}_{A1}$, system prices are larger with independent producers, implying that $CS^{SI} > CS^{SD}$, contrarily to the full leadership case. Then, breaking up integrated firms or prohibiting mergers when markets are characterized by shared quality leadership creates an anticommons problem and harms all consumers; an integrated duopoly is welfare superior to competition. The type of quality leadership characterizing a market is therefore a crucial factor

[13] These three effects are also at play with p_{B1}, with the difference that p^{SI}_{B2}, contrarily to p^{SI}_{A2}, is positive, so that the last two effects are weaker and dominated by the first. Overall then, $p^{SD}_{B1} > p^{SI}_{B1}$.

that antitrust authorities should take into account when assessing the impact of mergers on prices and, ultimately, on welfare.

Also, results above indicate that component prices are differently affected by the break-up, so that its impact on aggregate profits is not clear-cut. Our model suggests however that disintegration increases producer surplus, as indicated in the following Lemma.

Lemma 3. *Under shared leadership,* $PS^{SI} < PS^{SD}$.

Proof: See Appendix.

Finally, to complete our analysis, we check whether social welfare is higher when firms are integrated or when they are broken up.

Proposition 9. *No matter the form of quality leadership, total surplus is higher when firms are disintegrated.*

Proof: See Appendix.

Thus, our model implies that disintegration yields a higher total surplus. The consequences of such result are however very different according to the form of quality leadership. Under full leadership, disintegration harms producers, but the benefits of lower prices to consumers more than compensate firms' losses. Under shared leadership, the exact opposite occurs, since breaking up firms typically brings about a significant increase in prices hence profits at the detriment of consumer surplus. That is why, for policy purposes, we would not suggest to always favor break-ups, since doing that clearly damages either consumers or producers. In general, policy measures should always be undertaken weighting different social groups appropriately.[14]

[14] More specifically, two main cases are considered by antitrust authorities when analyzing the

2.6 The case for "partial disintegration"

In this Section we consider the intermediate cases in which only one firm in the market is integrated. Again, we distinguish between full and shared quality leadership. In this way, we can assess whether the conclusions obtained in Section 5 apply also to market structures in which integrated firms compete with independent firms that specialize in the production of a single component. In particular, we will be able to establish the implications for prices and profits of an antitrust authority requiring either one out of the two integrated producers A and B to divest in the market.

2.6.1 Full Leadership

Under full leadership, we may have two cases: one in which there is an integrated firm producing two high-quality complements A_1, A_2, competing with two independent low-quality firms producing B_1 and B_2 (from now on, we label this case FA) and one in which the opposite holds and the integrated firm produces two low-quality components (B_1 and B_2), henceforth labeled case FB. In both instances, however, conclusions are pretty straightforward and again depend on the results of Proposition 5. In other terms, when complementary components produced by the same firm exhibit "inverse" cross-price effects, as in full leadership, breaking up integrated firms increases competition and lowers both components' and systems' prices, no matter whether disintegration concerns all firms or is only partial. This result is summarized in the following Proposition, which indicates the existence of a clear direct relationship between the equilibrium prices and the

effect of mergers. First, the case in which the merger allows the merged firm to unilaterally raise prices. Second, the case in which the merger favors collusion. This implies that the primary concern of antitrust authorities are post-merger (or post-separation) prices and, ultimately, consumer welfare. See Motta (2003).

degree of integration.

Proposition 10. *Under full leadership, decreasing the degree of integration (i.e. breaking up integrated firms or prohibiting a merger) reduces all components' and systems' prices ($p_{iz}^{FD} < p_{iz}^{FA} < p_{iz}^{FI}$ and $p_{iz}^{FD} < p_{iz}^{FB} < p_{iz}^{FI}$, $i = A, B$; $z = 1, 2$). Moreover, $p_{iz}^{FB} \leq p_{iz}^{FA}$ ($i = A, B$; $z = 1, 2$).*

Proof: See Appendix.

Then, breaking up one firm only, reaching either case FA or FB, involves lower prices. Moreover, prices are further lowered moving from FA or FB, and breaking up the remaining integrated firm. Figure 2.5 illustrates this relationship for p_{A1} and $q_{BA} = 0.4$. These results clearly suggest that under full leadership the intervention of antitrust authorities trying to enhance competition (requiring either firm A or firm B to divest or, similarly, prohibiting the merger of two independent firms producing same-quality complements) favors consumers, notwithstanding the presence of complementary goods. In other terms, $CS^{FD} > CS^{FA} > CS^{FI}$ and $CS^{FD} > CS^{FB} > CS^{FI}$. Furthermore, restricting our analysis to partially disintegrated market structures, we notice that breaking up the quality leader (weakly) decreases prices more, so that $CS^{FB} > CS^{FA}$. Then, if the antitrust authority were to intervene on one firm only, it would be better to target the high-quality products.

By comparing the four cases FI, FA, FB and FD, we also notice that a direct relationship also exists between the degree of integration and individual profits. Breaking up one of the two firms of a fully integrated market decreases profits for all firms, including the one that remains integrated, and the reduction in profits is maximum when also the second firm is broken up. Then firms should always oppose anti-mergers policies, no matter whether they produce high- or

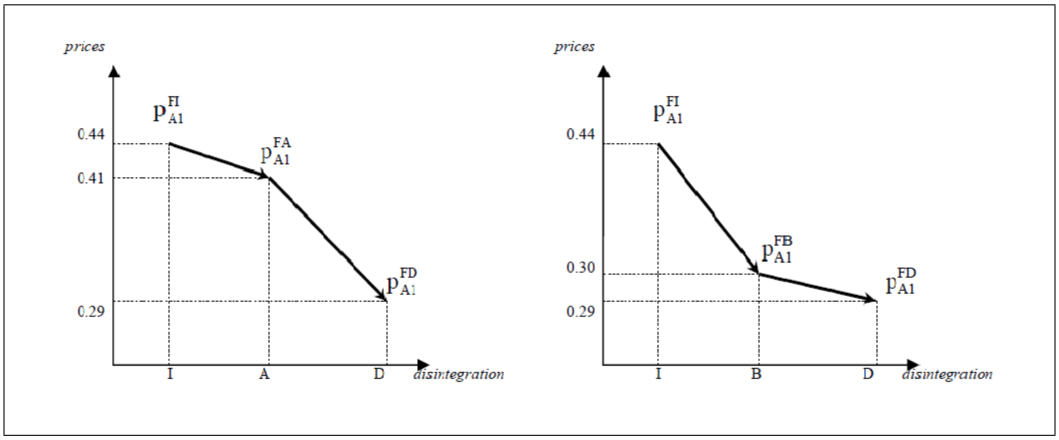

Figure 2.5: Price patterns for component p_{A1} as the degree of competitiveness in the market increases under full quality leadership. Simulation with $q_{BA} = 0.4$.

low- quality components and whether such policies hit them directly or their rivals only. This logic applies to aggregate profits, too, so that $PS^{FI} > PS^{FA} > PS^{FD}$ and $PS^{FI} > PS^{FB} > PS^{FD}$. Finally, a direct comparison of producer surplus in FA and FB shows that, not surprisingly, $PS^{FA} > PS^{FB}$.

2.6.2 Shared leadership

Under shared leadership, the "partial" break-up generates different and more complex effects on prices depending on which firm (A or firm B) remains integrated (cases SA and SB respectively, in analogy with subsection 2.6.1).

Proposition 11. *Starting from a fully integrated market with shared-quality leadership:*

a) *Breaking up Firm A (case SB) implies lower prices for components A_1 and B_1 and higher prices for A_2 and B_2. Also, prices of systems AA and BB increase, whereas prices of systems AB and BA decrease;*

b) *breaking up Firm B (case SA) implies higher prices for all components and therefore a higher price for all systems.*

Proof: See Appendix.

In part a) of Proposition 11 we notice that breaking up firm A produces the standard tragedy of the anticommons for component 2 (prices for A_2 and B_2 increase) but increases competition in the market for component 1 (both p_{A1} and p_{B1} decrease). The intuition for lower prices for A_1 and B_1 is similar to the one provided to explain the decrease of p_{A1} in Proposition 6: A_1 and B_2 are perceived as gross substitutes and so are B_1 and A_2. Also notice that the decrease in p_{A1} more than counterbalances the increase in p_{B2}, so that the price of system AB is actually lower. Then, breaking up firm A is not necessarily detrimental to all consumers. Particularly, while the intermediate portion of the demand (consumers buying systems AA and BB) always suffers from this policy, consumers with high valuation for quality might take advantage of the lower price for component 1, obtaining now system AB for less. In conclusion, the welfare effects of such a policy are not obvious and need a separate investigation, that we perform below.

In part b), the tragedy effect dominates and all prices increase. This happens because, under SB, the increase in p_{B2} after the breakup is less substantial than the increase in p_{A2} after the breakup of firm A and does not generate the same strong negative effect on p_{B1} and p_{A1}. Since it is a high-quality product, B_2 is priced above marginal cost also with an integrated firm B so that, *ceteris paribus*, the cross-price effect between complements is smaller. Moreover, as assumed, the second component contributes less to the system's value than the first, so that its price cannot increase too much after disintegration.

The analysis can be completed by studying policies of "sequential disintegration" starting from either SA or SB towards SD. The same intuitions provided for Proposition 11 explain the following results.

Proposition 12. *Under shared-quality leadership:*

a) *starting from SA and breaking up Firm A implies lower prices for components A_1 and B_1 and to higher prices for A_2 and B_2. All systems' prices increase with the exception of p_{AB} ($p_{AB}^{SA} < p_{AB}^{SD}$, $p_{BA}^{SA} > p_{BA}^{SD}$ and $p_{ii}^{SA} > p_{ii}^{SD}$, $i = A, B$);*

b) *starting from SB and breaking up firm B produces the standard tragedy of the anticommons, that is all systems' prices increase ($p_{iz}^{SB} < p_{iz}^{SD}$ where $i, z = A, B$).*

Propositions 6, 11 and 12 together offer a clear vision of how different the relationship is between the degree of integration and price levels in shared leadership. Starting from SI, sequentially disintegrating the two firms going first either to case SA and SB and then to SD, generates a monotonic increase in both p_{A2} and p_{B2}, whereas the impact on p_{A1} and p_{B1} is more complex. Figure 2.6 provides a full illustration of such relationship for $q_B = 0.4$. For instance, panel 2.6.2 shows that, starting from SI, breaking up firm A lowers p_{A1}, but then breaking up firm B as well, and reaching SD, increases p_{A1}, even if this second variation is of lower magnitude than the first. Similarly, p_{B1} decreases from SI to SB and then increases from SB to SD but, unlike the previous case, the second variation is larger in magnitude (see panel 2.6.4).

The results illustrated so far have several interesting policy implications, summarized in the following Corollary:

Corollary 3. *With shared-quality leadership and a fully integrated market:*

a) *breaking up both firms or breaking up firm B only (leading to cases SD or SA respectively) always generates the tragedy of the anticommons;*

b) *breaking up firm A (case SB) reduces the total prices of the highest and the lowest quality systems available;*

c) *starting with full integration, disintegrating A decreases B's profits, whereas disintegrating B increases A's profits.*

As for welfare, the more complex relationship between the degree of integration and firms' prices implies that policy recommendations may not be clear cut and require a more careful analysis than under full leadership. At this purpose, the

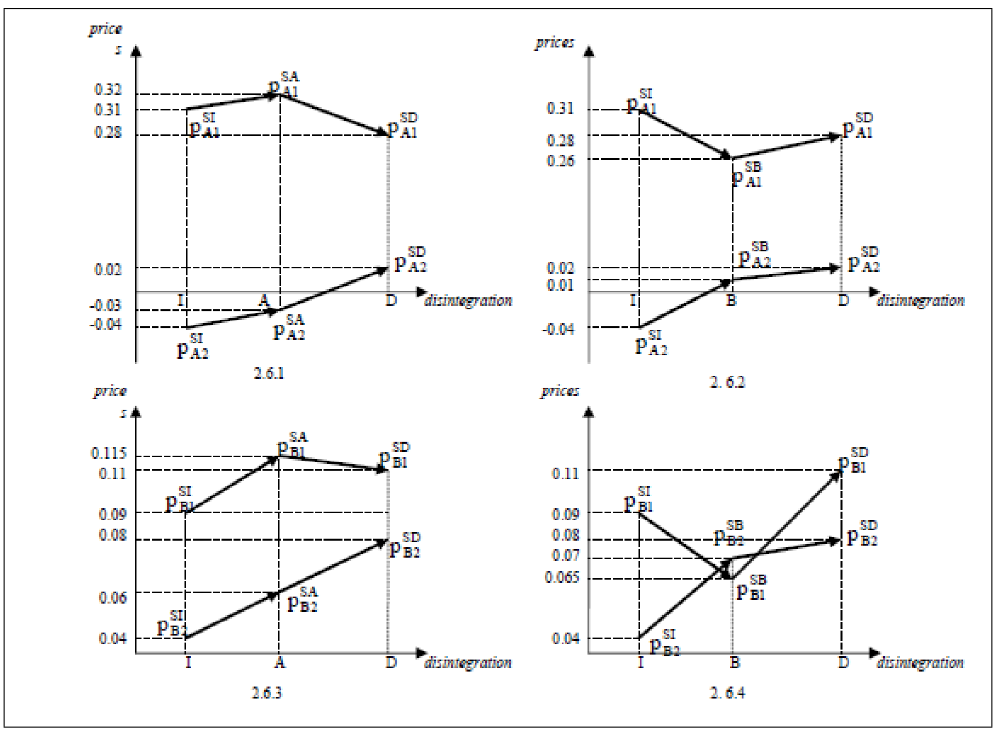

Figure 2.6: Price patterns for all components as the degree of competitiveness in the market increases under shared quality leadership. Simulation with $q_B = 0.4$.

following Lemma ranks consumer surplus.

Lemma 4. *Under shared leadership,* $CS^{SB} > CS^{SI} > CS^{SD}$ *and* $CS^{SI} > CS^{SA} > CS^{SD}$.

Proof: See Appendix B.

Particularly, comparing market configurations SB and SI, Proposition 12 indicated that, starting from an integrated market and breaking up firm A (that is, the firm producing the high-quality component that also provides the higher incremental value to utility levels), increases the prices of the systems previously produced by the same firm (AA and BB) and decreases the prices of "mixed systems" AB and BA. Notice that here AA and BB are the "intermediate systems" in terms of quality, whereas AB and BA are the highest and lowest quality systems, respectively (see Figure 2.3). Then, breaking up firm A benefits consumers at the extremes of market demand. This effect more than counterbalances the increases in p_{AA} and p_{BB}, so that, going from SI to SB, aggregate consumer surplus actually increases. In an oligopolistic setting, then, while the tragedy makes full integration always better for consumers than the presence of four independent firms, breaking up firm A only increases consumer surplus even further. To fully understand this result, recall that this firm, when integrated, sets a price below marginal cost on the second (low-quality) component. Doing this allows to set a very high price on A_1, thus reducing the possibility for consumers to purchase the highest-quality system AB. By breaking up firm A, the new firm producing A_2 will have to set a price equal or above marginal cost and A_1 will be forced to sell at a lower price. Consider, for example, the case of Adobe Writer and Adobe Reader. Adobe allows consumers to download its Reader for free from the Internet, pricing it at or below marginal cost. If we believe that the highest quality portable Writer in the market is indeed the Adobe one, our results would imply that breaking up Adobe in two firms, one producing the Reader and the other the Writer, would decrease the price charged for the Writer and increase consumer surplus.

Moreover, starting from a disintegrated market structure, Lemma 4 indicates that sometimes mergers of independent firms producing single components might be allowed, if not encouraged. Particularly, consumer surplus may increase if the merged firms produce goods of different quality levels (B_1 and B_2 in our case) and their high-quality product is not the one providing the largest incremental value. In our initial example of software markets, the integrated production of a low-quality operating system and an high-quality internet browser should then be judged favorably from an antitrust perspective.

As for producer surplus (PS), our model suggests that the tragedy of the anticommons prevails, as indicated in the following Lemma.

Lemma 5. *Under shared leadership, producer surplus decreases with the degree of integration. In particular, a)* $PS^{SI} < PS^{SA} < PS^{SD}$ *and b)* $PS^{SI} < PS^{SB} < PS^{SD}$.

Proof: See Appendix.

The first part of inequality a) is intuitive if analyzed together with panels 2.6.1 and 2.6.3 in Figure 2.6. Starting from an integrated market structure, breaking up firm B increases all firms' prices. This increases all firms' profits and, consequently, producer surplus.[15] Also, the second part of the inequality holds, notwithstanding the decrease in p_{A1} and p_{B1} obtained when shifting from

[15]This can be verified comparing the profits in SB (obtained in the proof of Proposition 11) with those in case SI.

SA to SD. In fact, the increase in aggregate profits due to higher p_{A2} and p_{B2} more than compensates for the lower profits provided by components A_1 and B_1, since three of the four available systems in the market end up costing more in SA than in SD anyway (see Proposition 12). Symmetrically, while the second part of inequality b) is a direct implication of the general price increase from SB to SD, the first part derives from the stronger effects on profits of components A_2 and B_2.[16]

Finally, it is interesting to figure out what an antitrust authority should do if it were to divest one and only one firm. In that case, following the same procedure adopted to prove Proposition 9, we have that $TS^{kB} > TS^{kA}$, $k = F, S$, so that, from a social welfare perspective, it is always better to break up the firm producing the high quality component that also provides the higher incremental value to utility levels. In fact, in such case, the reduction in equilibrium prices is more substantial, with a greater benefit to consumers that more than counterbalances the reduction in aggregate profits.[17]

2.7 Mixed Bundling and Disintegration: Whither the Tragedy?

The "mix & match" literature underlines how mergers and/or divestitures are not the only strategies available to firms producing complementary goods. In fact, profits can be increased further through pure or "mixed-bundling" practices. Particularly, under mixed bundling each integrated firm sells its components both

[16] In such shift, two of the four available systems cost more and two cost less (see Proposition 11).

[17] This implies $CS^{kB} > CS^{kA}$ and $PS^{kA} > PS^{kB}$ ($k = F, S$). While the former inequality can be verified by comparing the equilibrium prices in kA and kB, $k = F, S$, obtained in the proofs of Propositions 10 and 11, the latter requires a direct comparison of producer surplus.

separately and as a bundle, thus setting three different prices: p_{i1}, p_{i2}, and p_{ii} $(i = A, B)$, where p_{ii} is the price charged when a consumer buys both components from the same firm. Typically, as in Matutes and Regibeau (1992), the bundle costs less than the sum of the prices of components sold separately. Then, under full leadership, separation may harm consumers, since, when a firm is broken up, the sum of the prices set by the two newly separated firms could be higher than the price previously charged for the bundle. Conversely, under shared leadership, the tragedy of the anticommons implies that the new independent firms may charge lower prices than those set by the integrated firm when selling its components separately, so that competition may advantage consumers. In both cases, a simple price comparison might not be sufficient to establish which regime (integration or separation) is to be preferred, and a complete welfare analysis might become necessary.

2.7.1 Mixed bundling under full leadership

To obtain demand functions, we need to redefine the marginal consumers. When both firms engage in mixed bundling, $\theta_{AB}^{AA} = \frac{p_{AA}-p_{A1}-p_{A2}}{1-q_{AB}}$, $\theta_{BA}^{AB} = \frac{p_{A1}-p_{A2}+p_{B2}-p_{B1}}{q_{AB}-q_{BA}}$ and $\theta_{BB}^{BA} = \frac{p_{A2}+p_{B1}-p_{BB}}{q_{BA}}$. Then, A and B maximize the following profit functions

$$\Pi_A^{FM} = p_{AA}(1-\theta_{AB}^{AA}) + p_{A1}(\theta_{AB}^{AA} - \theta_{BA}^{AB}) + p_{A2}(\theta_{BA}^{AB} - \theta_{BB}^{BA}) \tag{2.19}$$

$$\Pi_B^{FM} = p_{BB}\theta_{BB}^{BA} + p_{B1}(\theta_{BA}^{AB} - \theta_{BB}^{BA}) + p_{B2}(\theta_{AB}^{AA} - \theta_{BA}^{AB}) \tag{2.20}$$

and the Bertrand equilibrium prices are $p_{AA}^{FM} = \frac{2}{3}$; $p_{A1}^{FM} = \frac{4q_{BA}}{3}$; $p_{A2}^{FM} = \frac{2q_{BA}}{3}$; $p_{BB}^{FM} = \frac{1}{3}$; $p_{B1}^{FM} = \frac{1-q_{BA}}{3}$ and $p_{B2}^{FM} = \frac{1-2q_{BA}}{3}$, where $p_{ii}^{FM} < p_{i1}^{FM} + p_{i2}^{FM}$ $(i = A, B)$, consistently with the results in previous literature. At these prices, it is immediate to verify that only systems AA and BB have positive demand in equilibrium, indicating that in our model allowing firms to engage in mixed bundling eliminates the incentives for consumers to mix and match. Equilibrium profits are $\Pi_A^{FM} = \frac{4}{9}$ and $\Pi_B^{FM} = \frac{1}{9}$.

Comparing prices under mixed bundling with those in the case of disintegration, the following result holds:

Proposition 13. *Under full leadership with mixed bundling, breaking up integrated firms involves lower prices for each single component:* $p_{iz}^{FD} < p_{iz}^{FM}$, $i = A, B$; $z = 1, 2$. *Moreover,* $p_{i1}^{FD} + p_{i2}^{FD} < p_{ii}^{FM}$. *Hence, total prices for all systems are lower when firms are separated* ($p_{ij}^{FD} < p_{ij}^{FM}$, $i = A, B$; $j = A, B$).

In case of full leadership, therefore, mixed bundling does not change the results that we previously obtained and separation is always beneficial for consumers.[18]

When analyzing producer surplus, notice first that each firm could choose whether to engage in mixed bundling or not, so that this can be thought as a simple simultaneous game, whose payoffs (profits) are given in the following matrix.[19]

Table 2.1: Mixed bundling under full leadership.

		B: MB	*B*: No MB
A	MB	Π_A^{FM}; Π_B^{FM}	$\frac{9+26q_{BA}-63q_{BA}^2}{36(1-q_{BA})}$; $\frac{q_{BA}(5-9q_{BA})}{9(1-q_{BA})}$
	No MB	$\frac{8q_{BA}}{9}$; $\frac{q_{BA}(2-3q_{BA})}{9(1-2q_{BA})}$	Π_A^{FI}; Π_B^{FI}

From Table 2.1, it is immediate to verify that mixed bundling (MB) is a dominant strategy for both firms and that $\Pi_i^{FM} > \Pi_i^{FI}$ ($i = A, B$). In Section 2.6.1

[18]It is important to notice that the results in Proposition 13 are general, in that they hold also for different relationships between q_{AB} and q_{BA}. In particular, they hold for all $q_{AB} = \alpha q_{BA}$, with $\alpha > 1$.

[19]Payoffs when A engages in mixed bundling and B does not are obtained by letting A choose prices so to maximize (2.19), whereas B maximizes Π_B^{SI}, where demands are computed using the definitions of marginal consumers given at the beginning of this Section. A similar procedure is followed to obtain profits when B engages in mixed bundling and A does not.

we argued that producer surplus is always higher when firms are integrated, even when they don't engage in mixed bundling. Then, we can conclude that producer surplus is *a fortiori* greater with mixed bundling than with a disintegrated market structure, so that, qualitatively speaking, our previous results in terms of social welfare under full leadership are actually reinforced: disintegration favors consumers and harms producers.

2.7.2 Mixed bundling under shared leadership

When both firms produce one high-quality component each and engage in mixed bundling, the marginal consumers are $\theta_{AA}^{AB} = \frac{p_{A1}+p_{B2}-p_{AA}}{1-q_A}$, $\theta_{BB}^{AA} = \frac{p_{AA}-p_{BB}}{q_A-q_B}$ and $\theta_{BA}^{BB} = \frac{p_{BB}-p_{B1}-p_{A2}}{q_B}$. Firms A and B maximize, respectively

$$\Pi_A^{SM} = p_{AA}(\theta_{BB}^{AA} - \theta_{BA}^{BB}) + p_{A1}(1-\theta_{AA}^{AB}) + p_{A2}\theta_{BA}^{BB} \tag{2.21}$$

$$\Pi_B^{SM} = p_{BB}(\theta_{BB}^{AA} - \theta_{BA}^{BB}) + p_{B1}\theta_{BA}^{BB} + p_{B2}(1-\theta_{AA}^{AB}) \tag{2.22}$$

The Bertrand equilibrium prices are $p_{AA}^{SM} = \frac{2q_B}{3}$; $p_{A1}^{SM} = \frac{1}{3}$; $p_{A2}^{SM} = 0$; $p_{BB}^{SM} = \frac{q_B}{3}$; $p_{B1}^{SM} = \frac{q_B}{3}$ and $p_{B2}^{SM} = \frac{1-2q_B}{3}$. As in full leadership, each integrated firm sells its bundle at a discount ($p_{ii}^{SM} < p_{i1}^{SM} + p_{i2}^{SM}$, $i = A, B$). In this case, systems AB, AA and BB have positive demand of the same size in equilibrium (they get $\frac{1}{3}$ of total demand each), whereas the lowest-quality system BA is not sold. Equilibrium profits are $\Pi_A^{SM} = \frac{1+2q_B}{9}$ and $\Pi_B^{SM} = \frac{1-q_B}{9}$.

The relationship between the prices of separate components under mixed bundling and under disintegration is rather complex. In fact, two opposite forces are at work in this case. Under mixed bundling, the integrated firms tend to raise the prices of each separate component compared to case SI, whereas the tragedy raises these prices under SD. Therefore, it is not possible to determine the impact of the tragedy on consumers simply looking at prices.[20] A direct comparison of CS^{SM} and CS^{SD} becomes necessary and is performed in the following Proposition.

Proposition 14. *Under shared leadership with mixed bundling,* $CS^{SD} > CS^{SM}$ *if and only if* $q_B < 0.39$.

Proof: See Appendix.

This result appears interesting as it reinforces our idea that the tragedy of the anticommons is not a necessary feature of complementary markets. In fact, when mixed bundling is allowed, disintegrating firms might increase consumer welfare as in the full leadership case and, in particular, this occurs for low levels of q_B. When instead q_B is high, systems AA and BB are close substitutes of the high-quality system AB and the negative impact of the tragedy will be higher. In fact, the increase in prices due to the creation of a tragedy of the anticommons is generally proportional to the degree of substitutability among competing systems, so that when systems are close substitutes (high q_B) and firms are integrated, each firm knows that even the smallest increase in the prices of its single components will have a strong negative impact on its overall profit and has therefore an incentive to keep them especially low.

Finally, it can be easily established, as in full leadership, that mixed bundling is a dominant strategy for both firms and $\Pi_i^{SM} > \Pi_i^{SI}$ $(i = A, B)$, so that $PS^{SM} > PS^{SI}$. However, as in the case of shared leadership without mixed bundling, producer surplus remains lower than with independent firms.[21]

[20] Specifically, for firm A, $p_{A1}^{SM} > p_{A1}^{SD}$, $p_{A2}^{SM} < p_{A2}^{SD}$ and $p_{AA}^{SM} < p_{A1}^{SD} + p_{A2}^{SD}$. As for firm B, $p_{B1}^{SM} > p_{B1}^{SD}$, $p_{B2}^{SM} > p_{B2}^{SD}$ if and only if $q_B < 0.36$ and $p_{BB}^{SM} < p_{B1}^{SD} + p_{B2}^{SD}$. Prices for both bundles AA and BB are then lower than under disintegration. However, $p_{AB}^{SM} < p_{AB}^{SD}$ if and only if q_B is large enough ($q_B > 0.44$).

[21] The proof involves tedious algebra and is therefore omitted.

2.8 Conclusions

While in the past competition policy has disregarded the tragedy of the anticommons, more recent decisions have explicitly considered it when assessing the effects of mergers and divestitures. The underlying belief shared by antitrust authorities is that complementary markets are characterized by a tradeoff between the tragedy of the anticommons and the lack of competition. In that sense, integration should be allowed only when the problem of complementary monopoly dominates the effect of the reduction in the number of active firms in the market.

In this paper we have analyzed such tradeoffs in oligopolistic complementary markets, where each component is produced by more than one (vertically differentiated) firm. Previous literature (Economides and Salop, 1992) argued that disintegrating firms producing complementary goods always leads to a tragedy of the anticommons, raising system prices. We prove that this may not be the case when goods are vertically differentiated. Particularly, we have shown that the relative strength of anticommons problems and lack of competition is crucially related to the type of quality leadership characterizing the market. In the presence of a quality leader, forcing firms to divest or prohibiting mergers leads to lower prices, lower profits and higher consumer surplus. On the contrary, if a market is characterized by shared leadership, integration (at least "partial") is to be preferred to competition, since the tragedy of the anticommons tends to prevail. This is because, with full quality leadership, complements produced by the same firm appear as substitutes in the consumers' demand functions, while with shared leadership, they exhibit the standard cross-price effect and the cross-price effect among complements produced by different firms that now has a reversed sign.

We have then considered the possibility that firms engage in mixed bundling.

In such case, firms might be able to charge higher prices to consumers who buy components separately and do "mix and match". The introduction of mixed bundling does not change the results under full-quality leadership, that is, not allowing mergers (or requiring firms to divest) always improves consumer surplus. It may however render disintegration desirable also in case of shared leadership, in particular for high degrees of substitutability among systems.

A possible extension would be to provide a full-fledged analysis of quality choices by firms producing complementary goods. In this paper we have assumed that the quality produced by each firm is exogenous and so is the relative position as "leaders" or "shared leaders". By letting firms choose their quality, we might not only analyze how they position themselves relative to competitors but also their strategic quality choices in response to expected (or prevailing) decisions by the antitrust authority.

Appendix

Proof of Lemma 3

Define

$PS^{SD} \equiv \sum_{i=A,B} \sum_{z=1,2} \Pi_{iz}^{SD} = \frac{q_{BB}(62319q_{BB}-178642q_{BB}^2+255403q_{BB}^3-182111q_{BB}^4+51805q_B^5-8674)}{(14-19q_{BB})^2(135q_{BB}-176q_{BB}^2+75q_{BB}^3-34)}$;

$PS^{SI} \equiv \Pi_A^{SI} + \Pi_B^{SI} = \frac{q_{BB}(7-9q_{BB})}{9(1-q_{BB})}$.

By comparing these two expressions for $q_{BB} \in (\frac{1}{3}, \frac{1}{2})$, it is possible to verify that $PS^{SD} > PS^{SI}$ in the relevant range of the parameters. □

Proof of Proposition 9

Define

$$CS^{Ft} \equiv \int_{\theta_{AB}^{AA}}^{1} (V + \theta - p_{A1}^{Ft} - p_{A2}^{Ft})d\theta + \int_{\theta_{BA}^{AB}}^{\theta_{AB}^{AA}} (V + \theta q_{AA} - p_{A1}^{Ft} - p_{B2}^{Ft})d\theta +$$
$$+ \int_{\theta_{BB}^{BA}}^{\theta_{BA}^{AB}} (V + \theta q_{BB} - p_{B1}^{Ft} - p_{A2}^{Ft})d\theta + \int_{0}^{\theta_{BB}^{BA}} (V - p_{B1}^{Ft} - p_{B2}^{Ft})d\theta$$

$(t = I, A, B, D)$. Substituting the expressions for equilibrium prices relative to each market configuration and rearranging,

$CS^{FI} = \frac{9-64q_{BA}+99q_{BA}^2}{18(1-q_{BA})} + V$ and $CS^{FD} = \frac{392-2536q_{BA}+6209q_{BA}^2-7010q_{BA}^3+2939q_{BA}^4}{2(14-19q_{BA})^2(2-3q_{BA})} + V.$

Then, $TS^{FI} = CS^{FI} + \Pi_A^{FI} + \Pi_B^{FI}$ and $TS^{FD} = CS^{FD} + \Pi_A^{FD} + \Pi_B^{FD}$. Using the expressions for Π_A^{FI}, Π_B^{FI}, Π_A^{FD} and Π_B^{FD},

$$TS^{FD} - TS^{FI} = \frac{q_{BA}(1 - 2q_{BA})(249 - 754q_{BA} + 569q_{BA}^2)}{18(1 - q_{BA})(14 - 19q_{BA})^2} \tag{2.23}$$

which is always positive for all $q_{BA} \in \left[\frac{1}{3}, \frac{1}{2}\right]$.

To prove that $TS^{SD} > TS^{SI}$, define

$$CS^{St} \equiv \int_{\theta_{AA}^{AB}}^{1} (V + \theta - p_{A1}^{St} - p_{B2}^{St})d\theta + \int_{\theta_{BB}^{AA}}^{\theta_{AA}^{AB}} (V + \theta q_{AA} - p_{A1}^{St} - p_{A2}^{St})d\theta +$$
$$+ \int_{\theta_{BA}^{BB}}^{\theta_{BB}^{AA}} (V + \theta q_{BB} - p_{B1}^{St} - p_{B2}^{St})d\theta + \int_{0}^{\theta_{BA}^{BB}} (V - p_{B1}^{St} - p_{A2}^{St})d\theta$$

$(t = I,\ A,\ B,\ D)$. Substituting the expressions for equilibrium prices relative to each market configuration and rearranging, we obtain

$CS^{SI} = \frac{9-28q_{BB}+27q_{BB}^2+18V(1-q_{BB})}{18(1-q_{BB})}$,
$CS^{SD} = \frac{(2939q_{BB}^4+392(1+2V)-q_{BB}^3(7010+2166V)+2q_{BB}^2(3145+2318V)-8q_{BB}(317+413V))}{2(14-19q_{BB})^2(2-3q_{BB})}$.

Using the expressions for Π_A^{SI}, Π_B^{SI}, Π_A^{SD} and Π_B^{SD}, we obtain TS^{SD} and TS^{SI}. It is relatively easy to ascertain that $TS^{SD} - TS^{SI} > 0$ for all $q_{BA} \in \left[\frac{1}{3}, \frac{1}{2}\right]$. □

Proof of Proposition 10

In case FA profits for each firm amount to $\Pi_A^{FA} = p_{A1}D_{A1}^F + p_{A2}D_{A2}^F$, $\Pi_{Bz}^{FA} = p_{Bz}D_{Bz}^F$ $(z = 1, 2)$. Differentiating Π_A^{FA} with respect to p_{A1} and p_{A2} and Π_{Bz}^{FA} with respect to p_{Bz}^{FA} and solving the first-order conditions simultaneously we obtain $p_{A1}^{FA} = \frac{q_{BA}\left(29-89q_{BA}+68q_{BA}^2\right)}{17-42q_{BA}+25q_{BA}^2}$, $p_{A2}^{FA} = \frac{q_{BA}\left(19-65q_{BA}+54q_{BA}^2\right)}{17-42q_{BA}+25q_{BA}^2}$, $p_{B1}^{FA} = \frac{q_{BA}(11q_{BA}-7)}{25q_{BA}-17}$ and $p_{B2}^{FA} = \frac{4q_{BA}(2q_{BA}-1)}{25q_{BA}-17}$.
Profits are $\Pi_A^{FA} = \frac{q_{BA}\left(461-2128q_{BA}+3257q_{BA}^2-1654q_{BA}^3\right)}{(25q_{BA}-17)^2(1-q_{BA})}$, $\Pi_{B1}^{FA} = \frac{q_{BA}(11q_{BA}-7)^2}{(25q_{BA}-17)^2}$ and $\Pi_{B2}^{FA} = \frac{16q_{BA}(2q_{BA}-1)(3q_{BA}-2)}{(25q_{BA}-17)^2}$ in equilibrium.

In case FB, profits for each firm amount to $\Pi_{Az}^{FB} = p_{Az}D_{Az}^F$ $(z = 1, 2)$, $\Pi_B^{FB} = p_{B1}D_{B1}^F + p_{B2}D_{B2}^F$. Equilibrium prices are $p_{A1}^{FB} = \frac{2q_{BA}(11q_{BA}-7)}{25q_{BA}-17}$, $p_{A2}^{FB} = \frac{8q_{BA}(2q_{BA}-1)}{25q_{BA}-17}$, $p_{B1}^{FB} = \frac{q_{BA}(11q_{BA}-7)}{25q_{BA}-17} = \frac{p_{A1}^{FB}}{2}$ and $p_{B2}^{FB} = \frac{4q_{BA}(2q_{BA}-1)}{25q_{BA}-17} = \frac{p_{A2}^{FB}}{2}$.
Equilibrium profits are $\Pi_{A1}^{FB} = \frac{4q_{BA}(11q_{BA}-7)^2}{(25q_{BA}-17)^2}$, $\Pi_{A2}^{FB} = \frac{64q_{BA}(2q_{BA}-1)(3q_{BA}-2)}{(25q_{BA}-17)^2}$ and $\Pi_B^{FB} = \frac{q_{BA}\left(41q_{BA}^2-66q_{BA}+25\right)}{(25q_{BA}-17)^2}$.

A simple direct comparison of prices in cases FA and FB clearly shows that $p_{iz}^{FD} < p_{iz}^{FA} < p_{iz}^{FI}$ and $p_{iz}^{FD} < p_{iz}^{FB} < p_{iz}^{FI}$, $i = A, B$; $z = 1, 2$, and $p_{Az}^{FB} < p_{Az}^{FA}$ and $p_{Bz}^{FB} = p_{Bz}^{FA}$ $(z = 1, 2)$, as indicated in the Proposition. □

Proof of Proposition 11

In case SA, profits for each firm amount to $\Pi_A^{SA} = p_{A1}D_{A1}^S + p_{A2}D_{A2}^S$, $\Pi_{Bz}^{SA} = p_{Bz}D_{Bz}^S$ $(z = 1, 2)$.

Equilibrium prices are $p^{SA}_{A1} = \frac{q_B\left(40q_B^2-55q_B+19\right)}{25q_B^2-42q_B+17}$, $p^{SA}_{A2} = -\frac{2q_B\left(10q_B^2-11q_B+3\right)}{25q_B^2-42q_B+17}$, $p^{SA}_{B1} = \frac{q_B(5q_B-4)}{25q_B-17}$ and $p^{SA}_{B2} = \frac{5q_B(2q_B-1)}{25q_B-17}$.
Equilibrium profits are $\Pi^{SA}_{A} = \frac{q_B\left(205-881q_B+1260q_B^2-600q_{BA}^3\right)}{(25q_B-17)^2(1-q_B)}$, $\Pi^{SA}_{B1} = \frac{q_B(5q_B-4)^2}{(25q_{BA}-17)^2}$ and $\Pi^{SA}_{B2} = \frac{25q_B(2q_B-1)(3q_B-2)}{(25q_{BA}-17)^2}$. In case SB, profits for each firm amount to $\Pi^{SB}_{Az} = p_{Az}D^S_{Az}$, $(z = 1, 2)$, $\Pi^{SB}_{B} = p_{B1}D^S_{B1} + p_{B2}D^S_{B2}$. Equilibrium prices are $p^{SB}_{A1} = \frac{q_B(16q_B-11)}{25q_B-17}$, $p^{SB}_{A2} = \frac{q_B(2q_B-1)}{25q_B-17}$, $p^{SB}_{B1} = \frac{q_B\left(16q_B-17q_B^2-3\right)}{25q_B^2-42q_B+17}$ and $p^{SB}_{B2} = \frac{q_B\left(26q_B^2-31q_B+9\right)}{25q_B^2-42q_B+17}$.
Equilibrium profits are $\Pi^{SB}_{A1} = \frac{q_B(16q_B-11)^2}{(25q_{BA}-17)^2}$, $\Pi^{SB}_{A2} = \frac{q_B(2q_B-1)(3q_B-2)}{(25q_{BA}-17)^2}$ and $\Pi^{SB}_{B} = \frac{q_B\left(117-540q_B+826q_B^2-419q_B^3\right)}{(25q_B-17)^2(1-q_B)}$.

It is now immediate to compare equilibrium prices in SA and SB with those obtained in Section 2.4 to reach the conclusions illustrated in the Proposition. □

Proof of Lemma 4

Propositions 11 and 12 imply that $CS^{SI} > CS^{SD}$, $CS^{SB} > CS^{SD}$ and $CS^{SI} > CS^{SA}$. To prove the remaining inequalities of the Lemma, define
$CS^{SA} = \frac{((2025q_{BB}^4-125q_{BB}^3(39+10V)+q_{BB}^2(4427+2950V)-2q_{BB}(909+1139V)+289(1+2V))}{2(17-25q_{BB})^2(1-q_{BB})}$;
$CS^{SB} = \frac{(1731q_{BB}^4-2q_{BB}^3(2118+625V)+q_{BB}^2(3983+2950V)-q_{BB}(1719+2278V)+289(1+2V))}{2(17-25q_{BB})^2(1-q_{BB})}$
using the expression for CS^{St} given in the proof of Proposition 9.

By comparing CS^{SI} to CS^{SB} and CS^{SA} to CS^{SD} for $q_{BB} \in (\frac{1}{3}, \frac{1}{2})$ (where CS^{SI} and CS^{SB} have been obtained in the Proof of Proposition 9) it is possible to conclude that $CS^{SD} > CS^{SA} > CS^{SI}$ and $CS^{SD} > CS^{SB} > CS^{SI}$ in the relevant range of the parameters. □

Proof of Lemma 5

Define
$PS^{SA} \equiv \Pi^{SA}_{A} + \sum_{z=1,2} \Pi^{SA}_{Bz} = \frac{q_{BB}(271-1162q_{BB}+1650q_{BB}^2-775q_{BB}^3)}{(17-25qb)\hat{}2(1-qb)}$;

$PS^{SB} \equiv \Pi_B^{SB} + \sum_{z=1,2} \Pi_{Az}^{SB} = \frac{q_{BB}(240-1022q_{BB}+1447q_{BB}^2-681q_{BB}^3)}{(17-25q_{BB})^2(1-q_{BB})}$.

By comparing PS^{SD}, PS^{SI} obtained in the Proof of Lemma 1 and either PS^{SA} or PS^{SB} for $q_{BB} \in (\frac{1}{3}, \frac{1}{2})$, it is possible to verify that $PS^{SD} > PS^{SA} > PS^{SI}$ and $PS^{SD} > PS^{SB} > PS^{SI}$ in the relevant range of the parameters. $\square$

Proof of Proposition 14

Using CS^{St} given in the Proof of Proposition 9 and the marginal consumers defined at the beginning of Section 2.7.2, we obtain $CS^{SM} = \frac{1+5q_B}{18} + V$. Comparing is with CS^{SD}, whose expression is again given in the Proof of Proposition 9, we have

$$CS^{SM} - CS^{SD} = \frac{(1-2q_B)(15933q_B^3 - 28832q_B^2 + 16860q_B - 3136)}{18(2-3q_B)(14-19q_B)^2} \qquad (2.24)$$

which is negative for $q_B < 0.39$ and positive otherwise in the interval $q_B \in \left[\frac{1}{3}, \frac{1}{2}\right]$.

$\square$

CHAPTER 3

Complementing Substitutes: Entry, Compatibility and Bundling *

3.1 Introduction

In this final chapter, we show that selling complementary goods, or equivalently, selling all components of a system, may not only be profit enhancing for a firm with monopoly power in the market for one complement, but also pro-competitive, with a positive impact on consumer surplus. We consider a setting in which two independent firms produce two perfectly complementary products, one each. Initially, both markets are monopolistic and the two firms replicate the standard Cournot's complementary monopoly problem (Cournot, 1838), in that the price of the system consisting of the two complementary goods is higher than the price set if the two firms were integrated in a unique monopoly. We show that, in such setting, a low quality rival producing one component only could never earn positive profits in a pure-strategy equilibrium.[1] Moreover, if the

*This chapter is extensively based on the paper "Bundling and Compatibility: Selling the Whole Package May Be Pro-Competitive "(with E.Carbonara), mimeo, University of Bologna, Italy. We are grateful to Luca Lambertini, Giuseppe Dari-Mattiacci, Massimo Motta, Francesco Parisi and participants to the 2008 MLEA (MidWest Law and Economics Association) Conference at Northwestern University, the 2008 SIDE-ISLE (Italian Society of Law and Economics) Conference at the University of Bologna and to seminars at Bologna University and Kassel University, for helpful suggestions.

[1]This result has been proved also by Casadesus-Masanell, Nalebuff and Yoffie (2007) in a less general setting.

quality produced by the entrant is not significantly lower than the incumbent's, a pure-strategy Nash equilibrium with all three firms in the market does not exist. In fact, when the entrant is in the market and one layer is a duopoly, the monopolist in the other layer prefers to price its product to kick the low-quality firm out. Conversely, when the entrant is out of the market, the monopolist in the other layer finds it profitable to allow entry.

Using these results as a starting point, we investigate the case in which one of the duopolists is given the option of producing both components. This is what happened, for instance, when Microsoft Windows started providing applications such as MsOffice, Internet Explorer and Windows Media Player or when Canon and Nikon started producing their own lenses. In such markets, if the components produced by different firms are incompatible, or firms engage in "pure bundling", each firm's system will compete with the others as in a traditional oligopoly, whereas, if components are compatible and firms do not bundle, consumers can assemble their own systems picking the components that better fit their taste so that firms will compete in the separate markets for each component.

Drawing from direct observation of several real-world markets and their historic evolution, we can assume, however that compatibility is only "partial". By this we mean that the integrated firm, while continuing to allow its original product to be combined with the others available in the market, makes the new component technically compatible only with its original product. When Microsoft started producing its own Internet browser, for instance, it continued to allow Windows to be technically compatible with other browsers but sold Internet Explorer only in combination with the OS. Similarly, when Nikon or Canon entered the market for lenses with their own product, they made them technically incompatible with the camera bodies produced by their rivals, while continuing to allow consumers to match their own camera bodies with lenses by other producers.

In all of these cases, we prove that a firm facing potential entry in its market finds it profitable to enter the complementary market, even if this has the effect of inducing entry. This strategy remains profitable also in case the firm chooses partial compatibility, allowing consumers to match its original product with complementary goods produced by rivals. The intuition for this result is that the newly integrated firm sells a high-quality product at an especially low quality-adjusted price (i.e., the price per "unit of quality") because the Cournot complementary monopoly problem is absent for a system entirely produced by the same firm. This ensures that the demand satisfied by the integrated firm is larger than the demand of the single component it produced before entering the other market and such increase in demand drives the result. We further prove that a qualitatively similar result holds when it's the low-quality firm that produces both components. Such firm might now be able to survive, enlarging consumer choice and increasing welfare. Finally, we consider entry in the related market by the monopolist not facing potential competition, proving that, in equilibrium, such firm will always price its system so to keep competitors out of the related market.

In conclusion, producing all components of a system is profit-maximizing for firms. Particularly, an integrated firm would actually maximize profits by engaging in pure bundling rather than in "partial compatibility" and the price of the bundle would be higher than the sum of the prices of the two components if they were sold separately. In both cases, however, producing all components may allow the low-quality producer to conveniently re-enter the market so that, with the low-quality duopolist now active in the market, consumers' welfare unambiguously increases. On the other side, pure strategy equilibria might still not exist and we may observe cycles in price competition, although for a restricted set of parameters.

The results in this paper shed new light on the ongoing debate raised around some important antitrust cases involving the production and sale of complementary goods. In the Microsoft case, the Court's holding in *United States v. Microsoft Corp.* required Microsoft to be divided into an operating systems company and a company that would hold the other branches of its business, such as application development.[2] The purpose of the break-up plan (later abandoned in the US) was two-fold: to prevent Microsoft from extending its market dominance to software applications and to facilitate entry and competition in the operating systems market. Being operating systems and applications complements, the likely result would have been the increase in the price of both goods. Far from being unaware of this potential issue, the Court ordered the break-up with the precise intent to prevent limit pricing by Microsoft. By raising prices, it was argued, separation would facilitate entry, possibly driving prices below pre-separation levels.[3] We prove instead that, by being active in two related markets, a conglomerate firm may actually facilitate entry rather than hinder it. Thus, forcing firms to divest may not only result in higher prices but may result in reduced competition and undermine market stability.

[2]United States v. Microsoft Corp., 97 F. Supp. 2d. 59 (D.D.C. 2000).

[3]Another famous case was the merger between General Electric and Honeywell, which was prohibited by the European Commission on the grounds that post merger prices would be so low to produce market foreclosure (European Commission Decision of 03/07/2001, declaring a concentration to be incompatible with the common market and the EEA Agreement Case, No. COMP/M.2220 - General Electric/Honeywell).

3.1.1 Related Literature

Previous literature has provided contrasting conclusions on the effects of entry into related markets on both the degree of competition and welfare. Initially, antitrust authorities (especially in the US), condemned such practices arguing that a firm with monopoly power in one market could use the "leverage" provided by its monopoly to either foreclose or reduce competition also in the related market, by imposing very low prices.[4] Such "leverage theory" was however attacked by the Chicago tradition (Posner, 1976). If the price of the bundled good is higher than the amount consumers would have to pay in the open market, the Chicago argument goes, they would demand less of it. This implies that there is only one monopoly to exploit, that on the original market. Such view has been challenged by Whinston (1990) and, more recently, by Choi and Stefanadis (2001), Carlton and Waldman (2002) and Nalebuff (2004). Whinston (1990) proves that tying may be an effective (and profitable) means for a monopolist in one market to affect the market structure of the tied-good market (i.e. "monopolize" it) by foreclosing potential entrants or excluding existing competitors. He proves however that, when goods are perfect complements, a monopolist in the market for one complement never finds it profitable to adopt strategies that reduce the level of competition in related markets. In this sense, for complementary goods, the Chicago view still holds. Choi and Stefanadis (2001) show that when an incumbent monopolist faces the threat of entry in all complementary components, tying may make the prospects of successful entry less certain, discouraging rivals from investing and innovating, so that tie-in sales may reduce consumer and total economic welfare. Nalebuff (2004) and, more recently, Peitz (2008) reinforce this result, showing how bundling is a powerful strategy to foreclose potential

[4]A long line of cases has developed under the Sherman and the Clayton Acts. The main ones date from the early 1900s to the late 1950's (see Whinston, 1990, for a discussion).

rivals. Particularly, if a firm is a monopolist in the market for two related goods, by bundling them together it can foreclose entry by rivals trying to enter the market for only one of these goods.[5] Interestingly, we reach the opposite conclusion: when entering the complement market, the monopolist facilitates entry in the original market, even if for its own good. In other words, a commitment to sell both components of a system as a bundle increases profits, *notwithstanding* the lower ability to foreclose the entry of low-quality competitors in the original market. We obtain the same results also when the complementary goods are not bundled but their producer allows partial compatibility.[6]

Another very relevant strand of literature is the one on "mix & match": firms producing all or some components of a system might sell them as a bundle or separately, allowing consumers to fully "mix and match" across firms (Matutes and Regibeau 1988). Einhorn (1992) considers two vertically differentiated firms, each producing both components of a system. He shows that, in equilibrium, they will both choose to be compatible with each other, allowing "mix and match". Denicolò (2000) considers the case of a "generalist" firm producing both components and two specialist firms each producing one component only. He finds that pure bundling may lead to higher profits than those created by compatibility. Both Einhorn (1992) and Denicolò (2000) consider covered market configurations.

Our model differs from the "mix and match" literature in several respects.

[5]Carlton and Waldman (2002) show, in a dynamic setting and in the presence of network externalities, how a firm holding monopoly power in the current period can use tying to preserve its monopoly in the future and how tying can be used to extend its power into a newly emerging market.

[6]Our contribution is also related to the literature on essential complements and entry. Cheng and Nahm (2007) consider double mark-up issues when one of the components is essential, meaning that it can be used alone even if it yields higher utility to the consumer when combined with a second component, supplied by an independent producer. They focus on instances in which there are no equilibria in pure strategies. Farrell and Katz (2000) study cases in which both goods are essential but one is produced by a monopolist and the other in a competitive market. They analyze the incentives for the monopolist to enter the competitive market to induce a reduction in the equilibrium price.

First, we focus on the possibility that integration can have pro-competitive effects, lowering barriers to entry and increasing consumer welfare. Second, we allow for partial compatibility only, believing that such assumption better represents a wide set of real-world markets, especially in the computing industry. In this case, contrarily to Matutes and Regibeau (1988) and Einhorn (1992), we find that selling both components of a system is more profitable through bundling rather than with compatible products. Third, while we assume, as in Einhorn (1992) that firms are differentiated across vertical characteristics, we also assume that they compete in an uncovered market configuration, i.e. in a setting where in equilibrium some consumers can choose not to purchase the system. This is not a purely technical assumption; in fact, it becomes crucial when performing welfare analysis, and compare consumer surplus in different equilibria. Finally, all of the literature on mix and match mentioned above assumes that consumers have a distribution of preferences over single components and not over systems. This assumption is indeed very restrictive, since it implies that the markets of single components are virtually independent when products are compatible. In other words, in the market for each component, demand never depends on the price of the complements so cross-price effects are totally absent. It is questionable whether these models are actually able to fully capture all of the features of complementary markets. In our analysis, we assume instead that consumer tastes are distributed over systems and not over single complements. Hence demands for complements are no longer independent and both the size of the market of each component and their prices vary with the prices set by all firms.

Finally, our contribution is related to the law and economics literature on the tragedy of the anticommons (Buchanan and Yoon 2000, Parisi et al. 2005, Dari-Mattiacci and Parisi 2007). In fact, our contribution shows that allowing firms to enter the market of other components might be welfare enhancing not simply because of the double-marginalization effect, but also because it increases the number of systems available to consumers through the entry of new firms.

The chapter is organized as follows. Section 2 presents the benchmark model in which three firms operate, each producing one component of a system: two firms produce component 1 and one firm produces component 2. Section 3 introduces the case in which the quality leader in the duopoly of component 1 has the option to enter the market of the other component. First, it analyzes the different market configurations that may arise when the newly integrated firm engages in "pure bundling" and when it allows for (partial) compatibility. Second, it characterizes the unique pure-strategy Nash equilibrium emerging in both cases. Section 4 analyzes the different market configurations that may arise when a firm which is not a quality leader produces both components. Section 5 analyzes the choice between pure bundling and partial compatibility. Section 6 presents some extensions and concludes. Proofs of Propositions and Lemmas can be found in the Appendix.

3.2 The Benchmark Model

Consider a market in which two perfectly complementary goods, 1 and 2, are produced. Goods 1 and 2 are of any value only when consumed together. Initially, we consider a benchmark model in which there is competition only in the market for good 1, whereas good 2 is sold by a single firm. In particular, we assume that good 1 is produced by two firms, A and C, producing a_1 and c_1 respectively, whereas firm B produces b_2, the second complementary good. All goods are produced at zero cost.

Consumers are free to choose either a_1 or c_1 and combine it with b_2. Hence, two systems are available on the market, $AB = (a_1 + b_2)$ and $CB = (c_1 + b_2)$. System AB is of better quality than CB, that is $q_{AB} > q_{CB}$. This is equivalent to assuming that firm A provides a superior complementary product. Let p_{A1}, p_{B2} and p_{C1} be the prices of the three components. Firms set their prices simultaneously.

Consumers differ in their valuation of the quality of the systems. The utility function of a type-θ consumer, is given by $V_{iB} = \theta q_{iB} - (p_{i1} + p_{B2})$, $i = A, C$, $\theta \in [0, 1]$. Let the cumulative distribution function and the continuous density function be given by $G(\theta)$ and $g(\theta)$, respectively. Define $F(\theta)$ as the proportion of consumers whose type is higher than θ and $f(\theta)$ as the corresponding density function, so that $F(\theta) = 1 - G(\theta)$ and $f(\theta) = -g(\theta) < 0$. We make the standard assumption that the distribution of θ satisfies the increasing hazard-rate condition, i.e., $-\frac{f(\theta)}{F(\theta)}$ is increasing in θ. [7]

Demand functions are obtained in a standard way. Consumers have three options: *i*) to buy system AB; *ii*) to buy system CB; *iii*) to buy neither and gain zero utility. Demand of AB is positive if and only if $0 < F\left(\theta_{CB}^{AB}\right) < 1$, where $\theta_{CB}^{AB} = \frac{p_{A1} - p_{C1}}{q_{AB} - q_{CB}}$ is the consumer type indifferent between buying system AB and system CB. Similarly, demand of CB is positive if $0 < F(\theta_{CB}) - F\left(\theta_{CB}^{AB}\right) < 1$, where $\theta_{CB} = \frac{p_{C1} + p_{B2}}{q_{CB}}$ is the consumer type indifferent between buying system CB or buying nothing.

In such a setting, when the degree of quality differentiation is sufficiently high, there exists a unique Nash equilibrium in pure strategies, as the following Proposition shows. Such equilibrium involves firms A and B only, whereas firm C is not able to gain access to the market even by setting a price equal to marginal

[7] The increasing hazard rate condition is satisfied by the most commonly used distributions, including the uniform, which we will use extensively below. Moreover, the increasing hazard rate condition yields strictly quasi-concave profit functions (see Cheng and Nahm, 2007).

cost.

Proposition 15. *When $0 < q_{CB} \leq \hat{q}_{CB}$, it exists a unique pure strategy Nash equilibrium in which firm C has no demand and firms A and B act like in a complementary monopoly.*

Proof: See Appendix.

For instance, when θ is uniformly distributed in the interval $[0, 1]$, that is $F(\theta) = 1 - \theta$, such unique pure-strategy equilibrium exists if and only if $q_{CB} \leq \frac{4}{9}q_{AB}$.[8] In this case, the demand of the only available system AB is $F(\theta_{AB})$, where $\theta_{AB} = \frac{p_{A1}+p_{B2}}{q_{AB}}$, and the Nash equilibrium prices are $p_{A1}^C = p_{B2}^C = \frac{q_{AB}}{3}$, so that profits are $\Pi_A^C = \Pi_B^C = \frac{q_{AB}}{9}$, as in the classical Cournot model with complementary goods (the superscript C stands for "standard Cournot model").[9] When instead $\frac{4}{9}q_{AB} < q_{CB} < q_{AB}$, no pure-strategy Nash equilibrium exists and this result can be explained loosely as follows. When C is out of the market, by lowering its price, B can deviate and change the market configuration, such that CB is also sold. This is indeed profitable when q_{CB} is relatively close to q_{AB} because, with high q_{CB}, demand for CB is very reactive to price changes (as AB and CB are close substitutes). Then, even a small reduction in p_{B2} triggers a large change in the consumption of CB (which was previously zero) and the gains provided by a larger demand more than counterbalance the loss due to a lower price. However, once C is in the market, A lowers its price to recover the share of its demand that has gone to C. Once A's price has decreased, it is then profitable for B to increase its price so that C falls out of the market again. This also clarifies why, when q_{CB} is relatively low, C is not an active player in the

[8]This application to the uniform case is also discussed in Casadesus-Masanell, Nalebuff and Yoffie (2007).

[9]See Cournot (1838), and Sonnenschein (1968).

game. Its product is not a close substitute for A so that B will not want to pursue the undercut strategy to bring C in.[10]

3.3 The quality leader produces both complements

In the previous Section we argued that whenever systems are sufficiently differentiated, the unique equilibrium is one in which C's entry is effectively foreclosed. The remaining two firms might then have the opportunity to further increase their profits by producing also the other component. In what follows, we analyze the case in which it's firm A that starts producing a_2. Interestingly, the entry of firm A, though making continued operation less profitable for rival firm B, does not lead to a monopolization of market 2. Moreover, firm C might now be able to re-enter market 1. As a result, when deciding whether to enter market 2, firm A faces a tradeoff. On the one side, it starts earning profits on the new market. On the other, it increases competition in the original market. We then need to check whether such practice is indeed profit-enhancing, considering the entire range of strategies available. Particularly, A can sell both goods a_1 and a_2 in a bundle (AA), rendering its components incompatible with the complements produced by other firms (we define this option as "pure bundling"), or to allow consumers to "mix & match" across components. In the first case, firm A will set a unique price p_A for its bundle and the available systems will be AA and, possibly, CB. In the second case, firm A sets two separate prices (p_{A1} and p_{A2}) for the two components and the available systems will be AA, AB and, possibly, CB.[11]

In both cases, we assume that firm A is the quality leader for both components,

[10] The underlying assumption of such results is that is that firms have no entry costs.

[11] As discussed in the Introduction we assume partial compatibility, so that the newly produced component is not compatible with complements produced by other firms.

so that $q_A > q_C$, $q_A > q_B$. Then, the quality of system AA (q_{AA}), is the highest in the market and $q_{AA} > q_{AB} > q_{CB}$ for all consumers. With no loss of generality, we normalize q_{AA} to 1.

3.3.1 Selling bundles?

We first analyze the case of pure bundling. In this case, the available systems will be AA and CB. Define $\theta_{CB}^{AA} = \frac{p_A - p_{C1} - p_{B2}}{1 - q_{CB}}$ the consumer indifferent between buying AA and CB, and $\theta_{CB} = \frac{p_{CB}}{q_{CB}}$ the consumer indifferent between buying CB and buying nothing. Demand functions are $D_{AA} = F(\theta_{CB}^{AA})$ and $D_{CB} = F(\theta_{CB}) - F(\theta_{CB}^{AA})$ whereas profits are $\Pi_A^* = p_A F(\theta_{CB}^{AA})$, $\Pi_B^* = p_{B2}\left[F(\theta_{CB}) - F(\theta_{CB}^{AA})\right]$ and $\Pi_C^* = p_{C1}\left[F(\theta_{CB}) - F(\theta_{CB}^{AA})\right]$.[12] The following proposition describes the unique equilibrium market configuration in this setting.

Proposition 16. *With pure bundling, there is a unique equilibrium in which both systems AA and CB are sold in positive amounts. Firm C has positive demand and earns positive profits.*

Proof: See Appendix.

Proposition 2 offers an interesting insight for competition policy. The quality leader's decision to enter the market of the second component and engage in pure bundling gives the opportunity to firm C to re-enter the market and earn positive profits. As opposed to Proposition 1, firm A no longer reacts by lowering the price of its bundle when firm C is in the market. Foreclosure is not a viable

[12]In order to guarantee satisfaction of second order conditions for profit maximization in this case, in addition to the increasing hazard-rate condition, we need to also impose the following: for any θ_1 and θ_2, with $\theta_1 > \theta_2$, $\frac{f'(\theta_2)}{q_{CB}^2} - \frac{f'(\theta_1)}{(1-q_{CB})^2} < 0$. The increasing hazard-rate condition yields a strictly quasi-concave profit function for firm A. The second inequality implies the same for B and C's profits.

option, since, in order to exclude C (and therefore B), A should set prices so low as to actually decrease its profits.[13]

In order to compare the results in the first two Propositions also in terms of profits and welfare, we will from here on assume that θ is uniformly distributed.

3.3.1.1 A uniform distribution for θ

In such case, the demand functions for the two available systems AA and CB are $D_{AA} = (1-\theta_{CB}^{AA}) = 1-\frac{p_A-p_{C1}-p_{B2}}{1-q_{CB}}$ and $D_{CB} = (\theta_{CB}^{AA}-\theta_{CB}) = \frac{p_A-p_{C1}-p_{B2}}{1-q_{CB}}-\frac{p_{C1}+p_{B2}}{q_{CB}}$. Profit functions are $\Pi_A^* = p_A\left(1-\frac{p_A-p_{C1}-p_{B2}}{1-q_{CB}}\right)$, $\Pi_B^* = p_{B2}\left(\frac{p_A-p_{C1}-p_{B2}}{1-q_{CB}}-\frac{p_{C1}+p_{B2}}{q_{CB}}\right)$ and $\Pi_C^* = p_{C1}\left(\frac{p_A-p_{C1}-p_{B2}}{1-q_{CB}}-\frac{p_{C1}+p_{B2}}{q_{CB}}\right)$, so that it is now immediate to obtain equilibrium prices and profits:

$$\begin{aligned} p_A^* &= \frac{3(1-q_{CB})}{2(3-q_{CB})} \\ p_{B2}^* &= p_{C1}^* = \frac{q_{CB}(1-q_{CB})}{2(3-q_{CB})} \end{aligned} \tag{3.1}$$

$$\begin{aligned} \Pi_A^* &= \frac{9(1-q_{CB})}{4(3-q_{CB})^2} \\ \Pi_B^* &= \Pi_C^* = \frac{q_{CB}(1-q_{CB})}{4(3-q_{CB})^2} \end{aligned} \tag{3.2}$$

A comparison of such equilibrium prices with those obtained in the benchmark model reveals that the entry of firm A into the complement market with a "pure bundling" strategy not only grants firm C positive demand and profits, but also leads to higher consumer surplus. In fact, when system AB only is available, it is sold at a quality-adjusted price equal to $\frac{2}{3}$.[14] With A's entry, however, the

[13] The proof of Proposition 2 in the Appendix also shows that it would never be optimal for firm A to lower p_A until $\theta_{AA} = \theta_{CB}$, and be the only seller in the market in a corner solution. In fact, profits would always be lower than Π_A^*.

[14] The quality-adjusted price of system AB is $\frac{p_{AB}}{q_{AB}} = \frac{2}{3}$. Roughly speaking, the quality-adjusted price measures the price per "unit of quality".

quality-adjusted price for system AA is p_A^* (since $q_{AA} = 1$) and $p_A^* < \frac{2}{3}$ for all $q_{CB} \in [0, 1]$. This result is not surprising, since now both components of system AA are offered by the same firm. This eliminates the Cournot complementary monopoly problem, so that the quality adjusted price for AA is lower than the price for AB, offered by different firms which did not coordinate their prices. Moreover, C's entry in market 1 exerts further downward pressure on A's price. Since system AA is at least as good in quality as AB but its quality-adjusted price is lower, consumers are better off regardless of how system CB is priced. As a consequence, consumer surplus must go up. This intuition can be confirmed by checking that the pre-entry marginal consumer in the market where AB only is sold (i.e., the consumer with $\theta = \theta_{AB}$, just indifferent between purchasing AB and nothing) is better off after $A's$ entry into the complement market. In fact, before entry $\theta_{AB} = \frac{2}{3}$, while after the introduction of a_2, such consumer will buy AA with a positive surplus, since $\theta_{AB} q_{AA} - p_A^* = \frac{2}{3} - \frac{3(1-q_{CB})}{2(3-q_{CB})} > 0$ for all $q_{CB} \in [0, 1]$ .Thus, those consumers previously buying AB now buy AA and are better off, whereas some of those consumers who were previously out of the market now buy either AA or CB, earning positive surplus.

Conclusions are less straightforward when profits are considered. In Section 2 we established that, in the uniform case and for a sufficiently low value of q_{CB}, equilibrium profits for firms A and B were Π_A^C and Π_B^C, respectively. Now, with the introduction of a_2, firm B gets obviously worse off, whereas firm C starts earning positive profits. As for firm A, the entry in market 2 involves a trade-off: selling the second component yields positive profits in market 2, but it brings also a competitor in market 1. It can be easily verified that the first positive effect prevails on the second, so that $\Pi_A^* > \Pi_A^C$.[15]. The reason is that

[15]In fact, $\Pi_A^* - \Pi_A^C \geq 0$ iff $q_{AB} \leq \frac{81(1-q_{CB})}{4(3q_{CB})^2}$, and $\frac{81(1-q_{CB})}{4(3-q_{CB})^2} \geq 1$ for every $q_{CB} \leq \frac{4}{9} q_{AB}$. When instead A produces component 1 only and $q_{CB} > \frac{4}{9} q_{AB}$, no pure-strategy Bertrand-Nash

now A is able to sell a better system at a lower quality-adjusted price, so that its total demand for AA is larger than the demand for AB satisfied before entry in market 2 ($D^*_{AA} > D^C_{AB}$). Moreover, there are the additional profits due to the fact that A sells two components. Finally, substituting system AB with system AA increases A's profits and such increase more than compensates for firm B's losses (in fact, $\Pi^*_A - \Pi^C_A > \Pi^C_B - \Pi^*_B$), so that even without considering the beneficial effects on firm C's profits, we can conclude that pure bundling also increases producer surplus.[16] Since both consumer and producer surplus are higher with pure bundling we can conclude that also total welfare is higher.

The results above are summarized in the following Proposition.

Proposition 17. *When firm A chooses pure bundling,*

1. *Aggregate consumer surplus is higher than in a Cournot monopoly case.*
2. *Pure bundling allows firm A to obtain higher profits than in a Cournot monopoly case. Firm B always loses, whereas firm C always gains. Aggregate industry profits and total surplus are higher under pure bundling.*

As a final remark, we should consider that in some markets firms C and B may have the option to respond to A's entry in market 2 by merging. In equilibrium, the merged firm would set its price at $p_{CB} = \frac{(1-q_{CB})q_{CB}}{4-q_{CB}} < p^*_{B2} + p^*_{C1}$ whereas

equilibrium exists, so that Π^C_A is not the right measure of profit to use in the comparison. In such case, Casadesus-Masanell et al.(2007) fully characterize the set of non-dominated strategies for firms A, B and C (i.e., the ranges in which their prices will fall in their competing interaction). Any possible combination of such strategies, will generate a market configuration (in particular, either one where both systems AB and CB are sold or one where only system AB is sold) and a corresponding set of profits. It can be shown that, for firm A, Π_A is in any case greater than the profit resulting from any combination of non-dominated strategies (and then greater than any expected profit from a mixed strategy equilibrium). Hence, entering the market of the second component is always convenient (proof available upon request).

[16] Interestingly, then, firm B would not even be able to offer A a side payment to prevent the production of a_2.

firm A would respond with $p_{AA} = \frac{2(1-q_{CB})}{4-q_{CB}} < p_A^*$ (see Shaked and Sutton, 1982). Profits for both firms would be $\Pi_{CB} = \frac{(1-q_{CB})q_{CB}}{(4-q_{CB})^2}$ and $\Pi_{AA} = \frac{4(1-q_{CB})}{(4-q_{CB})^2}$. It is then possible to verify that $\Pi_{CB} > \Pi_B^* + \Pi_C^*$, implying that B and C would always try to merge, if the option were available, when A enters the complementary market. Moreover, $\Pi_{AA} > \Pi_A^C$; even if B and C merge after A's entry in the related market, A's profit from such operation are always higher than in the standard Cournot complementary monopoly and A would always enter the second market. This reinforces the results of Proposition 2, widening the range of possible ways that C can exploit to gain a positive share of the market. In that follows, we rule out the merger of firms B and C because we want to focus on the worst possible scenario for C and still investigate whether entry is feasible.

Given the results on surplus of Proposition 17, the production of a_2 should not raise any antitrust concerns *per se*. However, an antitrust issue might indeed arise if we consider that "pure bundling" might not be the only available strategy after entry in the market for the second component. In fact, requiring firm A to keep a_1 compatible with b_2 after entry in market 2, consumer and then total surplus might increase further. What needs to be verified is whether firm A would have any incentive to follow such strategy and, if this were not the case, whether an antitrust authority should impose it whenever it observes the entry of firms in complementary markets. This is the main purpose of the following section.

3.3.2 The Compatibility Option

We now assume that consumers can "mix & match" across components. In other words, three systems are available to consumers: $AA = (a_1 + a_2)$, $AB = (a_1 + b_2)$ and potentially $CB = (c_1 + b_2)$. We define this scenario "partial compatibility", since we are not allowing consumers to form the system $CA = (c_1 + a_2)$. As illustrated in Section 1, we believe that this assumption can more accurately represent a wide set of real-world markets. In any case, compatibility implies that A has to set separate prices for a_1 and a_2 (i.e. p_{A1}, p_{A2}), even when they are purchased together.[17] To obtain demand functions, define V_{ij} the utility from buying system $ij, V_{ij} = \theta q_{ij} - (p_{i1} + p_{j2})$, $i = A, C$; $j = A, B$, $\theta \in [0, 1]$. Note that, given the assumptions of the previous section, the perceived qualities of the three systems will be ordered as follows: $q_{AA} > q_{AB} > q_{CB}$, and the graphical analysis of Figures 3.1, 3.2 and 3.3 indicates that the function $V_{AA}(\theta)$ is the steepest one, whereas $V_{CB}(\theta)$ is the flattest. Each function $V_{ij}(\theta)$ intersects the others only once. As an implication, any couple of prices (p_{A1}, p_{A2}) such that $p_{A1}+ p_{A2} < 1$ generates a positive demand for system AA for all values of p_{B2} and p_{C1}.[18] Such result also implies that the candidates for an equilibrium market configuration are just four: $H1$) All available systems AA, AB and CB, are sold in equilibrium in positive amounts (Figure 3.1); $H2$) Only systems AA and CB are sold and system AB has a null demand (Figure 3.2); $H3$) Only systems AA and AB are sold and system CB has a null demand (Figure 3.3); $H4$) Only system AA is sold and systems AB and CB have null demands.

It is however fairly immediate to argue that both $H3$ and $H4$ are unrealistic outcomes and can therefore be neglected. Under $H4$, neither B nor C have customers. This can only happen due to a coordination failure, in that both B and C charge high prices even though their demand is zero. In other words,

[17]The alternative assumption of "full compatibility" would render the analysis extremely complicated (we would have four different combinations of complements available to consumers) and would not add to our results. In fact, the entry of firm C (and its subsequent effects on welfare, explained below) already occurs in this form of partial compatibility, so that, increasing the choice space for consumers would only reinforce our results. We are also implicitly excluding "mixed bundling" practices for firm A. For the analysis of mixed bundling, see Matutes and Regibeau (1992), Economides (1993), Gans and King (2006) and Choi (2008) among others.

[18]Intuitively, with its system AA, firm A serves the high-end part of the market, consisting of consumers with a strong taste for quality and high willingness to pay for it. Firm A is then always able to set its two prices in order to keep that part of the market "under control".

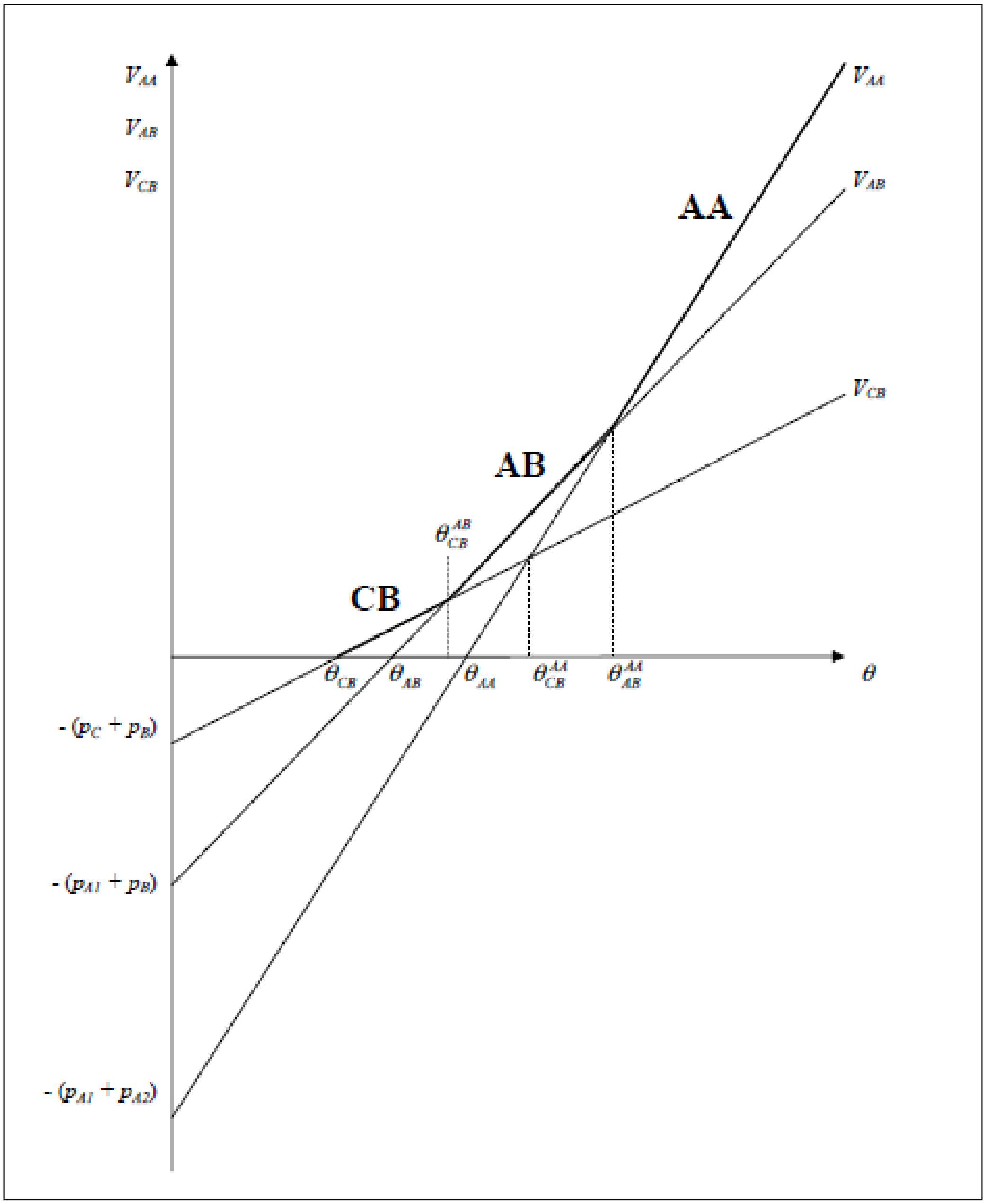

Figure 3.1: Market configuration $H1$: All available systems AA, AB and CB, are sold in equilibrium

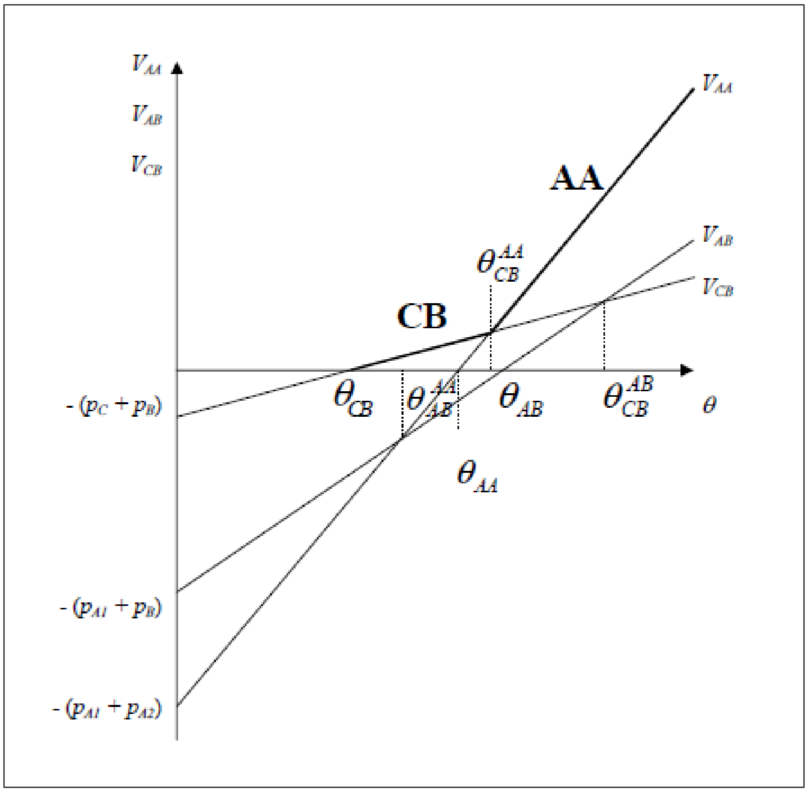

Figure 3.2: Market configuration $H2$: Only systems AA and CB are sold

lowering one firm's price to marginal cost (here zero) isn't enough to produce a positive demand for CB as the other complement's price is too high. It is then plausible to assume that firms B and C would try to react to such inefficient outcome by charging their price equal to marginal cost whenever their demand is null. In such case, $H4$ would not be possible because at $p_{B2} = p_{C1} = 0$ system CB has positive demand. A similar argument can be used to rule out an equilibrium in market configuration $H3$, which might emerge only when firm C charges a high price even when it is shut out of the market. We are therefore left with $H1$ and $H2$. The analysis of equilibrium prices and profits conditional to market configuration $H2$ is however redundant, as they coincide with those obtained in section 3.3.1 when A engages in pure bundling. In what follows, then, we will

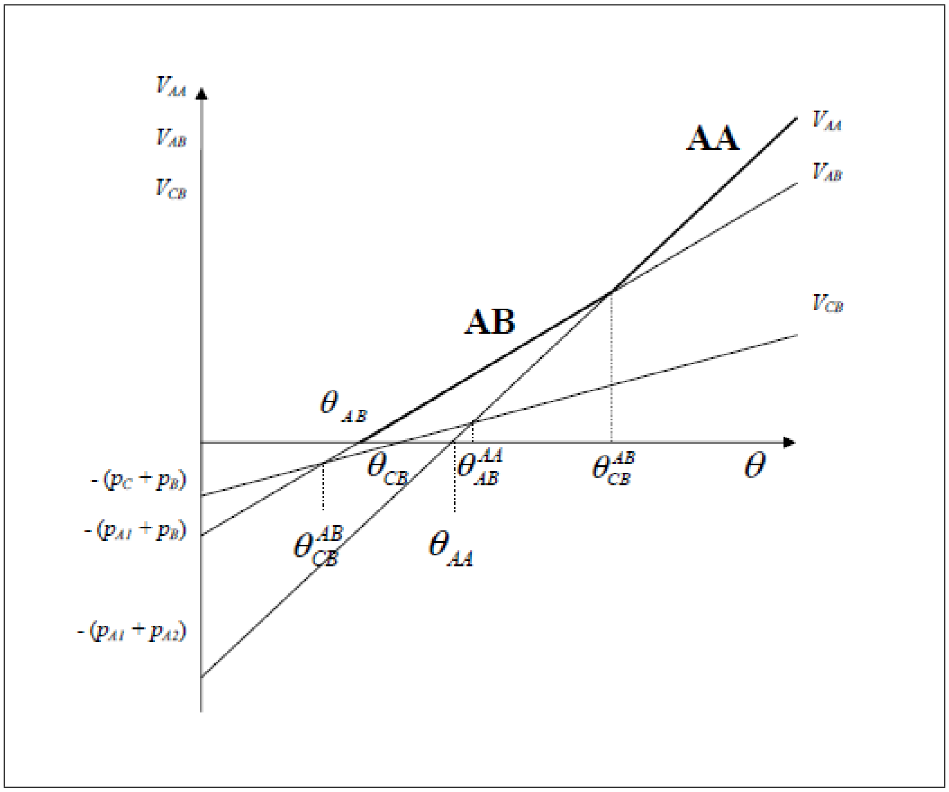

Figure 3.3: Market configuration $H3$: Only systems AA and AB are sold

focus on $H1$ with a primary interest on firm A's behavior and profits when system AA competes with both AB and CB.

3.3.2.1 Market Configuration H1: all three systems are sold

Define $\theta^{AA}_{AB} = \frac{p_{A2}-p_{B2}}{1-q_{AB}}$ the consumer indifferent between buying AA and AB, and $\theta^{AA}_{CB} = \frac{p_{A1}+p_{A2}-p_{C1}-p_{B2}}{1-q_{CB}}$ the consumer indifferent between AA and CB. When all systems AA, AB and CB are sold in equilibrium, consumers with $\theta \geq \theta^{AA}_{AB}$ buy system AA ($D_{AA} = 1 - \theta^{AA}_{AB} > 0$), those with $\theta^{AB}_{CB} \leq \theta < \theta^{AA}_{AB}$ buy AB; those with $\theta_{CB} \leq \theta < \theta^{AB}_{CB}$ buy CB ($D_{CB} = \theta^{AB}_{CB} - \theta_{CB} > 0$), whereas consumers with $0 < \theta < \theta_{CB}$ buy nothing. Profits for the three firms are $\Pi_A = p_{A1}D_{A1} + p_{A2}D_{A2}$, $\Pi_B = p_{B2}D_B$, and $\Pi_C = p_{C1}D_C$. Equilibrium prices and profits conditional to this market configuration are summarized in Table 3.1.[19]

Table 3.1: Bertrand Equilibrium Prices and Profits in $H1$.

Prices	$p_{A1}^{H1} = \frac{6q_{AB}^3+q_{CB}^2+5q_{AB}q_{CB}(1+q_{CB})-q_{AB}^2(6+11q_{CB})}{3(4q_{AB}^2+q_{CB}^2-4q_{AB}(1+q_{CB}))}$ $p_{A2}^{H1} = \frac{(1-q_{AB})(6q_{AB}^2+q_{CB}^2-q_{AB}(6+7q_{CB}))}{3(4q_{AB}^2+q_{CB}^2-4q_{AB}(1+q_{CB}))}$ $p_{B2}^{H1} = \frac{(1-q_{AB})q_{CB}(2q_{AB}+q_{CB})}{3(4q_{AB}(1+q_{CB})-4q_{AB}^2-q_{CB}^2)}$ $p_{C1}^{H1} = \frac{q_{CB}(q_{AB}(2+5q_{CB})-2q_{AB}^2-q_{CB}(2+3q_{CB}))}{3(4q_{AB}(1+q_{CB})-4q_{AB}^2-q_{CB}^2)}$
Profits	$\Pi_{A}^{H1} = \frac{12q_{AB}^4(3-q_{CB})-(1-q_{CB})q_{CB}^3-24q_{AB}^3(3+q_{CB}(2-q_{CB}))}{9(4q_{AB}(1+q_{CB})-4q_{AB}^2-q_{CB}^2)^2}+$ $-\frac{q_{AB}q_{CB}^2(23+q_{CB}(24+q_{CB}))-q_{AB}^2(36+60q_{CB}+35q_{CB}^2-11q_{CB}^3)}{9(4q_{AB}(1+q_{CB})-4q_{AB}^2-q_{CB}^2)^2}$ $\Pi_{B}^{H1} = \frac{q_{CB}(1-q_{AB})(1-q_{AB}+q_{CB})(2q_{AB}+q_{CB})^2}{9(4q_{AB}(1+q_{CB})-4q_{AB}^2-q_{CB}^2)^2}$ $\Pi_{C}^{H1} = \frac{q_{AB}q_{CB}(q_{AB}-q_{CB})(2-2q_{AB}+3q_{CB})^2}{9(4q_{AB}(1+q_{CB})-4q_{AB}^2-q_{CB}^2)^2}$

It can be noticed that all prices are positive and that consumers under $H1$ not only obtain the new, high-quality system AA at a lower quality-adjusted price than the pre-entry price for AB, but also pay for AB less than in an integrated monopoly in which a unique firm produces both a_1 and b_2.[20] We now proceed to the analysis of equilibria under compatibility.

3.3.3 Analysis of equilibria under compatibility

Given that one of our goals is to show that the decision of the quality leader to enter the market of the second component has the effect of allowing the entry of firm C also in the partial compatibility case, we will focus here on the case

[19]Here we actually focus on equilibrium prices in an interior optimum for all firms. In other words, we are not analyzing the set of corner solutions that might also generate this market configuration $H1$. In fact, it can be shown that, if a pure-strategy equilibrium exists, it always involves the three firms choosing an interior optimal price. The proof involves tedious algebra and is available upon request.

[20]In such case, in fact, the price for AB would be equal to $p_{AB}^{M} = \frac{q_{AB}}{2}$ and we can notice immediately that $(p_{A1}^{H1} + p_{B}^{H1}) - p_{AB}^{M} = \frac{q_{AB}q_{CB}(2-2q_{AB}+3q_{CB})}{2(4q_{AB}^2+q_{CB}^2-4q_{AB}(1+q_{CB}))} < 0$.

$q_{CB} < \hat{q}_{CB}$, condition that in the uniform case translates, as we know, to $q_{CB} \leq \frac{4}{9}q_{AB}$. This will allow us to compare the following results not only with the pure bundling case but also with the standard complementary monopoly. Using such parameter restriction, we can now state the main result of this Section.

Proposition 18. *Under partial compatibility;*

1. *If $q_{AB} \leq q^*(q_{CB})$, then there exists a unique pure strategy equilibrium at which all systems AA, AB and CB are sold.*

2. *If $q_{AB} \geq \frac{1+q_{CB}}{2}$, then there exists a unique pure strategy equilibrium at which only systems AA and CB are sold.*

3. *If $q^*(q_{CB}) < q_{AB} < \frac{1+q_{CB}}{2}$ then there is no pure-strategy equilibrium.*

   In particular, whenever a pure-strategy equilibrium exists, it involves firm C having positive demand and earning positive profits.

Proof: See Appendix.[21]

As for part 1 of this Proposition, the proof shows that neither A nor C ever want to move from market configuration $H1$ to configuration $H2$ when the quality of system AB, and then its price, is relatively low. In other words, none of them would unilaterally decrease the price of their components a_2 and c_1 so that both systems AA and CB become more attractive than system AB and D_{AB} becomes

[21]These results are qualitatively similar if generalized for any $q_{CB} < q_{AB}$. Again, depending on the range of the parameters, pure strategy equilibria may be either in $H1$ or $H2$ and there may be no pure-strategy equilibria. The area where $H1$ is an equilibrium is considerably enlarged. In particular, when $q_{CB} > \frac{4}{9}q_{AB}$, there exist two values $\tilde{q}(q_{CB})$ and $\bar{q}_{CB}$ in the interval $(\frac{4}{9}q_{AB}, q_{AB}]$ such that $\tilde{q}(q_{CB}) \leq q^*(q_{CB})$ for $q_{CB} \in [0, \bar{q}_{CB}]$ and $\tilde{q}(q_{CB}) \geq q^*(q_{CB})$ for $q_{CB} \in [\bar{q}_{CB}, q_{AB}]$. Then $H1$ is an equilibrium market configuration iff $q_{AB} < \max\{\tilde{q}_{AB}(q_{CB}), q^*_{AB}(q_{CB})\}$

null.[22] In fact, such decrease would need to be so large as to decrease their respective profits. Firm B could eliminate AB only increasing the price for b_2. Such increase is profitable only if the consequent reduction in the demand of system CB is small, so that the positive impact of an higher price prevails on B's profits. However, this happens only when q_{AB} is high enough ($q_{AB} > q^*(q_{CB})$). In such case, in fact, the quality of B's component is high too, so that the increase in price needed to eliminate D_{AB} is relatively high and the decrease in D_{CB} is relatively low.[23]

As for the second part of Proposition 18, the proof indicates that, starting from $H2$, A is not willing to deviate to $H1$ if q_{AB} is relatively high. In principle, AB could indeed be sold in the market if firm A decreased p_{A1}. Note, however, that when q_{AB} is relatively high, systems AA and AB are close substitutes and the contribution to the perceived quality of system AB brought by firm B (and its price p_{B2}, as well) is relatively high. Thus, A would have no incentive to implement such deviation because, given B's relatively high price, the decrease in p_{A1} would need to be so substantial to decrease $A's$ profits. Conversely, if q_{AB} is low, even a small reduction in p_{A1} creates a demand for AB and increases A's profits.

Finally, if $q^*(q_{CB}) < q_{AB} < \frac{1+q_{CB}}{2}$, when all firms set prices such that the market configuration is $H1$, firm B would find it profitable to deviate to $H2$. If all firms set prices such that the market configuration is $H2$, then it's firm

[22]Firm A might in principle find such strategy attractive even if it reduces the demand of a_1 used in combination with b_2 because this increases the demand of the higher-quality and higher-priced system AA. Note however that when q_{AB} is relatively low, consumers perceive systems AB and CB as close substitutes. Decreasing the prices of a_2 and c_1 would then gradually eliminate the demand for AB in favor of CB rather than AA, so that the negative effects of such decrease on firm A's profits would prevail.

[23]And this is actually what happens when $q_{AB} > q^*(q_{CB})$. Note also that, because q_{AB} is substantially higher than q_{CB}, AB and CB are not close substitutes. Thus, changes in the demand of AB do not significantly affect the demand of CB.

A that would find it convenient to deviate to $H1$. There is, therefore, space for mixed-strategy Nash equilibria. In any case, the conclusion is that the presence of a_2 generates instability in the game as both B and A become aggressive, B raising its price to gain back some of the profits lost because of a_2 and A lowering p_{A1} to gain market share.

Note that, as far as firm C is concerned, the equilibria defined by Proposition 18 are similar to those emerging in the "pure bundling" case. No matter the relative value of q_{AB} and q_{CB}, the entry of firm A in market 2 does not foreclose firm C, which is then able to get a positive demand and earn positive profits.[24]

As far as consumer surplus is concerned, a revealed-preference argument is enough to prove that it is always higher under partial compatibility than under the Cournot monopoly. As stated in proposition 18, the demand for c_1 is positive whenever a pure-strategy equilibrium exists under partial compatibility. In fact, the entry of firm A in the market for the second component allows some consumers, previously out of the market, to buy system CB at a lower price than the one at which system AB was available. Moreover, the price of system AB is now always lower than in a market without C, so that if consumers previously buying AB now shift towards CB or AA when AB is still available and cheaper, it's because they prefer either CB or AA to AB. Then, each single consumer is better off under partial compatibility than with C out of the market.[25] The following Proposition summarizes both this result and the ones obtained for producer and total surplus.[26]

[24] Even if this might happen in expected terms only, as in the mixed-strategy equilibria emerging for intermediate values of q_{AB}.

[25] This was not necessarily true when firm A chooses to produce a_2 under pure bundling, because in that case system AB is not available and some consumers might be forced to shift towards either AA or CB.

[26] Profit and welfare analysis in this and in the following Propositions focus on pure-strategy Nash equilibria. For a discussion of the issue of welfare analysis in mixed-strategy equilibria, see footnote 15.

Proposition 19. *Comparing partial compatibility with the Cournot monopoly case,*

1. *Consumer surplus is higher under partial compatibility.*

2. *Partial compatibility allows firm A to obtain higher profits than in a Cournot monopoly case. Firm B always loses, whereas firm C always gains.*

3. *Total welfare is higher under partial compatibility.*

Proof: See Appendix.

Not surprisingly, the effects on firms' profits are qualitatively similar to those illustrated for Proposition 18 for pure bundling. Producing the second component increases the profits of the quality leader at the expenses of firm B and, at the same time, allows firm C to obtain a positive market share. However, producer surplus can now be lower than in the complementary monopoly case, with the (greater) loss of firm B prevailing over the (smaller) gains of firms A and C. Overall, as in the pure bundling case, the three positive effects on consumer surplus and Firms A and C's profits dominate in any case over the loss of firm B, so that total surplus is higher than under a standard complementary monopoly.

3.4 Low-quality firms producing both components

In contrast with the analysis of section 3, this extension assumes that one of the lower-quality firms (either B or C) starts producing both components. The goal is to verify whether conclusions are qualitatively similar to those emerging with the quality leader selling both products and in particular whether firm C obtains positive demand and profits in this new setting, as well.

In case firm B enters the market of component 1, A would continue producing only a_1 and C only c_1. The number of systems that would be available to consumers depend again on whether firm B chooses to sell its components through "pure bundling" or it allows consumers to combine components from all firms. In any event, consistently with our previous assumptions, also in this setting we have $q_C < q_B$ and $q_C < q_A$, and we normalize the highest quality sold in the market to 1 ($q_{AB} = 1$), so that $q_{CC} < 1$.

Proposition 20. *If B produces both components, a pure-strategy equilibrium in which firm C has positive demand does not exist.*

Proof: See Appendix.

Thus, contrarily to the previous case, the production of b_1 does not allow firm C to obtain positive demand. This should not be surprising. In this new setting, B is still a monopolist for the second component and b_1 has only two effects: 1) it makes competition for component 1 fiercer; 2) it allows the (quality-adjusted) price of system BB to be lower than the price for CB. In fact, the simultaneous setting of p_{B1} and p_{B2} allows Firm B to eliminate the Cournot complementary monopoly problem. Both effects would make it more difficult for firm C to gain positive demand even in comparison with the setting of Proposition 15, so that it continues being effectively foreclosed.

When it's firm C that produces both components c_1 and c_2, we distinguish again the case in which C chooses pure bundling from the case in which it maintains technical compatibility between c_1 and b_2.

Under pure bundling, consumers can choose between systems AB and CC. Firm C sets a unique price p_C for its system CC and the only two possible candidates for an equilibrium market configuration are: $L1$) both systems AB

and CC are sold; $L2$) only AB is sold.[27]

A consumer buys the high-quality system AB if $\theta_{CC}^{AB} \leq \theta < 1$, where $\theta_{CC}^{AB} = \frac{p_{A1}+p_{B2}-p_C}{1-q_{CC}}$ is the consumer indifferent between buying AB and CC. Similarly, a consumer buys CC if $\theta_{CC} \leq \theta < \theta_{CC}^{AB}$, where $\theta_{CC} = \frac{p_C}{q_{CC}}$ is the consumer indifferent between buying CC and buying nothing. Demand functions for the two systems are $D_{AB} = 1 - \frac{p_{A1}+p_{B2}-p_C}{1-q_{CC}}$ and $D_{CC} = \frac{p_{A1}+p_{B2}-p_C}{1-q_{CC}} - \frac{p_C}{q_{CC}}$. Profits are $\Pi_A = p_{A1}\left(1 - \frac{p_{A1}+p_{B2}-p_C}{1-q_{CC}}\right)$, $\Pi_B = p_{B2}\left(1 - \frac{p_{A1}+p_{B2}-p_C}{1-q_{CC}}\right)$ and $\Pi_C = p_C\left(\frac{p_{A1}+p_{B2}-p_C}{1-q_{CC}} - \frac{p_C}{q_{CC}}\right)$, respectively. Bertrand equilibrium prices are

$$p_{A1}^{L1} = p_{B2}^{L1} = \frac{1-q_{CC}}{3-q_{CC}} \tag{3.3}$$

$$p_C^{L1} = \frac{q_{CC}(1-q_{CC})}{3-q_{CC}} \tag{3.4}$$

Equilibrium profits are $\Pi_A^{L1} = \Pi_B^{L1} = \frac{1-q_{CC}}{(3-q_{CC})^2}$ and $\Pi_C^{L1} = \frac{q_{CC}(1-q_{CC})}{(3-q_{CC})^2}$.

Market configuration $L2$ is instead emerging when $\theta_{AB} < \theta_{CC}$ and would coincide with the equilibrium in the standard double-marginalization case illustrated in section 2 so that $p_{A1}^{L2} = p_{B2}^{L2} = \frac{1}{3}$ and $\Pi_A^{L2} = \Pi_B^{L2} = \frac{1}{9}$ respectively.

We can now prove the following result.

Proposition 21. *When the low-quality firm produces both components and sells them as a bundle, there is a unique equilibrium in which both systems AB and CC are sold in positive quantities. Thus, producing the second component in addition to the first allows firm C to stay in the market earning positive profits.*

Proof: See Appendix.

Thus, pure bundling allows firm C to avoid foreclosure in the original market.

[27] A parallel argument to that used in analyzing Figures 3.1, 3.2 and 3.3 in section 3.2 excludes a market configuration with CC being sold in positive quantity with system AB having instead null demand.

When c_2 is instead offered under partial compatibility, consumers can buy CC, AB, or they can mix and match and buy CB. Also, given the assumptions stated above on q_i, $i = A, B, C$, the perceived qualities of the three systems are ordered as follows: $1 = q_{AB} > q_{CB} > q_{CC}$.[28] As in section 3.2 there are four possible candidates for an equilibrium market configuration; $L1$) systems AB and CC are sold; $L2$) only system AB is sold; $L3$) systems AB and CB are sold; $L4$) all three systems are sold. However the following Proposition simplifies the analysis substantially.[29]

Proposition 22. *If a pure-strategy equilibrium exists when the low-quality firm produces both components under partial compatibility, then it must be under market configuration* $L1$.

Proof: See Appendix.

The main implication of such a result is very similar to the one indicated for Proposition 18: if a pure strategy equilibrium exists, it involves the presence of firm C.[30] Producing both components of the system allows the low-quality duopolist to obtain a positive demand and thereby positive profits. In this case, thanks to the production of c_2, firm C is not forced to sell c_1 only in combination with b_2 and this relaxes the competition with firm A when selling the first component. Moreover, the production of c_2 eliminates the Cournot complementary monopoly problem for the low quality system, so that the quality-adjusted price for CC is lower than the one for CB. Qualitatively speaking, the only difference

[28] Moreover, in order to make results comparable to Proposition 15 applied to the uniform case, we keep the assumption $q_{CB} \leq \frac{4}{9} q_{AB} = \frac{4}{9}$.

[29] When the low-quality firm sells both components, none of such four candidates can be *a priori* excluded through pure logical arguments, as it was the case of $H3$ and $H4$.

[30] And a pure-strategy equilibrium in $L1$ indeed exists and is unique when q_{CC} is sufficiently low. The proof is available upon request.

with the results in Proposition 18 is that an equilibrium in which system CB is purchased in positive amounts never exists. As shown in the Appendix, this is mainly due to the different behavior of firm B. When the quality leader produces both components, in equilibrium, firm B might set p_{B2} so that AB has positive demand. It is true that the quality-adjusted price of such system is higher than the one for system AA, but it might be profitable for firm B to price its good so that the lower tail of the demand buys both AB and CB. Particularly, when q_{AB} is relatively low, so that systems AA and AB are not close substitutes, firm B would never increase p_{B2} to eliminate AB. On the contrary, when firm C sells both components, firm B always prefers to be coupled with the quality leader and would never find profitable to decrease p_{B2} to attract customers for CB. Differently from the first case, firm B now has access to the top portion of the demand and the gains that can be extracted through high prices ouweight the loss from not being matched with c_1.

3.5 The choice between pure bundling and partial compatibility

It happens quite often that a firm in one layer extends its product boundary to include functions traditionally provided by its complementors. In the previous sections we showed that such decision might also have a pro-competitive impact when it allows an (imperfect) substitute good to be sold on the market (in our case component c_1). Now we want to compare the impact of the production of a new component (be that either a_2 or c_2) on firms' prices and profits and on consumer surplus in both cases of pure bundling and partial compatibility. Particularly, we check whether these results change with the degree of substitutability across systems, that is with the values of q_{AB} and q_{CB}.

Most of the previous literature on "mix and match" with imperfectly substitute goods does not include a welfare analysis (Denicolò 2000 and Einhorn 1992). All results are obtained assuming a covered market configuration (that is, a market in which all consumers in equilibrium buy one of the available systems) so that choosing either pure bundling or partial compatibility has only distributive effects on a market with fixed demand. In other words, no choice of firm A or firm C would have a clear impact on consumers' surplus: some consumers will be better off, others worse off. In our case, market configurations are always "uncovered", i.e., the size of the demand is not fixed; then it becomes possible to compare the welfare consequences of different selling strategies.

When firm A starts producing a_2 but chooses to render its component a_1 compatible with b_2, results are qualitatively similar to those obtained under pure bundling, with a weaker positive impact on firm A's profits and a stronger positive impact on consumer surplus, as indicated in the following Proposition

Proposition 23. *Under partial compatibility;*

1. *Consumer surplus is never lower and the prices of systems AA and CB are never higher than under pure bundling.*

2. *Pure bundling never yields lower profits to firm A than partial compatibility.*

3. *Producer surplus is always higher under pure bundling.*

4. *Partial compatibility yields higher total surplus than pure bundling.*

Proof: See Appendix.

A revealed-preference argument is actually sufficient to prove that consumers weakly prefer partial compatibility to pure bundling and are actually strictly better off under partial compatibility when three systems are sold in equilibrium. First, some consumers who would be out of the market under pure bundling buy CB (whose price has decreased) under partial compatibility. Second, some consumers buying CB under pure bundling have now access to the same combination of goods at a lower price but choose to purchase AB, so that, by revealed preference, they are better off. Finally, there is a third portion of consumers buying AA under pure bundling but AB under partial compatibility even when system AA is available at a lower price. As a result each single consumer is actually never worse off under partial compatibility than in pure bundling.

Partial compatibility is also proven to yield higher total welfare than pure bundling. From a policy perspective, therefore, partial compatibility should then be encouraged.

3.5.1 Firm C produces both complements

Propositions 17 and 19 indicate that allowing the production of a_2 always increases consumer welfare. One might think that this strong result is uniquely justified by the high quality of such new product. In fact, when the average quality of the products increases, consumers should indeed be better off. We believe that our result actually indicates something different and possibly more general: no matter the average quality of the products, what benefits consumers is the increased variety of systems available due to the entry of new firms. This idea is confirmed by the welfare analysis conducted in the case in which firm C, the lower-quality duopolist, enters the market producing both components, as indicated in the following Proposition and Corollary.

Proposition 24. *1. Producing c_2 increases C's profits (from zero) but decreases profits for A and B. 2. Aggregate industry profits increase iff $q_{CC} < \frac{3}{11}$. 3.*

The entry of firm C into the market of the second component always increases consumer welfare.

Proof. Part 1 follows directly from observation that $\Pi_C^{L1} > 0$ and $\Pi_A^{L1} = \Pi_B^{L1} < \Pi_A^C = \Pi_B^C = \frac{1}{9}$. Part 2 follows from checking that $\Pi_A^{L1} + \Pi_B^{L1} + \Pi_C^{L1} - \frac{2}{9} = \frac{q_{CC}(3-11q_{CC})}{9(3-q_{CC})^2}$, which is positive iff $q_{CC} < \frac{3}{11}$. As for part 3, a revealed- preference argument similar to the one in Proposition 19 could be also used here as proof. In any case, note that $CS^L = \int_{\frac{p_C}{q_{CC}}}^{\frac{p_{A1}+p_B-p_C}{1-q_{CC}}} [\theta q_{CC} - p_C] d\theta + \int_{\frac{p_{A1}+p_B-p_C}{1-q_{CC}}}^{1} [\theta - p_{A1} - p_B] d\theta = \frac{1+3q_{CC}}{2(3-q_{CC})^2}$, which is always greater than consumer surplus in market configuration AB, that is $CS^C = \frac{1}{18}$. □

Corollary 4. *If $q_{CC} \geq \frac{3}{11}$, firms A and B may be able to bribe firm C to stay out of the market.*[31]

When comparing pure bundling and partial compatibility for firm C we first notice, using Proposition 23 and 24, that equilibrium market configurations, profits and consumer surplus are all the same. In other terms, firm C always gains from producing both components, no matter how it sells them, and this at the benefit of consumers. Also the impact on total surplus is independent on the selling strategy. When $q_{CC} < \frac{3}{11}$, both consumer and producer surplus increase because of the introduction of c_2, so that total surplus clearly increases. Moreover, using the same methodology adopted in the proof for Proposition 19, it can be shown that this result actually holds for any value of q_{CC}, so that the possible losses in producer surplus caused by c_2 (due the the lower profits for both firms A and B) are always more than counterbalanced by the increase in producer surplus. In conclusion, total welfare is always higher when C produces

[31] In fact, the gain in profits from switching to a market configuration inwhich only system AB is sold is $\left[\frac{1}{9} - \Pi_j^{L1}\right] = \frac{q_{CC}(3+q_{CC})}{9(3-q_{CC})^2}$, for firm $j = A, B$. If the aggregate gain is greater than or equal to firm C's profits from entering the market for both components, A and B will be able to bribe C to stay out of the market. And $2 \times \left[\frac{1}{9} - \Pi_j^{L}\right] \geq \Pi_C^{L1}$ iff $q_{CC} \geq \frac{3}{11}$.

both components and this matches qualitatively what previously obtained with the introduction of a_2.

3.6 Conclusions

In this chapter we have proven that, when the quality leader starts producing both components all pure-strategy Nash equilibria entail the presence of C, the low quality producer of component 1, which in the absence of a_2 would be out of the market. In general, while entering into the complement market lowers the ability of firm A to foreclose the entry of other (lower-quality) competitors in the original market, it remains a profit-enhancing strategy in any form it might be implemented, "pure bundling" or "partial compatibility". Also, such strategy, maybe not surprisingly (as explained in section 3) unambiguously increases consumer welfare. Such increase with respect to the standard double-marginalization case is actually greater under partial compatibility than in pure bundling, so that it might be reasonable from an antitrust perspective to require components to be compatible when observing a firm trying to sell both products of a system. Note however that such strategy does not eliminate the possibility of cycles in competition as in Casadesus-Masanell et al. (2007). In fact, when the quality produced by B is "intermediate" the presence of a_2 causes instability in the market and the absence of pure strategy Bertrand-Nash equilibria.

It has also been interesting to verify that when the option to sell both components is assigned to the low-quality producer of component 1, results have a similar flavor. In fact, if a pure-strategy Nash equilibrium exists, it always entails the presence of firm C in the market with a positive demand for both components c_1 and c_2. Again, consumer welfare increases with respect to the benchmark model, even if now aggregate producer surplus might decrease because of the losses imposed on firms A and B's profits by such entry[32]. As a result, total surplus might decrease and an antitrust intervention preventing the entry of a low-quality competitor producing all components of a system when a high-quality version of it is currently produced by two independent firms might again look reasonable. There is one important qualitative difference with respect to the first equilibrium, and it is related to the fact that $q_C < q_A$ so that, *ceteris paribus*, C is a "weaker competitor" than A. In equilibrium, all systems AB, CB and CC are never sold concurrently. In contrast to firm C in the first case, now firm A can in fact profitably adopt a pricing strategy that prevents bundle CB from being sold.

These conclusions might also represent a new contribution to the recent debate about the break-up of firms and the approval of mergers that recently developed both in the literature and on the part of antitrust agencies in the United States and the European Union. As opposed to Nalebuff (2004), in fact, our results do not involve a necessary trade-off between pure bundling and the creation of barriers to entry. Allowing one firm to produce both components always might in fact have pro-competitive effects.

An interesting extension of our model would be to devise a setting in which both duopolists end up producing both components and possibly allow for full mutual compatibility across complements. In that case, not only may compatibility entail higher prices than pure bundling (as in Einhorn, 1992 and Denicolò 2000), with negative effects on welfare, but there may also be lower consumer surplus than in the standard Cournot complementary monopoly scenario.

[32]When firm A produces both components, firm B is harmed but firm C actually benefits, since it re-enters the market and obtains positive profits. As a result, total surplus increases.

Appendix

Proof of Proposition 15

Both systems AB and CB have positive demand iff $0 \leq \theta_{CB} < \theta_{CB}^{AB} < 1$, where θ_{CB} and θ_{CB}^{AB} have been defined in Section 2 above. Consumers with $\theta \geq \theta_{CB}^{AB}$ buy system AB, those with $\theta_{CB} \leq \theta < \theta_{CB}^{AB}$ buy CB and the remaining buy nothing. If $\theta_{CB} > \theta_{CB}^{AB}$, then only system AB is sold and consumers with $\theta \geq \theta_{AB}$ buy it (where $\theta_{AB} = \frac{p_{A1}+p_{B2}}{q_{AB}}$ is the consumer indifferent between buying AB and nothing), whereas the remaining buy nothing.

Firms A, B and C can price their goods so that AB and CB are both sold, that is if $\frac{p_{B2}+p_{C1}}{q_{CB}} < \frac{p_{A1}-p_{C1}}{q_{AB}-q_{CB}}$. Demands are $D_{AB} = 1 - G(\theta_{CB}^{AB}) = F(\theta_{CB}^{AB})$ and $D_{CB} = G(\theta_{CB}^{AB}) - G(\theta_{CB}) = F(\theta_{CB}) - F(\theta_{CB}^{AB})$. If instead firms A and B price their goods such that $\theta_{CB} > \theta_{CB}^{AB}$ (i.e., if $\frac{p_{B2}+p_{C1}}{q_{CB}} > \frac{p_{A1}-p_{C1}}{q_{AB}-q_{CB}}$) then AB only is sold and $\widetilde{D}_{AB} = 1 - G(\theta_{AB}) = F(\theta_{AB})$. In this second case, we assume that C charges a price equal to marginal cost, that is, $p_{C1} = 0$. Define $R_A^{ABC}(p_{B2}, p_{C1})$, $R_B^{ABC}(p_{A1}, p_{C1})$ and $R_C^{ABC}(p_{A1}, p_{B2})$ the best-response functions when both AB and CB are sold. $R_A^{AB}(p_{B2}, 0)$ and $R_B^{AB}(p_{A1}, 0)$ are their counterparts in the AB case. Define also p_{A1}^{ABC}, p_{B2}^{ABC}, p_{C1}^{ABC} p_{A1}^{AB} and p_{B2}^{AB} the optimal prices when AB and CB are sold (label ABC) and when AB only is sold (label AB), respectively.

We can now proceed to characterize the best-response functions.

Best Response Functions

Firm C. Firm C is active only when both AB and CB are sold, i.e., when $p_{C1} \leq \frac{q_{CB}p_{A1}-(q_{AB}-q_{CB})p_{B2}}{q_{AB}}$. When both systems are sold, profit for C is $\Pi_C^{ABC} = p_{C1}\left[F(\theta_{CB}) - F(\theta_{CB}^{AB})\right]$. The first-order condition for C is

$$\frac{\partial \Pi_C^{ABC}}{\partial p_{C1}} = \left[F(\theta_{CB}) - F(\theta_{CB}^{AB})\right] + p_{C1}\left[\frac{f(\theta_{CB})}{q_{CB}} - \frac{f(\theta_{CB}^{AB})}{q_{AB} - q_{CB}}\right] \tag{3.5}$$

Evaluating condition (3.5) at the kink point $p_{C1} = \frac{q_{CB}p_{A1} - (q_{AB} - q_{CB})p_{B2}}{q_{AB}}$, it becomes

$$\frac{\partial \Pi_C^{ABC}}{\partial p_{C1}} = \frac{q_{CB}p_{A1} - (q_{AB} - q_{CB})p_{B2}}{q_{AB}}\left[\frac{f(\theta_{CB})}{q_{CB}\,(q_{AB} - q_{CB})}\right] \tag{3.6}$$

Given $p_{C1} \leq \frac{q_{CB}p_{A1} - (q_{AB} - q_{CB})p_{B2}}{q_{AB}}$, then the first term of the product in (3.6) and the overall product is negative, since $f(\theta) < 0$. In other terms, $\frac{\partial \Pi_C^{ABC}}{\partial p_{C1}} < 0$ at the kink. Then $R_C^{ABC}(p_{A1}, p_{B2}) < \frac{q_{CB}p_{A1} - (q_{AB} - q_{CB})p_{B2}}{q_{AB}}$ and $p_{C1} = R_C^{ABC}(p_{A1}, p_{B2})$. If $p_{A1} < \frac{(q_{AB} - q_{CB})p_{B2} + q_{AB}\,p_{C1}}{q_{CB}}$, $R_C^{ABC}(p_{A1}, p_{B2})$ is not defined and $p_{C1} = 0$.

Firm B. This case is more interesting, because B's function is discontinuous. If $p_{B2} \leq \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$, then B maximizes $\Pi_B^{ABC} = p_{B2}F(\theta_{CB}) = p_{B2}F\left(\frac{p_{B2} + p_{C1}}{q_{CB}}\right)$. If $p_{B2} > \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$, B maximizes $\Pi_B^{AB} = p_{B2}F(\theta_{AB}) = p_{B2}F\left(\frac{p_{A1} + p_{B2}}{q_{AB}}\right)$.

The first order conditions of these maximization problems are

$$\frac{\partial \Pi_B^{ABC}}{\partial p_{B2}} = F\left(\frac{p_{B2} + p_{C1}}{q_{CB}}\right) + \frac{p_{B2}}{q_{CB}}f\left(\frac{p_{B2} + p_{C1}}{q_{CB}}\right) = 0 \tag{3.7}$$

$$\frac{\partial \Pi_B^{AB}}{\partial p_{B2}} = F\left(\frac{p_{A1} + p_{B2}}{q_{AB}}\right) + \frac{p_{B2}}{q_{AB}}f\left(\frac{p_{A1} + p_{B2}}{q_{AB}}\right) = 0 \tag{3.8}$$

Evaluated at the kink $p_{B2} = \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$, the conditions above become

$$\frac{\partial \Pi_B^{ABC}}{\partial p_{B2}} = \frac{F\left(\frac{p_{A1} - p_{C1}}{q_{AB} - q_{CB}}\right)}{f\left(\frac{p_{A1} - p_{C1}}{q_{AB} - q_{CB}}\right)} + \frac{p_{A1}\,q_{CB} - p_{C1}\,q_{AB}}{q_{CB}\,(q_{AB} - q_{CB})} \tag{3.9}$$

$$\frac{\partial \Pi_B^{AB}}{\partial p_{B2}} = \frac{F\left(\frac{p_{A1} - p_{C1}}{q_{AB} - q_{CB}}\right)}{f\left(\frac{p_{A1} - p_{C1}}{q_{AB} - q_{CB}}\right)} + \frac{p_{A1}\,q_{CB} - p_{C1}\,q_{AB}}{q_{AB}\,(q_{AB} - q_{CB})} \tag{3.10}$$

Because $\frac{F(\theta)}{f(\theta)}$ is increasing in θ for the increasing hazard-rate condition, equations (3.9) and (3.10) are monotonically increasing in p_{A1} for any given $p_{C1} \geq 0$. Thus, given q_{AB} and q_{CB}, there exist $\underline{p}_{A1}(p_{C1})$ and $\bar{p}_{A1}(p_{C1})$ that solve (3.9) and (3.10) respectively, $\underline{p}_{A1}(p_{C1}) < \bar{p}_{A1}(p_{C1})$. We can distinguish three cases, as follows;

B1. if $p_{A1} \leq \underline{p}_{A1}(p_{C1})$, $\frac{\partial \Pi_B^{ABC}}{\partial p_{B2}} \geq 0$ and $\frac{\partial \Pi_B^{AB}}{\partial p_{B2}} > 0$ when evaluated at $p_{B2} = \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$. Hence $R_B^{ABC} \geq \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$ and R_B^{ABC} violates the constraint $p_{B2} \leq \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$, whereas $R_B^{AB} > \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$ does not violate the constraint $p_{B2} > \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$. Because $\Pi_B^{ABC}\left(\frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}\right) = \Pi_B^{C}\left(\frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}\right)$, the overall profit function has a single peak at R_B^{AB} and firm B's global optimum price is given by R_B^{AB}.

B2. if $p_{A1} \geq \bar{p}_{A1}(p_{C1})$, $\frac{\partial \Pi_B^{ABC}}{\partial p_{B2}} < 0$ and $\frac{\partial \Pi_B^{AB}}{\partial p_{B2}} \leq 0$ evaluated at $p_{B2} = \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$. Then $R_B^{ABC} < \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$ and $R_B^{AB} \leq \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$. Hence R_B^{ABC} does not violate the constraint, whereas R_B^{AB} does. B's global optimum price is R_B^{ABC}.

B3. if $\underline{p}_{A1}(p_{C1}) < p_{A1} < \bar{p}_{A1}(p_{C1})$, then, evaluated at $p_{B2} = \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$, $\frac{\partial \Pi_B^{ABC}}{\partial p_{B2}} < 0$ and $\frac{\partial \Pi_B^{AB}}{\partial p_{B2}} > 0$, so that $R_B^{ABC} < \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$ and $R_B^{AB} > \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}}$. Neither R_B^{ABC} nor R_B^{AB} violate the respective constraints and B's overall profit function has two peaks, one at R_B^{ABC} and the other at R_B^{AB}, where $R_B^{ABC} < \frac{q_{CB}\,p_{A1} - q_{AB}\,p_{C1}}{q_{AB} - q_{CB}} < R_B^{AB}$. B's best-response function is therefore discontinuous in p_{B2}.

Firm A. If $p_{A1} \geq \frac{(q_{AB} - q_{CB})\,p_{B2} + q_{AB}\,p_{C1}}{q_{CB}}$, then AB and CB are sold and firm A maximizes $\Pi_A^{ABC} = p_{A1} F(\theta_{CB}^{AB}) = p_{A1}\,F(\frac{p_{A1} - p_{C1}}{q_{AB} - q_{CB}})$. If $p_{A1} < \frac{(q_{AB} - q_{CB})\,p_{B2} + q_{AB}\,p_{C1}}{q_{CB}}$, then AB only is sold and firm A maximizes $\Pi_A^{AB} = p_{A1} F(\theta_{AB}) = p_{A1} F(\frac{p_{A1} + p_{B2}}{q_{AB}})$.

The first order conditions of these maximization problems are

$$\frac{\partial \Pi_A^{ABC}}{\partial p_{A1}} = F\left(\frac{p_{A1}-p_{C1}}{q_{AB}-q_{CB}}\right) + \frac{p_{A1}}{q_{AB}-q_{CB}} f\left(\frac{p_{A1}-p_{C1}}{q_{AB}-q_{CB}}\right) = 0 \tag{3.11}$$

$$\frac{\partial \Pi_A^{AB}}{\partial p_{A1}} = F\left(\frac{p_{A1}+p_{B2}}{q_{AB}}\right) + \frac{p_{A1}}{q_{AB}} f\left(\frac{p_{A1}+p_{B2}}{q_{AB}}\right) = 0 \tag{3.12}$$

Evaluated at $p_{A1} = \frac{(q_{AB}-q_{CB})\,p_{B2}+q_{AB}\,p_{C1}}{q_{CB}}$, the conditions above become

$$\frac{\partial \Pi_A^{ABC}}{\partial p_{A1}} = \frac{F\left(\frac{p_{B2}+p_{C1}}{q_{CB}}\right)}{f\left(\frac{p_{B2}+p_{C1}}{q_{CB}}\right)} + \frac{(q_{AB}-q_{CB})p_{B2}+q_{AB}\,p_{C1}}{q_{CB}(q_{AB}-q_{CB})} \tag{3.13}$$

$$\frac{\partial \Pi_A^{AB}}{\partial p_{A1}} = \frac{F\left(\frac{p_{B2}+p_{C1}}{q_{CB}}\right)}{f\left(\frac{p_{B2}+p_{C1}}{q_{CB}}\right)} + \frac{(q_{AB}-q_{CB})p_{B2}+q_{AB}\,p_{C1}}{q_{CB}\,q_{AB}} \tag{3.14}$$

For any given $p_{C1} \geq 0$, equations (3.13) and (3.14) are monotonically increasing in p_{B2}, so there exist $\underline{p}_{B2}(p_{C1}$ and $\bar{p}_{B2}(p_{C1})$ that solve (3.13) and (3.14) respectively, $\underline{p}_{B2}(p_{C1}) < \bar{p}_{B2}(p_{C1})$. Again, we can distinguish three cases;

A1. if $p_{B2} \leq \underline{p}_{B2}(p_{C1})$, $\frac{\partial \Pi_A^{ABC}}{\partial p_{A1}} \geq 0$ and $\frac{\partial \Pi_A^{AB}}{\partial p_{A1}} > 0$ when $p_{A1} = \frac{(q_{AB}-q_{CB})\,p_{B2}+q_{AB}\,p_{C1}}{q_{CB}}$. Thus $R_A^{ABC} \geq \frac{(q_{AB}-q_{CB})\,p_{B2}+q_{AB}\,p_{C1}}{q_{CB}}$ and $R_A^{AB} > \frac{(q_{AB}-q_{CB})\,p_{B2}+q_{AB}\,p_{C1}}{q_{CB}}$. This implies R_A^{AB} violates the constraint $p_{A1} < \frac{(q_{AB}-q_{CB})\,p_{B2}+q_{AB}\,p_{C1}}{q_{CB}}$, whereas R_A^{ABC} does not violate the constraint $p_{A1} > \frac{(q_{AB}-q_{CB})\,p_{B2}+q_{AB}\,p_{C1}}{q_{CB}}$. Then firm's A global optimum price is R_A^{ABC}.

A2. if $p_{B2} \geq \bar{p}_{B2}(p_{C1})$, $\frac{\partial \Pi_A^{ABC}}{\partial p_{A1}} < 0$ and $\frac{\partial \Pi_A^{AB}}{\partial p_{A1}} \leq 0$ when $p_{A1} = \frac{(q_{AB}-q_{CB})\,p_{B2}+q_{AB}\,p_{C1}}{q_{CB}}$. Thus $R_A^{ABC} < \frac{(q_{AB}-q_{CB})\,p_{B2}+q_{AB}\,p_{C1}}{q_{CB}}$ and $R_A^{AB} \leq \frac{(q_{AB}-q_{CB})\,p_{B2}+q_{AB}\,p_{C1}}{q_{CB}}$ and R_A^{AB} only does not violate the constraint. Then firm's A global optimum price is R_A^{AB}.

A3. if $\underline{p}_{B2}(p_{C1}) < p_{B2} < \bar{p}_{B2}(p_{C1})$, then, $\frac{\partial \Pi_A^{ABC}}{\partial p_{A1}} < 0$ and $\frac{\partial \Pi_A^{AB}}{\partial p_{A1}} > 0$ evaluated at $p_{A1} = \frac{(q_{AB}-q_{CB})p_{B2}+q_{AB}p_{C1}}{q_{CB}}$. Here $R_A^{ABC} < \frac{(q_{AB}-q_{CB})p_{B2}+q_{AB}p_{C1}}{q_{CB}}$ and $R_A^{AB} > \frac{(q_{AB}-q_{CB})p_{B2}+q_{AB}p_{C1}}{q_{CB}}$ and both best-response prices violate the respective constraints. A corner solution would prevail, such that $R_A^{ABC} = R_A^{AB} = \frac{(q_{AB}-q_{CB})p_{B2}+q_{AB}p_{C1}}{q_{CB}}$.

Analysis of Nash Equilibria

From the preceding discussion, we know that the two candidate equilibrium price sets are $\{p_{A1}^{ABC}, p_{B2}^{ABC}, p_{C1}^{ABC}\}$ and $\{p_{A1}^{AB}, p_{B2}^{AB}, 0\}$.

Conditions under which $\{p_{A1}^{AB}, p_{B2}^{AB}, 0\}$ is a Nash equilibrium. A necessary condition for these prices to be a Nash equilibrium is $\theta_{CB} > \theta_{CB}^{AB}$ or, given $p_{C1} = 0$, $\frac{p_{B2}^{AB}}{q_{CB}} > \frac{p_{A1}^{AB}}{q_{AB}-q_{CB}}$, i.e.,

$$q_{CB} < \frac{q_{AB}\, p_{B2}^{AB}}{p_{A1}^{AB} + p_{B2}^{AB}}. \tag{3.15}$$

Let us check whether firms have any incentive to deviate from these prices when condition (3.15) holds.

Firm C could obtain a positive demand for its component only through a reduction of its price, given that D_{CB} is decreasing in p_{C1} for given p_{A1} and p_{B2}. Being $p_{C1} = 0$ (i.e., equal to marginal cost) already, there is no way that C can profitably deviate from it and enter the market.

When firm B sets its price at p_{B2}^{AB} and $p_{C1} = 0$, firm A maximizes Π_A^{AB} if $p_{A1} < \frac{q_{AB}-q_{CB}}{q_{CB}} p_{B2}$ and Π_A^{ABC} if $p_{A1} > \frac{q_{AB}-q_{CB}}{q_{CB}} p_{B2}$. As shown above, firm A's best response function is continuous and, given p_{B2}, has a unique maximum. Therefore, if p_{A1}^{AB} maximizes Π_A^{AB} without violating the constraint $p_{A1} \leq \frac{q_{AB}-q_{CB}}{q_{CB}} p_{B2}^{AB}$, then it is firm A's globally optimal price. When inequality (3.15) holds, we have that $p_{A1}^{AB} \leq \frac{q_{AB}-q_{CB}}{q_{CB}} p_{B2}^{AB}$, so that p_{A1}^{AB} maximizes Π_A^{AB} without violating the constraint and is A's globally optimal price when $p_{B2} = p_{B2}^{AB}$ and $p_{C1} = 0$. Hence, firm A does not deviate.

When firm A sets $p_{A1} = p_{A1}^{AB}$, firm B maximizes $\Pi_B^{ABC} = p_{B2} F\left(\frac{p_{B2}}{q_{CB}}\right)$ if $p_{B2} \leq \frac{p_{A1}\, q_{CB}}{q_{AB}-q_{CB}}$ and $\Pi_B^{AB} = p_{B2} F\left(\frac{p_{A1}^{C}+p_{B2}}{q_{AB}}\right)$ if $p_{B2} > \frac{p_{A1}\, q_{CB}}{q_{AB}-q_{CB}}$. Let $\Sigma_1^{ABC}(q_{CB})$ and Σ_2^{AB} represent the maximum values of the profit functions given the respective constraints. Because q_{CB} satisfies inequality (3.15), $\Sigma_2^{AB} = \Pi_B^{AB} = p_{B2}^{AB} F\left(\frac{p_{A1}^{AB}+p_{B2}^{AB}}{p_{B2}^{AB}}\right)$, which does not depend on q_{CB}.[33] Let us define $\Sigma(q_{CB}) = \Sigma_1^{ABC}(q_{CB}) - \Sigma_2^{AB}$. The function $\Sigma(q_{CB})$ is continuous and monotonically increasing in q_{CB}, since $\Sigma_1^{ABC}(q_{CB})$ is monotonically increasing in q_{CB} and Σ_2^{AB} is invariant. Given condition (3.15), $q_{CB} \in [0, \frac{q_{AB}\, p_{B2}^{AB}}{p_{A1}^{AB}+p_{B2}^{AB}}]$. We then study the sign of $\Sigma(q_{CB})$ at the extremes of such interval. It is immediate to check that, if $q_{CB} = 0$, $\Sigma_1^{ABC}(0) = 0$, and $\Sigma(q_{CB}) < 0$. At $q_{CB} = \frac{q_{AB}\, p_{B2}^{AB}}{p_{A1}^{AB}+p_{B2}^{AB}}$, we have $p_{B2}^{AB} = \frac{q_{CB}\, p_{A1}^{AB}}{q_{AB}-q_{CB}}$. Thus, Π_A^{AB} is optimized at the kink $p_{B2} = \frac{q_{CB}\, p_{A1}^{AB}}{q_{AB}-q_{CB}}$. As implied by point B2 above, when $\frac{\partial \Pi_B^{AB}}{\partial p_{B2}} = 0$ at the kink, then $\frac{\partial \Pi_B^{ABC}}{\partial p_{B2}} < 0$. Thus, $\Sigma_1^{ABC}(q_{CB}) > \Sigma_2^{AB}$ and $\Sigma(q_{CB}) > 0$. Because $\Sigma(q_{CB})$ is continuous and monotonically increasing in q_{CB}, there exists a unique $0 < \hat{q}_{CB} < \frac{q_{AB}\, p_{B2}^{C}}{p_{A1}^{C}+p_{B2}^{C}}$, such that firm B is not willing to deviate to p_{B2}^{ABC} for $q_{CB} \leq \hat{q}_{CB}$.

In summary, for $q_{CB} \leq \hat{q}_{CB}$ neither firm is willing to deviate from $\{p_{A1}^{AB}, p_{B2}^{AB}, 0\}$. These prices constitute a Nash equilibrium, system AB only is sold and C is out of the market.

Proof that $\{p_{A1}^{ABC}, p_{B2}^{ABC}, p_{C1}^{ABC}\}$ is not a Nash equilibrium. A necessary condition for these prices to be a Nash equilibrium is $\theta_{CB} < \theta_{CB}^{AB}$ or $\frac{p_{B2}^{ABC}+p_{C1}^{ABC}}{q_{CB}} <$

[33] In fact, neither first-order condition $\frac{\partial \Pi_A^{AB}}{\partial p_{A1}} = 0$ not $\frac{\partial \Pi_B^{AB}}{\partial p_{B2}} = 0$ depend on q_{CB}, so that p_{A1}^{AB} and p_{B2}^{AB} are independent of q_{CB}.

$\frac{p_{A1}^{ABC}-p_{C1}^{ABC}}{q_{AB}-q_{CB}}$, i.e.,

$$q_{CB} > \frac{q_{AB}\,(p_{B2}^{ABC}+p_{C1}^{ABC})}{p_{A1}^{ABC}+p_{B2}^{ABC}}. \tag{3.16}$$

In order to prove that the prices $\{p_{A1}^{ABC}, p_{B2}^{ABC}, p_{C1}^{ABC}\}$ do not constitute a Nash equilibrium, it suffices to prove that firm B is always willing to deviate. Denote $\tau(q_{CB}) = \tau_1^{ABC}(q_{CB}) - \tau_2^{AB}(q_{CB})$, where $\tau_1^{ABC}(q_{CB})$ and τ_2^{AB} are defined in analogy with $\Sigma_1^{ABC}(q_{CB})$ and Σ_2^{AB}. Particularly, since q_{CB} satisfies inequality (3.16), $\tau_1^{ABC}(q_{CB}) = p_B^{ABC}\, F\left(\frac{p_{B2}^{ABC}+p_{C1}^{ABC}}{q_{CB}}\right)$ and is increasing in q_{CB}. In this case, $\tau_2^{ABC}(q_{CB})$ varies with q_{CB}, since p_{A1}^{ABC} is decreasing in q_{CB}, whereas p_{C1}^{ABC} is increasing. Given condition (3.16), $q_{CB} \in \left[\frac{q_{AB}\,(p_{B2}^{ABC}+p_{C1}^{ABC})}{p_{A1}^{ABC}+p_{B2}^{ABC}}, q_{CB}^{'}\right]$, where $q_{CB}^{'} < q_{AB}$ is the maximum value of q_{CB} compatible with $D_{AB} \geq 0$, i.e., such that $\theta_{CB}^{AB} = 1$.

If $q_{CB} = \frac{q_{AB}\,(p_{B2}^{ABC}+p_{C1}^{ABC})}{p_{A1}^{ABC}+p_{B2}^{ABC}}$, then Π_B^{ABC} is optimized at the kink. From B1 above, we know that, when $\frac{\partial\,\Pi_B^{ABC}}{\partial\,p_{B2}} = 0$ at the kink point, $\frac{\partial\,\Pi_B^{AB}}{\partial\,p_{B2}} > 0$. Thus, given p_{A1}^{ABC}, firm B's optimal response is $R_B^{AB}(p_{A1}^{ABC}, 0)$, i.e., the best response to A's price when AB only is sold. Therefore, $\tau(q_{CB}) < 0$ and B is willing to deviate at the kink. We now need to check the sign of $\tau(q_{CB})$ at $q_{CB} = q_{CB}^{'}$. In order to do that, we need to find the global optimal price for B, given p_{A1}^{ABC} and p_{C1}^{ABC}. Again, we need to check the sign of $\frac{\partial\,\Pi_B^{AB}}{\partial\,p_{B2}}$ at the kink. From (3.10), $\frac{\partial\Pi_B^{AB}}{\partial p_{B2}} = F\left(\frac{p_{A1}^{ABC}-p_{C1}^{ABC}}{q_{AB}-q_{CB}}\right) + f\left(\frac{p_{A1}^{ABC}-p_{C1}^{ABC}}{q_{AB}-q_{CB}}\right)\frac{q_{CB}\,p_{A1}^{ABC}-q_{AB}\,p_{C1}^{ABC}}{q_{AB}(q_{AB}-q_{CB})}$. Using (3.11), the first order condition for the maximization of Π_A^{ABC} (which is of course satisfied at p_{A1}^{ABC}), we obtain $F\left(\frac{p_{A1}^{ABC}-p_{C1}^{ABC}}{q_{AB}-q_{CB}}\right) = -\frac{p_{A1}^{ABC}}{q_{AB}-q_{CB}} f\left(\frac{p_{A1}^{ABC}-p_{C1}^{ABC}}{q_{AB}-q_{CB}}\right)$. Substituting into $\frac{\partial\Pi_B^{AB}}{\partial p_{B2}}$ evaluated at the kink, we obtain $\frac{\partial\Pi_B^{AB}}{\partial p_{B2}} = -f\left(\frac{p_{A1}^{ABC}-p_{C1}^{ABC}}{q_{AB}-q_{CB}}\right)\frac{p_{A1}^{ABC}(q_{AB}-q_{CB})+q_{AB}\,p_{C1}}{q_{AB}(q_{AB}-q_{CB})}$, which is positive for all $q_{CB} < q_{AB}$ (hence for $q_{CB} = q_{CB}^{'}$) since $f(\theta) < 0$. From point B1 above, when $\frac{\partial\Pi_B^{AB}}{\partial p_{B2}} > 0$ at the kink, B's global optimum price is given by $R_B^{AB}(p_{A1}^{ABC}, p_{C1}^{ABC})$ and B wants to deviate.

Hence, we have proven that B is always willing to deviate from p_{B2}^{ABC} when $p_{A1} = p_{A1}^{ABC}$ and $p_C = p_{C1}^{ABC}$. Thus, $\{p_{A1}^{ABC}, p_{B2}^{ABC}, p_{C1}^{ABC}\}$ is never a Nash equilibrium in pure strategies and firm C is never in the market.[34]. □

Proof of Proposition 16

Define the best-response prices as $R_A(p_{B2}, p_{C1})$, $R_B(p_A, p_{C1})$ and $R_C(p_A, p_{B2})$.

Both systems AA and CB are sold when $\theta_{CB}^{AA} \geq \theta_{CB}$. Thus, given p_A and p_{C1}, firm B can obtain positive demand and profits by setting $0 \leq p_{B2} \leq p_A q_{CB} - p_{C1}$ and this is feasible if and only if $p_A q_{CB} \geq p_{C1}$. If $p_{C1} > p_A q_{CB}$, even $p_{B2} = 0$ (marginal cost) would not serve such purpose, but in this case we assume that this would be indeed B's best response: $R_B(p_A, p_{C1}) = 0$.[35]

As just stated, when feasible, firm B maximizes Π_B^* subject to $\theta_{CB}^{AA} \geq \theta_{CB}$, or equivalently to $p_{B2} \leq p_A q_{CB} - p_{C1}$. The first-order condition is

$$\frac{\partial \Pi_B^*}{\partial p_{B2}} = \frac{1}{2}(q_{CB} p_A - p_{C1}) - p_{B2} = 0 \tag{3.17}$$

This condition evaluated at the kink $p_{B2} = p_A q_{CB} - p_{C1}$ becomes

$$\frac{\partial \Pi_B^*}{\partial p_{B2}} = \frac{p_{C1} - p_A q_{CB}}{2}$$

Notice however that Π_B^* is defined only for $\theta_{AA} \geq \theta_{CB}$, i.e., $p_A \geq \frac{p_{B2}+p_{C1}}{q_{CB}}$, so that, at the kink $\frac{\partial \Pi_B^*}{\partial p_{B2}} < 0$. Thus $0 \leq R_B(p_A, p_{C1}) \leq p_A q_{CB} - p_{C1}$, and, from (3.17),

$$p_{B2} = R_B(p_A, p_{C1}) = \frac{1}{2}(q_{CB} p_A - p_{C1}) \tag{3.18}$$

[34] The fact that the best-response function of B is discontinuous, suggests the possible existence of mixed-strategies Nash equilibria for values of $q_{CB} > q_{\hat{C}B}$. For a characterization of mixed-strategy equilibria in vertically differentiated, complementary markets see Cheng and Nahm (2007) or Casadesus-Masanell et al. (2007) for an application to the uniform case.

[35] Any $R_B(p_A, p_{C1}) > 0$ when $p_{C1} > p_A q_{CB}$ would be counterintuitive because it would imply that B is not trying to do its best to gain a positive market share.

As an implication, firm's B profits function is continuous in p_{B2} (in fact, from (3.18), when $p_{C1} = p_A q_{CB}$, $R_B(p_A, p_{C1}) = 0$) and has one single peak at the price p^*_{B2} that solves (3.17).

The behavior of firm C is totally symmetrical to the behavior of B. When $p_{B2} > p_A q_{CB}$, C's best response is $R_C(p_A, p_{B2}) = 0$. When instead $p_{B2} \leq p_A q_{CB}$, C maximizes Π^*_C subject to $p_{C1} \leq p_A q_{CB} - p_{B2}$. Then, at the kink, $\frac{\partial \Pi^*_C}{\partial p_{C1}} < 0$ and $R_C(p_A, p_{B2}) \leq p_A q_{CB} - p_{B2}$, so that $p_{C1} = R_C(p_A, p_{B2}) = \frac{1}{2}(q_{CB} p_A - p_B)$. Also firm $C's$ profit function is continuous in p_{C1} and has one single peak at p^*_{C1}.

Now consider A, which maximizes

$$\Pi^M_A(p_A) = p_A(1 - p_A) \quad if \quad p_A < \frac{p_{CB}}{q_{CB}}$$
$$\Pi^*_A(p_A; p_{CB}) = p_A(1 - \frac{p_{A1} - p_{CB}}{1 - q_{CB}}) \quad if \quad p_A \geq \frac{p_{CB}}{q_{CB}}$$

The two first-order conditions are

$$\begin{aligned} \frac{\partial \Pi^M_A}{\partial p_A} &= 1 - 2p_A = 0 \\ \frac{\partial \Pi^*_A}{\partial p_A} &= 1 - q_{CB} - 2p_A + p_{CB} = 0 \end{aligned} \tag{3.19}$$

which, evaluated at the kink $p_A = \frac{p_{CB}}{q_{CB}}$, become, respectively:

$$\frac{\partial \Pi^M_A}{\partial p_A} = 1 - 2\frac{p_{CB}}{q_{CB}}$$
$$\frac{\partial \Pi^*_A}{\partial p_A} = 1 - q_{CB} - 2\frac{p_{CB}}{q_{CB}} + p_{CB}$$

so that

$$\frac{\partial \Pi^M_A}{\partial p_A} \geq 0 \quad iff \quad p_{CB} \leq \bar{p}_{CB} = \frac{q_{CB}}{2}$$
$$\frac{\partial \Pi^*_A}{\partial p_A} \geq 0 \quad iff \quad p_{CB} \leq \underline{p}_{CB} = \frac{q_{CB}(1 - q_{CB})}{2 - q_{CB}}$$

Notice that $0 < \underline{p}_{CB} < \bar{p}_{CB}$. We thus have the following 3 cases:

1. $p_B + p_C \leq \underline{p}_{CB}$. In this case $\frac{\partial \Pi^M_A}{\partial p_A} > 0$; $\frac{\partial \Pi^*_A}{\partial p_A} \geq 0$. Hence p^*_A does not violate the constraint, whereas p^M_A violates the constraint. Thus firm $A's$ global optimum is, from (3.19), $p_A = R_A(p_{B2}, p_{C1}) = \frac{1 + p_{CB} - q_{CB}}{2}$.

2. $p_B + p_C \geq \bar{p}_{CB}$. Here $\frac{\partial \Pi_A^M}{\partial p_A} \leq 0$; $\frac{\partial \Pi_A^*}{\partial p_A} < 0$ and p_A^M does not violate the constraint, whereas p_A^* violates the constraint. Thus firm $A's$ global optimum is $p_A = p_A^M = \frac{1}{2}$;

3. $\underline{p}_{CB} < p_B + p_{C1} < \bar{p}_{CB}$. In this case $\frac{\partial \Pi_A^M}{\partial p_A} > 0$; $\frac{\partial \Pi_A^*}{\partial p_A} < 0$. Both first order conditions, evaluated at the kink, violate the respective constraint, so that firm $A's$ optimum is the corner solution $p_A = \frac{p_{CB}}{q_{CB}}$.

Notice, from the analysis above, that firm A's profit function is always single peaked for any $p_A > 0$.

Combining the above cases, we can see that firm A's optimal price is continuous in $p_{CB} = p_{C1} + p_{B2}$. Then, if a pure-strategy Nash equilibrium exists, in equilibrium either only system AA is sold and prices are (p_A^M, $p_{B2} = p_{C1} = 0$), or the two systems AA CB are sold at prices $(p_A^* = R_A(p_{B2}^*, p_{C1}^*), p_{B2}^* = R_B(p_A^*, p_{C1}^*), p_{C1}^* = R_C(p_A^*, p_{B2}^*))$. Let's now verify the conditions under which each triplet of prices is a Nash equilibrium.

a. (p_A^M, $p_{B2} = p_{C1} = 0$). A necessary condition for this equilibrium to hold is $\theta_{CB} > \theta_{CB}^{AA}$, which in this case is equivalent to $\frac{p_A^M}{1-q_{CB}} < 0$, which is never possible for any $p_A^M \geq 0$. Then, an equilibrium in which only firm A operates never exists.

b. $(p_A^* = R_A(p_{B2}^*, p_{C1}^*), p_{B2}^* = R_B(p_A^*, p_{C1}^*), p_{C1}^* = R_C(p_A^*, p_{B2}^*))$. The necessary condition for these prices to be a Nash equilibrium is $\theta_{CB} < \theta_{CB}^{AA}$, or, equivalently, $q_{CB} > \frac{p_{B2}^* + p_{C1}^*}{p_A^*}$. We have shown above that B's profit function is continuous and has only one peak. Moreover, p_{B2}^* always grants positive profits Π_B^* without violating the constraint. Then firm B will never deviate since any price $p_{B2} \neq p_{B2}^*$ would yield a lower profit. Then p_{B2}^* is firm B's globally optimal best response to p_A^* and p_{C1}^*. The same argument applies to firm C. It is then only firm A that could potentially deviate from p_A^* to a market configuration in which only AA is sold when $q_{CB} > \frac{p_{B2}^*+p_{C1}^*}{p_A^*}$. Note however that firm A's has a single peak for every p_A, as shown above. In fact, when firms B and C set their prices at p_{B2}^* and p_{C1}^*, firm A maximizes $\Pi_A^*(p_A; p_{B2}^*, p_{C1}^*, q_{CB})$ if $p_A \geq \frac{p_{B2}^*+p_{C1}^*}{q_{CB}}$ and $\Pi_A^M(p_A)$ if $p_A \leq \frac{p_{B2}^*+p_{C1}^*}{q_{CB}}$. Thus, if p_A^* maximizes Π_A^* without violating the constraint $p_A \geq \frac{p_{B2}^*+p_{C1}^*}{q_{CB}}$, then it is firm A's globally optimal price. When $q_{CB} > \frac{p_{B2}^*+p_{C1}^*}{p_A^*}$ we have $\frac{p_A^*-(p_{B2}^*+p_{C1}^*)}{1-q_{CB}} > \frac{p_{B2}^*+p_{C1}^*}{q_{CB}}$. Then, as established in case 1, p_A^* maximizes Π_A^* without violating the constraint and is therefore firm A's globally optimal price. $\square$

Proof of Proposition 18

Part 1.

In market configuration $H1$, consumer types with $\theta \geq \theta_{AB}^{AA} = \frac{p_{A2}-p_B}{1-q_{AB}}$ and $\theta > \theta_{CB}^{AA}$ buy system AA, provided that $V_{AB}(\theta) \geq 0$ (i.e., $\theta > \theta_{AA}$). Those with $\theta_{CB}^{AB} \leq \theta < \theta_{AB}^{AA}$ buy AB, provided that and $V_{AB}(\theta) \geq 0$ (i.e., $\theta > \theta_{AB}$). Finally, consumers with $\theta < \theta_{CB}^{AA}$ and $\theta < \theta_{CB}^{AB}$ buy CB, provided that $V_{CB}(\theta) \geq 0$ (i.e. $\theta > \theta_{CB}$).

Therefore, this case is defined when prices are such that

$$\theta_{CB} < \theta_{AB} < \theta_{AA} < 1 \tag{3.20}$$

$$D_{AB} = \theta_{AB}^{AA} - \theta_{CB}^{AB} > 0 \tag{3.21}$$

as it can be verified also in Figure 3.1. Substituting the equilibrium prices in Table 3.1 into the consistency conditions (3.20) and (3.21), we verify that, while the first is always satisfied, the second holds if and only if

$$q_{AB} < \frac{1 + q_{CB}}{2} \tag{3.22}$$

Market configuration $H1$ forms an equilibrium with interior optima if neither firm is willing to deviate from the prices set in Table 3.1.

a)**Deviations to market configuration H2.** Market configuration $H2$ yields the same prices and profits as when firm A engages in pure bundling. From equation (3.1) in Section 3.1, then $p_A^{H2} = p_A^*$, $p_{B2}^{H2} = p_{B2}^*$, $p_{C1}^{H2} = p_{C1}^*$ and $\Pi_A^{H2} = \Pi_A^*$, $\Pi_B^{H2} = \Pi_B^*$, $\Pi_C^{H2} = \Pi_C^*$. In order for these equilibrium prices to be feasible, we need to check whether at these prices

$$D_{AB} = \theta_{AB}^{AA} - \theta_{CB}^{AB} \leq 0. \tag{3.23}$$

In this process, unlike Section 3.1, we need to specify p_{A1} and p_{A2} separately, since the definition of θ_{CB}^{AB} requires the use of p_{A1}. From the maximization problem, however, we are only able to define the price of the system, p_A. In order to resolve this issue, we find the lower bound for p_{A1} that satisfies condition (3.23). In particular, condition (3.23) is satisfied if and only if

$$p_{A1} \geq p_{A1}^2 = \frac{q_{AB}(3 - 2q_{CB}) - (2 - q_{CB})q_{CB}}{2(3 - q_{CB})} \tag{3.24}$$

If p_{A1} does not satisfy this inequality in equilibrium, $H2$ is not a candidate for the equilibrium market configuration.

Firm A

Given p_{B2}^{H1} and p_{C1}^{H1}, A could set p_{A1} and p_{A2} such that market configuration $H2$ arises. We prove that such deviation is never profitable for firm A. Firm A could set price $p_A = (p_{A1} + p_{A2})$ for its system such that (3.23) is satisfied. In this case only systems AA and CB would be sold in the market with demands equal to those in Section 3.1.1. Based on this market configuration, firm A would then set the price p_A in order to maximize $\Pi_A^{DEV} = p_A \left[1 - \frac{p_A - p_{C1}^{H1} - p_{B2}^{H1}}{1-q_{CB}}\right]$, obtaining $p_A^{DEV} = \frac{(q_{AB}^2(6-4q_{CB})+2q_{CB}^2-2q_{AB}(3+q_{CB}(1-2q_{CB}))}{3(4q_{AB}^2+q_{CB}^2-4q_{AB}(1+q_{CB}))}$. Substituting p_A^{DEV} into Π_A^{DEV}, it can be verified that $\Pi_A^{DEV} < \Pi_A^{H1}$ always, hence A never deviates to an interior solution.[36]

Firm C

Provided that system AA is always sold, given p_{A1}^{H1}, p_{A2}^{H1} and p_{B2}^{H1}, C could set p_{C1} to deviate to $H2$. However, we now prove that a profitable deviation for C is never feasible.

Conditional to market configuration $H2$, firm C would set its price p_{C1} in order to maximize $\Pi_C^{DEV} = p_{C1}\left[\frac{p_{A1}^{H1}+p_{A2}^{H1}-p_B^{H1}-p_{C1}}{1-q_{CB}} - \frac{p_{C1}+p_B^{H1}}{q_{CB}}\right]$, obtaining an interior optimum equal to $p_{C1}^{DEV} = \frac{q_{CB}\left[4q_{AB}^2(1-q_{CB})+q_{CB}(1+2q_{CB})-q_{AB}(4+q_{CB}(3-4q_{CB}))\right]}{6(4q_{AB}^2+q_{CB}^2-4q_{AB}(1+q_{CB}))}$. At these prices, condition (3.23) is always violated. Hence C is never able to deviate to $H2$ through an interior optimum. However, C may still set p_C such that (3.23) holds as a strict equality. In such case $p_{C1}^{CORN} = \frac{(q_{AB}-q_{CB})q_{CB}(1+2q_{CB})}{3(4q_{AB}(1+q_{CB})-4q_{AB}^2-q_{CB}^2)}$. Substituting p_{C1}^{CORN} into Π_C^{DEV}, we have $\Pi_C^{CORN} = p_{C1}^{CORN}\left[\frac{p_{A1}^{H1}+p_{A2}^{H1}-p_{B2}^{H1}-p_{C1}^{CORN}}{1-q_{CB}} - \frac{p_{C1}^{CORN}+p_{B2}^{H1}}{q_{CB}}\right]$. It is possible to show that $\Pi_{C1}^{CORN} < \Pi_{C1}^{H1}$, hence C never wants to deviate to such corner solution.[37]

Firm B

Provided that system AA is always sold, given p_{A1}^{H1}, p_{A2}^{H1} and p_{C1}^{H1}, B could set p_{B2} to deviate to $H2$. Conditional to market configuration $H2$, firm B would set its price p_B to maximize $\Pi_B^{DEV} = p_{B2}\left[\frac{p_{A1}^{H1}+p_{A2}^{H1}-p_{B2}-p_{C1}^{H1}}{1-q_{CB}} - \frac{p_{C1}^{H1}+p_{B2}}{q_{CB}}\right]$, obtaining an

[36]Firm A might still deviate with a price that generates configuration $H2$ with a corner solution. However, if profits under deviation are lower when A can optimize without constraints, *a fortiori* profits will be lower in a corner solution.

[37]Deviations to corner solutions can be proved to be never feasible. For the sake of brevity, we will omit proofs from now on.

interior optimum equal to $p_{B2}^{DEV} = \frac{q_{CB}(4q_{AB}^2(1-q_{CB})-q_{CB}(2+q_{CB})+q_{AB}(q_{CB}(4q_{CB}+3)-4))}{6(4q_{AB}^2+q_{CB}^2-4q_{AB}(1+q_{CB}))}$. At these prices, condition (3.23) is satisfied only for $q_{AB} \geq \tilde{q}(q_{CB})$, where

$$\tilde{q}(q_{CB}) = \frac{6 + q_{CB}(1 - 4q_{CB}) - \sqrt{4 + 12q_{CB} + q_{CB}^2 - 24q_{CB}^3 + 16q_{CB}^4}}{8 - 8qcb} \quad (3.25)$$

Hence, deviation is feasible only for $\tilde{q}(q_{CB}) < q_{AB} \leq 1$. Profits under deviation are obtained substituting p_{B2}^{DEV} into Π_B^{DEV}. Define $S(q_{AB}, q_{CB}) = \Pi_B^{DEV} - \Pi_B^{H1}$, which is a polynomial of degree 4 in q_{AB}. Solving $S(q_{AB}, q_{CB}) = 0$ with respect to q_{AB}, we find four real solutions but only one is in the admissible range for q_{AB}, that is $q^*(q_{CB}) = \frac{(3-q_{CB}-2q_{CB}^2)}{6(1-q_{CB})}$. We can see that $S(q_{AB}, q_{CB}) > 0$ at $q_{AB} > q^*(q_{CB})$. We find that $q^*(q_{CB}) \geq \tilde{q}(q_{CB})$ for all $q_{AB} \geq \frac{9}{4}q_{CB}$. Then, if $\frac{1+q_{CB}}{2} > q_{AB} > q^*(q_{CB})$, deviation to $H2$ is feasible and profitable for firm B.

b) **Deviations to market configuration H3.** This case requires prices such that

$$\theta_{AB} \leq \min\{\theta_{AA}, \theta_{CB}\} \quad (3.26)$$

Consumers with $\theta \geq \theta_{AB}^{AA}$ buy system AA ($D_{AA} = 1 - \theta_{AB}^{AA}$), those with $\theta_{AB} \leq \theta < \theta_{AB}^{AA}$ buy AB ($D_{AB} = \theta_{AB}^{AA} - \theta_{AB}$) whereas consumers with $\theta < \theta_{AB}$ buy nothing. Firm C would never set its price as to generate such configuration so its behaviour will be ignored.

Firm A

In order to deviate to this market configuration, firm A would have to set prices in order to maximize $\Pi_A^{DEV} = p_{A1}\left[1 - \frac{p_{A1}+p_{B2}^{H1}}{q_{AB}}\right] + p_{A2}\left[1 - \frac{p_{A2}-p_{B2}^{H1}}{1-q_{AB}}\right]$. In an interior optimum,

$$p_{A1}^{DEV} = \frac{12q_{AB}^3 + q_{CB}^2 + 2q_{AB}q_{CB}(1 + q_{CB}) - 2q_{AB}^2(6 + 7q_{CB})}{6(4q_{AB}^2 + q_{CB}^2 - 4q_{AB}(1 + q_{CB}))}$$

and

$$p_{A2}^{DEV} = \frac{(1 - q_{AB})(6q_{AB}^2 + q_{CB}^2 - q_{AB}(6 + 7q_{CB}))}{3(4q_{AB}^2 + q_{CB}^2 - 4q_{AB}(1 + q_{CB}))}$$

Note however that, at these prices, $\theta_{CB} < \theta_{AB}$, so that condition (3.26) is violated.

Firm B

In $H3$, firm B would set its price to maximize $\Pi_B^{DEV} = p_{B2}\left[\frac{p_{A2}^{H1}-p_{B2}}{1-q_{AB}} - \frac{p_{A1}^{H1}+p_{B2}}{q_{AB}}\right]$, obtaining an interior optimum equal to $p_{B2}^{DEV} = \frac{(1-q_{AB})q_{CB}(4q_{AB}^2-q_{CB}-q_{AB}(5+4q_{CB}))}{6(4q_{AB}^2+q_{CB}^2-4q_{AB}(1+q_{CB}))}$. However, at these price (3.26) is violated and, in particular, $\theta_{CB} < \theta_{AB}$. Hence demand for CB remains positive.

Intuitively, deviation to $H3$ does not work for B because eliminating C from the market involves increasing p_{B2} until demand for B decreases too much and outweighs the increase in profits due to an higher p_{B2}.

c) **Deviations to market configuration H4.** This case occurs when p_A satisfies

$$\theta_{AA} \leq \min\{\theta_{AB}, \theta_{CB}\} \tag{3.27}$$

Consumers with $\theta \geq \theta_{AA}$ buy bundle AA, whereas all those with $\theta < \theta_{AA}$ buy nothing. The only firm that might find convenient to deviate to $H4$ is A, so that possible deviations by B and C will be ignored here.

Firm A

In an interior optimum in $H4$, $p_A^{DEV} = p_A^M = \frac{1}{2}$ and $\Pi_A^{DEV} = \Pi_A^M = \frac{1}{4}$. It can be checked that $\Pi_A^M > \Pi_A^{H1}$, but, at this price, equation (3.27) is always violated: while $\theta_{AA} < \theta_{AB}$ always holds through a proper choice of p_{A1} and p_{A2} (maintaining $p_{A1} + p_{A2} = \frac{1}{2}$), we have infact that $\theta_{CB} < \theta_{AA}$.

Part 2)

From the application of Proposition 16 to the uniform case, we know equilibrium prices and profits conditional to market configuration $H2$. Market configuration

$H2$ is an equilibrium if neither firm is willing to deviate from the prices set in (3.1).

$a)$ **Deviations to market configuration H1.**

Firm A

Conditional to $H1$, firm A would set its prices p_{A1} and p_{A2} to maximize $\Pi_A^{DEV} = p_{A1}\left(1 - \frac{p_{A1}-p_{C1}^{H2}}{q_{AB}-q_{CB}}\right) + p_{A2}\left(1 - \frac{p_{A2}-p_{B2}^{H2}}{1-q_{AB}}\right)$, obtaining an interior optimum equal to $p_{A1}^{DEV} = \frac{(6q_{AB}-5q_{CB}-2q_{AB}q_{CB}+q_{CB}^2)}{4(3-q_{CB})}$ and $p_{A2}^{DEV} = \frac{6-2q_{AB}(3-q_{CB})-q_{CB}(1-q_{CB})}{4(3-q_{CB})}$. At these prices profits are indeed higher, that is $\Pi_A^{DEV} > \Pi_A^{H2}$ and the conditions in (3.20) are always satisfied. Conversely, the condition in (3.21) is satisfied if and only if $q_{AB} < \frac{1+q_{CB}}{2}$, which is the necessary and sufficient condition for the feasibility of an equilibrium in market configuration $H1$ stated in (3.22). Hence, whenever $q_{AB} < \frac{1+q_{CB}}{2}$, $H2$ is not an equilibrium market configuration. In other terms, whenever market configuration $H1$ is feasible, firm A is able to deviate profitably to it, no matter what the other firms do.

For this range of parameters, the analysis of the incentives for firms B and C to deviate to $H1$ can then be ignored for the proof. Firm C's behaviour could actually be ignored totally, since market configuration $H1$ represents the only potentially profitable deviation C can do. As far as firm B is concerned, we simply need to check whether it wants to deviate to market configuration $H3$.

$b)$ **Deviations to market configuration H3**

Firm A

Firm A would set prices p_{A1} and p_{A2} to maximize

$\Pi_A^{DEV} = p_{A1}\left[1 - \frac{p_{A1}+p_{B2}^{H2}}{q_{AB}}\right] + p_{A2}\left[1 - \frac{p_{A2}-p_{B2}^{H2}}{1-q_{AB}}\right]$,

obtaining $p_{A1}^{DEV} = \frac{6q_{AB}-q_{CB}-2q_{AB}q_{CB}+q_{CB}^2}{4(3-q_{CB})}$ and $p_{A2}^{DEV} = \frac{6-2q_{AB}(3-q_{CB})-q_{CB}(1+q_{CB})}{4(3-q_{CB})}$.

At these prices profits are indeed higher, that is $\Pi_A^{DEV} > \Pi_A^{H2}$. However, the condition $\theta_{CB} > \theta_{AB}$ in (3.26) is always violated and, consequently, the demand for CB is positive.

Firm B

Firm B would set its price to maximize $\Pi_B^{DEV} = p_{B2}\left[\frac{p_{A2}^{H2}-p_{B2}}{1-q_{AB}} - \frac{p_{A1}^{H2}+p_{B2}}{q_{AB}}\right]$, that is $p_{B2}^{DEV} = \frac{3q_{AB}(1-q_{CB})-2p_{A1}(3-q_{CB})}{4(3-q_{CB})}$. At this value, $\Pi_B^{DEV} > \Pi_B^{H2}$ if and only if

$$p_{A1} < p_{A1}^3 = \frac{3q_{AB}(1-q_{CB})\sqrt{(1-q_{AB})q_{AB}(1-q_{CB})q_{CB})}}{3-q_{CB}} \tag{3.28}$$

However, $p_{A1}^3 > p_{A1}^2$, defined in equation (3.24). Therefore, every time an equilibrium in $H2$ is feasible, it is never profitable for B to deviate.

c) **Deviation to market configuration H4.**

Firm A

Conditional to $H4$, firm A's interior optimum is $p_A^{H4} = \frac{1}{2}$ and $\Pi_A^{H4} = \frac{1}{4}$.As before, it can be checked that $\Pi_A^{H4} > \Pi_A^{H2}$, but, at this price, condition (3.27) is always violated: while $\theta_{AA} < \theta_{AB}$ always holds through a proper choice of p_{A1} and p_{A2} (maintaining $p_A = p_{A1} + p_{A2} = \frac{1}{2}$), we have that $\theta_{CB} < \theta_{AA}$.

Part 3)

Part 3) follows from the results above. □

Proof of Proposition 19

1. The revealed-preference argument in the text works as a proof of this part.

2. As demonstrated in Proposition 18, if a pure-strategy equilibrium with partial compatibility exists, it is characterized by either market configuration $H1$ or $H2$, depending on the relationship between q_{AB} and q_{CB}. In such two cases, equilibrium profits for firm A are Π_A^{H1} and Π_A^* respectively and both need to be compared with Π_A^C. The same procedure applies to firm B.

As for firm A, as already shown in footnote 15, $\Pi_A^* - \Pi_A^C \geq 0$ in the relevant parameters' range, that is $q_{CB} \in \left[0, \frac{4}{9}q_{AB}\right]$. Moreover, it can be checked that $\Delta\Pi = \Pi_A^{H1} - \Pi_A^C = g_A(q_{AB}, q_{CB}) > 0$ in the relevant parameters' range, where $g(\cdot)$ is a polynomial of fourth degree in both q_{AB} and q_{CB}.

As for firm B, we find that $\Pi_B^C - \Pi_B^* \geq 0$ iff $q_{AB} \geq \frac{9q_{CB}(1-q_{CB})}{4(3-q_{CB})^2}$, which is always true, since $\frac{9q_{CB}(1-q_{CB})}{4(3-q_{CB})^2}$ is lower than the minimum value for q_{AB} in the relevant parameters' range, that is $\frac{9}{4}q_{CB}$. Moreover, $\Pi_B^C - \Pi_B^{H1} = g_B(q_{AB}, q_{CB})$, which is again a polynomial of fourth degree in both q_{AB} and q_{CB} and is positive in the relevant parameters' range.

Finally, firm C goes from a situation in which it is not able to sell (the standard double-marginalization case) to a situation in which it has positive demand and price higher than marginal cost, hence positive profits.

3. Defining ΔW_2 the difference between total welfare with partial compatibility in market configuration $H2$ and total welfare in the Cournot monopoly, $\Delta W_2 = (CS^* + \Pi_A^* + \Pi_B^*) - (CS^C + \Pi_A^C + \Pi_B^C)$, such difference is always positive given our results in part 1 of this Proposition and in part 2 of Proposition 17. A similar conclusion can be reached for market configuration H_1. In fact, in this case $\Delta W_1 = (CS^{H1} + \Pi_A^{H1} + \Pi_B^{H1} + \Pi_C^{H1}) - (CS^C + \Pi_A^C + \Pi_B^C) = w(q_{AB}, q_{CB}) > 0$ in the relevant parameters' range, where $w(\cdot)$ is a polynomial of fifth degree in q_{AB} and of fourth degree in q_{CB}. $\square$

Proof of Proposition 20

Notice first that under pure bundling this result would be trivial, since B is the only producer of component 2. Under partial compatibily, there are three possible market configurations in which firm C has positive demand: $BB1$) prices are such that all available systems (AB, BB and CB) are sold in positive amounts; $BB2$)

prices are such that A has zero demand (and only systems BB and CB are sold); $BB3$) Component $B1$ has zero demand (and only systems AB and CB are sold).

Under $BB1$), profits for the three firms would be $\Pi_A = p_{A1}D_{AB}$, $\Pi_B = p_{B1}D_{BB} + p_{B2}(D_{AB} + D_{BB} + D_{CB})$, $\Pi_C = p_C D_{CB}$, where $D_{AB} = (1 - \theta_{BB}^{AB})$, $D_{BB} = (\theta_{BB}^{AB} - \theta_{CB}^{BB})$ and $D_{CB} = (\theta_{CB}^{BB} - \theta_{CB})$, $D_{iB} > 0$, $i = A, B, C$, and where, consistently with our previous notation, $\theta_{BB}^{AB} = \frac{p_{A1}-p_{B1}}{q_{AB}-q_{BB}}$, $\theta_{CB}^{BB} = \frac{p_{B1}-p_{C1}}{q_{BB}-q_{CB}}$, $\theta_{CB} = \frac{p_{C1}+p_{B2}}{q_{CB}}$. Solving for the three firms' first order conditions conditional to this market configuration, their reaction functions result $p_{A1} = \frac{1+p_{B1}-q_{BB}}{2}$, ($p_{B1} = \frac{[p_{C1}(1-q_{BB})+p_{A1}(q_{BB}-q_{CB})]}{2(1-q_{CB})}$, $p_{B2} = \frac{q_{CB}-p_{C1}}{2}$) and $p_{C1} = \frac{p_{B1}q_{CB}-p_{B2}(q_{BB}-q_{CB})}{2q_{BB}}$ respectively. Solving the system of such four equations yields
$p_{A1}^{\circ} = \frac{q_{BB}(6-5q_{CB}+q_{CB}^2)-q_{BB}^2(6-5q_{CB})-q_{CB}^2}{3[2q_{BB}(2-q_{CB})-q_{BB}^2-q_{CB}^2]}$, $p_{B1}^{\circ} = \frac{(1-q_{BB})(3q_{BB}^2-4q_{BB}q_{CB}+q_{CB}^2)}{3[2q_{BB}(2-q_{CB})-q_{BB}^2-q_{CB}^2]}$,
$p_{B2}^{\circ} = \frac{q_{CB}[q_{BB}(7-5q_{CB})-q_{BB}^2-q_{CB}]}{3[2q_{BB}(2-q_{CB})-q_{BB}^2-q_{CB}^2]}$, $p_{C1}^{\circ} = \frac{q_{CB}[q_{CB}(2-3q_{CB})+q_{BB}(4q_{CB}-2)-q_{BB}^2]}{3[2q_{BB}(2-q_{CB})-q_{BB}^2-q_{CB}^2]}$.
Notice however that, at these prices, $D_{CB} < 0$ for any couple of q_{BB} and q_{CB} such that $q_{BB} > q_{CB} > 0$. Thus, whenever all firms are playing their best response, no consumer would purchase system CB. Even setting p_{C1} at marginal cost (here $p_{C1} = 0$) would not be enough to get positive demand for component $C1$. In such case, when both firm A and B set their prices according to their reaction functions, $D_{CB} = \frac{2+q_{BB}-3q_{CB}}{2(-4+q_{BB}+3q_{CB})} < 0$, so that an equilibrium in this market configuration would never exist.

Under $BB2$), profits for firms B and C are $\Pi_B = p_{B1}D_{BB} + p_{B2}(D_{BB} + D_{CB})$ and $\Pi_C = p_{C1}D_{CB}$, where $D_{BB} = (1 - \theta_{CB}^{BB})$ and $D_{CB} = (\theta_{CB}^{BB} - \theta_{CB})$, $D_{iB} > 0$, $i = B, C$, and where $\theta_{CB}^{BB} = \frac{p_{B1}-p_{C1}}{q_{BB}-q_{CB}}$, $\theta_{CB} = \frac{p_{C1}+p_{B2}}{q_{CB}}$. Solving for the two firms' first order conditions conditional to this market configuration, their reaction functions result $p_{B1} = \frac{(p_{C1}+q_{BB}-q_{CB})}{2}$, $p_{B2} = \frac{p_{C1}-q_{CB}}{2}$) and $p_{C1} = \frac{p_{B1}q_{CB}-p_{B2}(q_{BB}-q_{CB})}{2q_{BB}}$. Solving the system of such three equations yields $p_{B1}^{\circ} = \frac{q_{BB}-q_{CB}}{2}$, $p_{B2}^{\circ} = \frac{q_{CB}}{2}$ and $p_{C1}^{\circ} = 0$. Thus, if an equilibrium exists in such market configuration, it entails firm C playing at its marginal cost. In this case, after substitution, we notice that $D_{CB} = 0$ as well, so that if such equilibrium exists, it implies that firm C is active in the market but with zero demand and profits. In other terms, producing both components would allow firm B to become the only producer in the market.

Under $BB3$), firm B would price $B1$ so high that no consumer would be willing to purchase it. The resulting market configuration is the same analyzed in Proposition 15 for the uniform case and conditional to such market configuration either it exists a pure strategy equilibrium with firm C out of the market ($q_{CB} \leq \frac{4}{9}q_{AB}$) or a pure strategy equilibrium does not exist ($q_{CB} > \frac{4}{9}q_{AB}$). $\square$

Proof of Proposition 21

The proof is in two steps. The first focuses on existence and the second on uniqueness.

Step 1) Market configuration $L1$ is defined when prices are such that $\theta_{AB} > \theta_{CC}$.[38] $L1$ is an equilibrium market configuration if no firm is willing and able to deviate to $L2$ with an alternative pricing strategy. Clearly, firm C would never want to deviate to $L2$ since it would make zero profits. We then need to check whether deviation for either A or B is profitable and/or feasible.

We can focus on firm A (firm B's behavior would be symmetric since prices and profits are the same for the two firms). Given p_{B2}^{L1} and p_C^{L1}, firm A would deviate to $L2$ by choosing p_{A1} such that $\theta_{AB} < \theta_{CC}$. In this case, all consumers with $\theta_{AB} \leq \theta < 1$ would buy AB and all those with $0 \leq \theta < \theta_{AB}$ would buy nothing. Firm A's profits would be $\Pi_A^{DEV} = p_{A1}(1 - p_{A1} - p_B^{L1}) = p_{A1}(1 - p_{A1} - \frac{1-q_{CC}}{3-q_{CC}})$ and the profit-maximizing price would be $p_{A1}^{DEV} = \frac{1}{3-q_{CC}}$. Then

[38] This inequality implies $\theta_{CC}^{AB} > \theta_{AB}$, so that both systems have indeed positive demand.

$\Pi_A^{DEV} = \frac{1}{(3-q_{CC})^2} > \Pi_A^{L1}$ and such deviation would always be desirable for A.

We need however to check whether p_{A1}^{DEV} satisfies the condition $\theta_{AB} < \theta_{CC}$. Substituting p_{A1}^{DEV}, p_{B2}^{L1} and p_C^{L1} we obtain that $\theta_{AB} - \theta_{CC} = \frac{1}{3-q_{CC}} > 0$. Hence, this deviation is not feasible. Also, it can be easily checked that a corner solution, in which A sets p_A such that $\theta_{AB} = \theta_{CC}$ always yields lower profits than the alternative market configuration $AB\ CC$.

Step 2) The proposed equilibrium is unique only if there are not other equilibria emerging in the alternative market configuration $L2$, where only system AB is sold. We now prove that when only system AB is sold, firm C would always find profitable to deviate to $L1$.

Under $L2$, which is defined only if prices are such that

$$\theta_{AB} < \min\{\theta_{CB}, \theta_{CC}\}, \tag{3.29}$$

$p_{A1}^C = p_B^C = \frac{1}{3}$ and profits are $\Pi_A^C = \Pi_B^C = \frac{1}{9}$. Given such prices, firm C's profits under $L1$ would be $\Pi_C^{DEV} = p_C(\frac{\frac{2}{3}-p_C}{1-q_{CC}} - \frac{p_C}{q_{CC}})$ and the profit-maximizing price would be $p_C^{DEV} = \frac{q_{CC}}{3}$, whereas $\Pi_C^{DEV} = \frac{q_{CC}}{9(1-q_{CC})} > 0$. Note that $L1$ is defined only when

$$\theta_{CC} < \min[\theta_{AB}, \theta_{CC}] \tag{3.30}$$

$$\theta_{CC}^{CB} \geq \theta_{CC}^{AB} \tag{3.31}$$

and these feasibility conditions are always satisfied at these prices. □

Proof of Proposition 22

The second part of the proof of Proposition 21 has already established that $L2$ can never emerge as an equilibrium market configuration. In fact, $L2$ would produce the same prices and profits obtained for $L2$ under pure bundling. The remaining part of the proof can then be divided into two steps. In the first step we show that $L4$ can never emerge as an equilibrium market configuration. In the second step we show that $L3$ can never emerge as an equilibrium market configuration. Step 1) $L4$ is defined when prices are such that

$$\theta_{CC} < \theta_{CB} < \theta_{AB} \tag{3.32}$$

and

$$\theta_{CB}^{AB} > \theta_{CC}^{CB}. \tag{3.33}$$

Consumers with $\theta_{CB}^{AB} \leq \theta < 1$ buy AB, those with $\theta_{CC}^{CB} \leq \theta < \theta_{CB}^{AB}$ buy CB. Finally, consumers with $\theta_{CC} \leq \theta < \theta_{CC}^{CB}$ buy CC. The demands for the three systems are $D_{AB} = 1 - \frac{p_{A1}-p_{C1}}{1-q_{CB}}$, $D_{CB} = \frac{p_{A1}-p_{C1}}{1-q_{CB}} - \frac{p_{B2}-p_{C2}}{q_{CB}-q_{CC}}$ and $D_{CC} = \frac{p_{A1}-p_{C1}}{1-q_{CB}} - \frac{p_{C1}+p_{C2}}{q_{CC}}$. Profits are $\Pi_A^{L4} = p_{A1}\left(1 - \frac{p_{A1}-p_{C1}}{1-q_{CB}}\right)$; $\Pi_B^{L4} = p_{B2}\left(\frac{p_{A1}-p_{C1}}{1-q_{CB}} - \frac{p_{B2}-p_{C2}}{q_{CB}-q_{CC}}\right)$ and $\Pi_C^{L4} = p_C\left(\frac{p_{A1}-p_{C1}}{1-q_{CB}} - \frac{p_{C1}+p_{C2}}{q_{CC}}\right)$. Conditional on this market configuration, we solve for Bertrand equilibrium prices in an interior optimum:[39]

$$p_{A1}^{L4} = \frac{2\left(1-q_{CB}\right)}{4-q_{CC}} \tag{3.34}$$

$$p_{B2}^{L4} = \frac{2\left(q_{CB}-q_{CC}\right)}{4-q_{CC}} \tag{3.35}$$

$$p_{C1}^{L4} = \frac{q_{CC}\left(1-q_{CB}\right)}{4-q_{CC}} \tag{3.36}$$

$$p_{C2}^{L4} = \frac{q_{CC}\left(q_{CB}-q_{CC}\right)}{4-q_{CC}} \tag{3.37}$$

Profits are $\Pi_A^{L4} = \frac{4(1-q_{CB})}{(4-q_{CC})^2}$, $\Pi_B^{L4} = \frac{4(q_{CB}-q_{CC})}{(4-q_{CC})^2}$ and $\Pi_C^{L4} = \frac{q_{CC}(1-q_{CC})}{(4-q_{CC})^2}$. In such proposed equilibrium, however, firm B would always find profitable to deviate to $L1$. In fact, under $L1$ firm B's profits would be $\Pi_B^{DEV} = p_{B2}(1 - \frac{p_{A1}^{L4}+p_{B2}-p_{C1}^{L4}-p_{C2}^{L4}}{1-q_{CC}})$ and the profit-maximizing price would be $p_{B2}^{DEV} = \frac{1+q_{CB}-2q_{CC}}{4-q_{CC}}$.Then $\Pi_B^{DEV} =$

[39] Also in the case where the low-quality firm produces both components, if a pure-strategy equilibrium exists, it always involves the three firms choosing an interior optimal price, as in the previous case (see footnote 19).

$\frac{(1+q_{CB}-2q_{CC})^2}{(1-q_{CC})(4-q_{CC})^2} > \Pi_B^{L4}$: deviation would always be desirable for B. Substituting p_{A1}^{L4}, p_{B2}^{DEV} and p_{C1}^{L4}, p_{C2}^{L4} into (3.30) and (3.31) it can also be checked that the feasibility conditions for $L1$ are satisfied.

Step 2) $L3$ requires

$$\theta_{CB} < \min\{\theta_{AB}, \theta_{CC}\}. \tag{3.38}$$

Consumers with $\theta_{CB}^{AB} < \theta < 1$ buy the system AB, those with $\theta_{CB} < \theta < \theta_{CB}^{AB}$ buy CB. The demands for the two systems are $D_{AB} = 1 - \frac{p_{A1}-p_{C1}}{1-q_{CB}}$ and $D_{CB} = \frac{p_{A1}-p_{C1}}{1-q_{CB}} - \frac{p_{B2}+p_{C1}}{q_{CB}}$. Profits are $\Pi_A^{L3} = p_{A1}\left(1 - \frac{p_{A1}-p_{C1}}{1-q_{CB}}\right)$; $\Pi_B^{L3} = p_{B2}\left(1 - \frac{p_{B2}+p_{C1}}{q_{CB}}\right)$ and $\Pi_C^{L3} = p_C\left(\frac{p_{A1}-p_{C1}}{1-q_{CB}} - \frac{p_{B2}+p_{C1}}{q_{CB}}\right)$. Interior optimum prices are $p_{A1}^{L3} = \frac{1-q_{CB}}{2}$, $p_{B2}^{L3} = \frac{q_{CB}}{2}$, and $p_{C1} = 0$, so that equilibrium profits are $\Pi_A^{L3} = \frac{1-q_{CB}}{4}$, $\Pi_B^{L3} = \frac{q_{CB}}{4}$ and $\Pi_C^{L3} = 0$.

In such case, firm C would however find profitable to deviate to $L1$. In fact, under $L1$ firm C's profits would be $\Pi_C^{DEV} = p_C(\frac{p_{A1}^{L3}+p_B^{L3}-p_C}{1-q_{CC}} - \frac{p_C}{q_{CC}})$ and the profit-maximizing price would be $p_C^{DEV} = \frac{q_{CC}}{4}$. Then $\Pi_C^{DEV} = \frac{q_{CC}}{16(1-q_{CC})} > 0$ always. It is immediate to check that the feasibility conditions (3.30) and (3.31) are satisfied.
□

Proof of Proposition 23

Given the prices p_{A1}^{H1}, p_{A2}^{H1}, p_B^{H1} and p_{C1}^{H1} obtained in Table 3.1 and p_{A1}^*, p_{A2}^*, p_{B2}^* and p_{C1}^* in (3.1), $p_{A1}^{H1} + p_{A2}^{H1} - (p_{A1}^* + p_{A2}^*) = \frac{q_{CB}^2(8q_{AB}(1+q_{CB})-3-8q_{AB}^2-5q_{CB})}{6(3-q_{CB})(4q_{AB}(1+q_{CB})-4q_{AB}^2-q_{CB}^2)}$, which is always positive in the admissible range of parameters. Similarly, $p_{C1}^{H1} + p_{B2}^{H1} - (p_{C1}^* + p_{B2}^*) = \frac{q_{CB}^2(3+8q_{AB}^2+5q_{CB}-8q_{AB}(1+q_{CB}))}{3(3-q_{CB})(4q_{AB}(1+q_{CB})-4q_{AB}^2-q_{CB}^2)}$, and this expression is again always positive in the admissible range of parameters.

As for the second part of the Proposition, we know from Proposition 18 that a pure-strategy equilibrium arises either in $H1$ or in $H2$. In the latter case, prices and profits for the three firms are the same as in the pure bundling case. Hence, when $q_{AB} \geq \frac{1+q_{CB}}{2}$ profits are the same with and without compatibility. When $\frac{9}{4}q_{CB} < q_{AB} < q^*(q_{CB})$ the equilibrium market configuration with compatibility is $H1$.In such case, define $\Delta\Pi = \Pi_A^* - \Pi_A^{H1} = q_{CB}^2 f(q_{AB}, q_{CB})$.The function $f(q_{AB}, q_{CB})$ is always positive for any value of q_{AB} in the relevant parameter's range ($q_{AB} \in [q_{CB}, 1]$), so that, *a fortiori,* $f(q_{AB}, q_{CB}) > 0$ and $\Delta\Pi > 0$ when $q_{AB} \in \left[\frac{9}{4}q_{CB}, q_{AB}^*(q_{CB})\right]$, for any $q_{CB} > 0$. Thus, pure bundling yields strictly higher profits than partial compatibility when three systems are sold in equilibrium.

In order to prove part 3, define $\Delta PS = (\Pi_A^* + \Pi_B^* + \Pi_C^*) - (\Pi_A^{H1} + \Pi_B^{H1} + \Pi_C^{H1}) = q_{CB}^2\, \pi(q_{AB}, q_{CB})$, where $\pi(\cdot)$ is a polynomial of degree four in both q_{AB} and q_{CB}, which can be checked to be always positive in the relevant parameters range.

Finally, for part 4 define $\Delta W = (CS^{H1} + \Pi_A^{H1} + \Pi_B^{H1} + \Pi_C^{H1}) - (CS^* + \Pi_A^* + \Pi_B^* + \Pi_C^*) = q_{CB}^2\, w(q_{AB}, q_{CB})$, where $w(\cdot)$ is a polynomial of degree four in q_{AB} and q_{CB}. Solving $w(q_{AB}, q_{CB})$ numerically, it can be verified that $w(q_{AB}, q_{CB}) > 0\ \forall q_{CB} \in \left[0, \frac{4}{9}q_{AB}\right]$ if $q_{AB} < 0.9$. If $q_{AB} \in [0.9, 1]$, then there exists always $\overline{\overline{q}}_{CB}(q_{AB}) \in \left[0, \frac{4}{9}q_{AB}\right]$, such that $w(q_{AB}, q_{CB}) < 0$ if $q_{CB} > \overline{\overline{q}}_{CB}(q_{AB})$. However, $0.9 > q^*(q_{CB})\ \forall q_{CB} \in \left[\overline{\overline{q}}_{CB}(q_{AB}), \frac{4}{9}q_{AB}\right]$, so that, when $w(q_{AB}, q_{CB}) < 0$, $H1$ is not an equilibrium. $\square$

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